7th International Conference on Modelling, Identification and Control (ICMIC 2015)
Sousse, Tunisia - December 18-20, 2015

Design and Modeling of Planar Magnetic Inductors for Power Converters Applications

Aymen Ammouri[#1], Tarek Ben Salah [#1,2], Ferid Kourda[#1]

[#1] *Université de Tunis El Manar, ENIT-L.S.E, BP 37-1002, Tunis le Belvédère, Tunis, Tunisie*

[#2] *Technical and Vocational Training Corporation, College Of Technology in Tabuk, KSA*

***Abstract*— Planar inductors devices are essential parts in power converters which are used in the different stage of energy conversion. Designed and simulated power converters needs prototype-less approaches. However, Models of planar inductors devices are still not available in simulator tools. There is, thus, a specific limitation during the simulation process of integrated power converters. Thus, this paper focuses on the modeling of planar inductors with magnetic core. Based on an analytical field solution, an accurate method to calculate the reluctance magnetic core geometry is presented. Also, this model includes the impact of high frequency effects such as the skin, the proximity in the metal lines and the parasitic capacitances effects. This model is based on the geometrical parameters and physical characteristics of the planar inductors. The proposed model is compared to 3D finite element method simulations and reported results of several planar inductors with magnetic core. The analytical model of planar inductor results shows good agreement with the 3D FEM simulation.**

***Keywords*— Square Planar Inductor, Modeling, Magnetic core, 3D finite element methods (FEM).**

I. INTRODUCTION

Planar inductor with magnetic core is attractive option used to increasing inductance density and canalized the magnetic flux lines. Where the advantages of planar technology are; low profile, excellent repeatability, economical assembly, mechanical integrity and superior thermal characteristics [1]. From a design point of view, planar inductor electrical behavior cannot be precisely predicted by the conventional transformer models. In particular, the skin, the proximity and the parasitic capacitances and core loss effects are complex devices to model, particularly during large frequency domain.

Based on finite method analysis, an accurate model is introduced. However, this method is not only complicated to implement in circuit simulator but also is time-consuming. Therefore, an analytical approach used to analyze the planar inductor can be a solution aided to design a prototype-less approaches. Based on the geometry parameters of this device, an equivalent circuit model is preferred. This method allows reducing the time simulation and the cost of achieving a prototype.

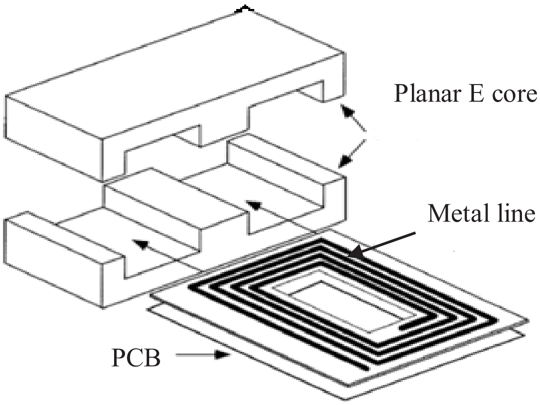
In this paper, an accurate planar magnetic core inductors models are developed. The conventional model is based in the simple expression of the magnetic core reluctance [1-4]. Nevertheless, it is not convenient to represent an inductance of planar devices [5-6]. In this paper, an analytical based planar inductor model with magnetic core is presented. This model describes the problems of magnetic field lines in the corner sections comparing to the idealized simple magnetic core. The effects of high frequency such as the skin, the proximity in the metal lines and the parasitic capacitances effects are taken into account. 3D FEM simulations are used to verify the results obtained from the analytical model. A comparative study between analytical model and simulation results is presented and discussed.

II. MODEL DESCRIPTION

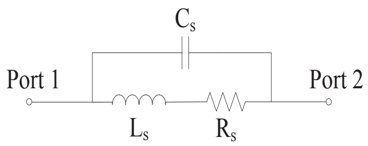
Figure 1 shows the proposed magnetic planar inductor with Ferrite Cores. Planar inductor consists of two main elements: the planar windings printed on PCB and planar magnetic core for channeling the magnetic field line. It is important to note that, this winding comprises more layers with multiple turns in each layer. The planar winding of inductor is designed by following geometric parameters. N_{layer} is the number of layer, the number of turns per layer n, the thickness of the isolated between layer T_{sub}, the width of the trace w, the thickness of the metal trace and t and the spacing between turns S. The ferrite magnetic core is described by following geometric parameters; A_c *is the* effective cross sectional area of the core, b_c *is the* width of the outer leg of the core, b_w is the width of the core window, l_c is the effective length of the core, l_w is the depth of the magnetic core and μ_r is the relative permeability of the core.

