numerical convolution for interpolated data   2011.5.10

$$f(x) \cdot g(x) = \int_{-\infty}^{\infty} f(\tau)\cdot g(x-\tau)\,d\tau$$

$$\boxed{f(x) \to \{x_0, x_1, \cdots, x_n\} \qquad f(x) = 0 \begin{cases} x \le x_0 \\ x \ge x_n \end{cases}}$$

$$\int_{-\infty}^{\infty} f(\tau)\cdot g(x-\tau)\,d\tau = \sum_{i=0}^{n-1}\int_{x_i}^{x_{i+1}} f(\tau)\cdot g(x-\tau)\,d\tau.$$

in $[x_i, x_{i+1}]$ $f(x)$ can be approximated by a polynominal

$$\begin{cases} f(\tau) = a_0 + a_1 \Delta x + \cdots \\ g(x-\tau) = b_0 + b_1 \Delta x + \cdots \end{cases} \qquad (\Delta x = x - x_i)$$

◎ if f, g is linear ...

$$\int_{x_i}^{x_{i+1}} f(\tau)\, g(x-\tau)\,d\tau = \int_0^{\overset{x_{i+1}-x_i}{\overset{\parallel}{\Delta x_i}}} \{a_0 \cdot b_0 + (a_1 b_0 + a_0 b_1)x + a_1 b_1 x^2\}\,dx$$

$$= a_0 b_0 \Delta x_i + \frac{1}{2}(a_1 b_0 + a_0 b_1)\Delta x_i^2 + \frac{1}{3} a_1 b_1 \Delta x_i^3$$

$$\boxed{\begin{array}{l} i = 0 \sim i' \\ \quad \& \quad x_{i'} < x-\tau < x_{i'+1} \end{array}}$$

◎ if f, g is cubic spline..

$$\int_{x_i}^{x_{i+1}} f(\tau)\, g(x-\tau)\,d\tau = \frac{1}{1!} a_0 b_0 \Delta x_i + \frac{1}{2!}\sum a_i b_{1-i} \Delta x_i^2 + \frac{1}{3!}\sum a_i b_{2-i} \Delta x_i^3 + \cdots$$

$$= \sum_n \frac{\Delta x_i^n}{n!} \sum_{i=0}^{n-1} a_i b_{n-i-1}$$