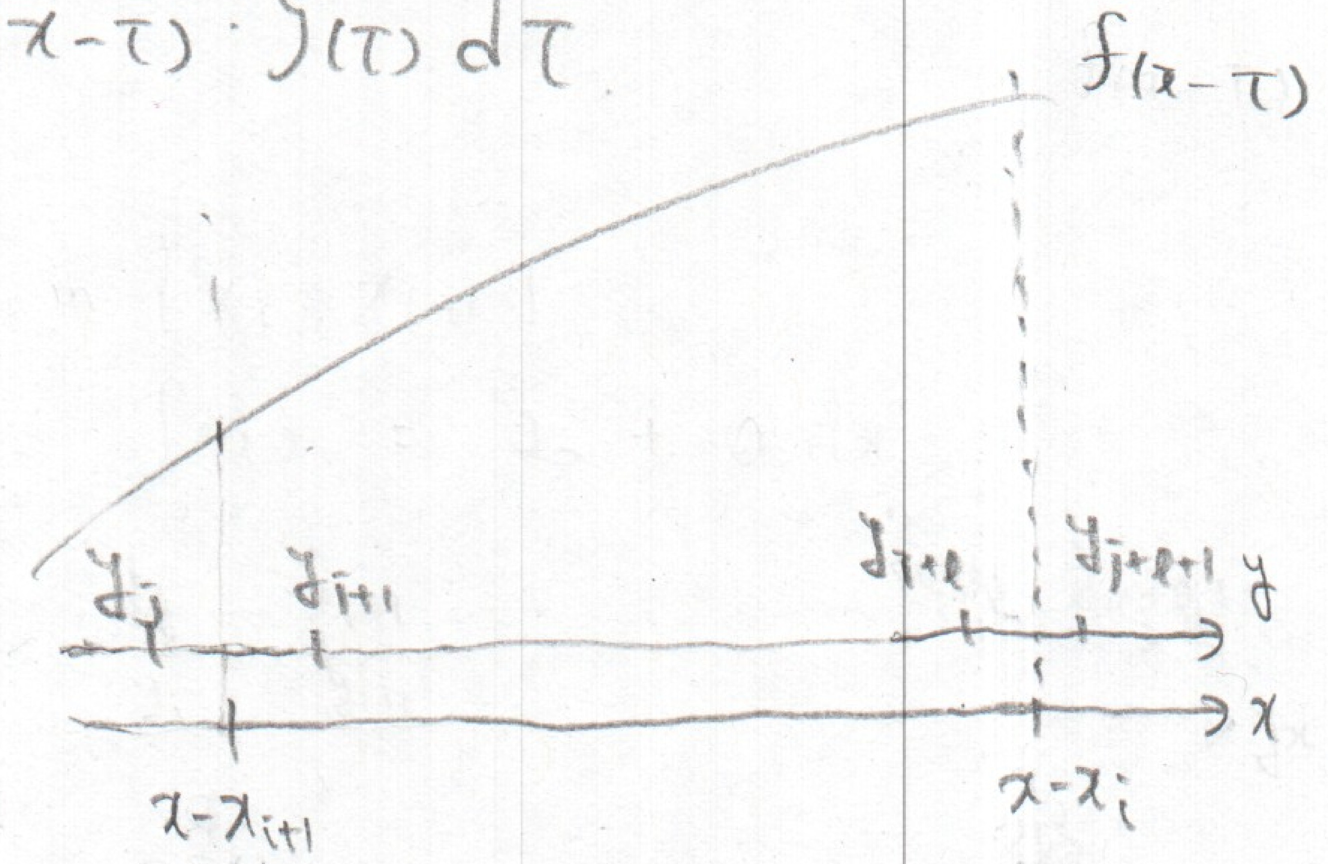
# numerical convolution for interpolated data

convolution: $f(x)\circ g(x) = \int_{-\infty}^{\infty} f(x-\tau)\cdot g(\tau)\,d\tau$

$\{f_0, f_1, \cdots, f_n\}\ \{x_0, \cdots, x_{n-1}\}$

$\{g_0, \cdots, g_{m-1}\}\ \{y_0, \cdots, y_{m-1}\}$




$$f(x)\circ g(x) = \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} f(\tau'(x))\cdot g(\tau)\,d\tau' \quad \text{------} \ (1)$$

$\boxed{\tau' = x-\tau}$

$f(\tau')$ is spline function in $[x_i \sim x_{i+1}]$

$\boxed{\Delta\tau' = \tau' - x_i}$

$$f(x_i+\Delta\tau') = f(x_i) + \frac{d}{d\tau} f(x_i)\,\Delta\tau' + \left(\frac{d}{d\tau}\right)^2 f(x_i)\,\Delta\tau'^2 + \left(\frac{d}{d\tau}\right)^3 f(x_i)\,\Delta\tau'^3$$

$$\int_{x_i}^{x_{i+1}} f(\tau')\,g(\tau)\,d\tau = -\int_{y_j}^{x-x_{i+1}} f(\tau')\,g(\tau)\,d\tau + \sum_{j}^{j<j+l} \int_{y_j}^{y_{j+l}} f(\tau')\,g(\tau)\,d\tau + \int_{y_{j+l}}^{x-x_i} f(\tau')\,g(\tau)\,d\tau$$

$\Delta\tau = \tau - y_j = x - \tau' - y_j$

$= x - x_i - y_j - \Delta\tau' \qquad \Delta\tau' = x - x_i - y_j - \Delta\tau$

$$f(y_j+\Delta\tau) = f(x_i + (\underbrace{x - x_i - y_j}_{s} - \Delta\tau))$$

$$= f(x_i) + \frac{d}{d\tau} f(x_i)\,(s-\Delta\tau) + \left(\frac{d}{d\tau}\right)^2 f(x_i)\,(s-\Delta\tau)^2 + \left(\frac{d}{d\tau}\right)^3 f(x_i)\,(s-\Delta\tau)^3$$

the integration range:

$\min[\,y_0,\ x-x_{n-1}\,] \sim \min[\,y_{m-1},\ x-x_0\,]$