Figure 2.b shows the equivalent circuit model of a planar inductor [1,2,7]. This model includes the total inductance, L_s, corresponding of the self-inductance of the magnetic core, the series resistance used to describe the high frequency copper losses, R_{ac}, depended to the skin and proximity effect in the

metal wire. The parasitic capacitances effect coupling between the ports of the inductor are presented by C_s.



(a)



(b)

Fig. 1 The proposed magnetic planar inductors: (a) picture and (b) Equivalent circuit model.

A. Inductance Model

The inductance contribution by the ferrite core can be estimated by simple equation (1). However, this approximation does not take into account the corner section effects. The flux tends to concentrate mainly at the inner bend, so shortening the mean magnetic path. Based on the results in [1], an improved expression incorporating corner section of planar inductor model has been derived.

$$L_s = \mu_0 \mu_r \frac{A_e}{l_e} N^2 \tag{1}$$

Where μ_0 is the permeability of free air and μ_r is the relative permeability of the core material. A_e is the core cross-sectional area and l_e is the magnetic path length and N is the number of turns.
In this work, an improved expression incorporating corner section effects proposed by [5,6], is used to calculate the inductance value of the planar inductors. The magnetic path length and the cross-sectional area for different types of section are illustrated and calculated in Figure 2.
The parameter calculation of the straight sections is quite simple whereas the calculation of the parameters of the corner sections is more complicated. The mean magnetic path length l_e and the cross-section A_e of the each corner segment is approximated by the geometry given in Figure 2.

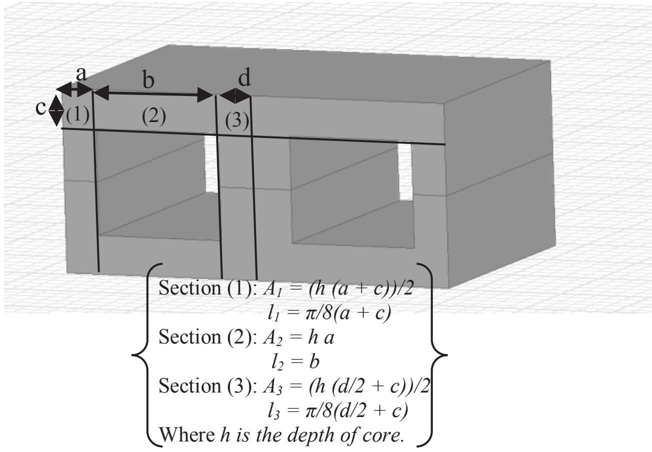


Fig 2. Example of EE core geometry for reluctance calculation.

For the test structure, EE magnetic core was used. Hence, the inductance contribution from the two cores can be estimated by equation (2):

$$L = \frac{N^2}{\sum_i R_{ci}} \tag{2}$$

Where N is the number of turns and R_i is the magnetic reluctance of each section.
The reluctance R_i of the each core section is expressed by:

$$R_{ci} = \frac{l_i}{\mu_0 \mu_r A_i} \tag{3}$$

Where l_i and A_i are the magnetic path length and cross-section area of the core i^{th} section, respectively.

B. Winding Loss Model

The copper loss in the winding is a crucial problem in the design of planar devices. Moreover, when the planar inductor used in the dynamic mode, the current density redistribution inside the winding becomes non-uniform. Therefore, at high frequency, the copper losses increase due to the skin and proximity effect. Therefore, a comprehensive collection of analytically formulation for copper loss is summarized on the paper submitted by Dowell [8]. Based on this textbook, Ferreira proposes an improved model to calculate the AC resistance at high frequency [9]. It is identified a fill factor of the picture proportion of copper in the width of the window in Figure 3. This factor is known as porosity defined by equation (4).

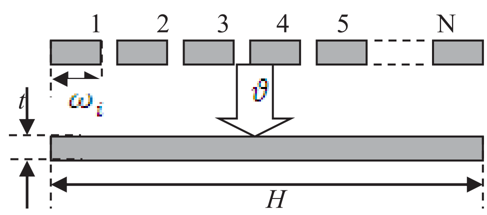


Fig.3. Porosity factor definition.

$$\eta = \frac{N \times \omega_i}{H} \tag{4}$$

$$\sigma_m = \eta \times \sigma \tag{5}$$

The AC resistance of the m layer can be calculated by equation (x), the parameter m is the number of layers in a winding section. It is important to note that, in equation (6), the first term corresponds to the skin effect whereas the second term represents the proximity effect [9].

$$\frac{R_{ac_m}}{R_{dc_m}} = \chi \left[\frac{\sinh(2\chi) + \sin(2\chi)}{\cosh(2\chi) - \cos(2\chi)} + 2\chi \left(\frac{m^2 - 1}{3} \right)^2 \cdot \frac{\sinh(\chi) - \sin(\chi)}{\cosh(\chi) + \cos(\chi)} \right] \tag{6}$$

Where χ is the ratio between the conductor's thickness and the skin depth.

$$\chi = \frac{t}{\delta} \tag{7}$$

$$\delta = \frac{1}{\sqrt{\pi \mu \eta \sigma f}} \tag{8}$$

Where δ is the skin depth in the conductor, t is the thickness of conductor and μ and σ are the permeability and conductivity of the metal conductor, respectively.

The DC series resistance for the multilayer planar inductor depends on the cross sectional area, the total length of the winding and the resistivity of the metal wire. Therefore, at lower frequencies, the current distribution inside the cross-section of conductor is uniform. The Rdc resistance of multilayer planar inductor is given by equation (9) [9].

$$R_{dc} = \rho \frac{l}{wt} \tag{9}$$

Where ρ is the resistivity of the winding material, l is the total length of the winding inductor, w and t are the width and the thickness of the metal trace, respectively.

III. Capacitance Model

The proposed HF planar inductor model with magnetic core integrates parasitic capacitances named Cs. These parasitic parameters are located between the input and the output ports of the planar devices i.e., the capacitance in inter-layer capacitance in winding of the planar inductor.

It is important to noted that, the parasitic capacitance between turns in each layer is neglected compared the capacitance between layers.

The parasitic capacitance between two layers in winding is modeled using the parallel-plate capacitance. Its value can be calculated by the following expression [10]:

$$C_l = \frac{(n+1)(2n+1)}{6n} C_0 \tag{10}$$

Where C_0 is the equivalent parallel plate capacitance of a two adjacent layers, is expressed by equation (11):

$$C_0 = \varepsilon_0 \varepsilon_r \frac{S}{H} \tag{11}$$

Where, ε_o is the permittivity of air space, ε_r is the relative permittivity of dielectric material, 4.4 for FR-4. H is the distance between two parallel plate and S is the area of conductor plates.

Where the planar inductor winding consists of m uniform layers connected in series, the equivalent capacitance can be calculated by equation (12) [11]:

$$C_s = \frac{4}{3} \frac{(m-1)}{m^2} C_l \tag{12}$$

IV. Validation and Discussion

In this section, several planar inductors with ferrite core are design to validate the proposed model. The obtained results are compared with 3D FEM simulation for validating the analytical approach.

During simulation step, several spiral inductors with ferrite core are designed with different number of turns, trace width, spacing between turns and length of the conductor. The substrate material is FR4-PCB, with relative permittivity constant 4.4 and thickness 1.6 mm.

The used of planar E or I core shape is easier for the core manufacturer to fabricate the magnetic planar inductors. However, the greater winding breadth of the E shape is an important advantage as stated before. The high frequency planar inductors were built using the core EE-38/10/25-3F4 and EI-43/10/28–3F3 planar core from Ferroxcube, the relative permeability of magnetic core are 750 and 1600, respectively. All design parameters of the resonant converter are depicted in Table I.

Table 1. Geometrical design of planar inductor parameters used during simulation steps.

	EE planar core E38/8/25–3F4		EI planar core EI43/10/28	
Geometrical Parameters	Sample#1	Sample#2	Sample#3	Sample#4
w (mm)	2.6	1.8	3	2.2
s (mm)	0.5	0.5	0.8	0.5
t (µm)	35	35	35	35
N_l	4	4	3	3
n_t	3	4	3	4
N	12	16	9	12

TABLE 2. SIMULATION AND ANALYTICAL CALCULATION RESULTS FOR PLANAR INDUCTORS: INDUCTANCE VALUE

Method	3D FEM Simulation (µH)	Analytical Inductance value (µH)	Calculation (eqn.1) (µH)
Sample#1	529.39	528.1	472.1
Sample#2	937.43	939.1	839.2
Sample#3	778.5	769.6	730
Sample#4	1390	1368.3	1297.7

In order to validate the accuracy of the proposed model, an error function named δ is used. The error is calculated according to the following expression [7]:

$$\gamma = \left|\frac{f^{FEM}(x_i) - f^{S}(x_i)}{f^{FEM}(x_i)}\right| \qquad (12)$$

Where $f^{FEM}(x_i)$, $f^{S}(x_i)$ are the electrical quantities extracted from FEM simulation and analytical results, respectively. Based on 3D Finite Element Method (FEM) simulation tools, we can find the inductance, series resistance and parasitic capacitance of the planar inductor.
Table 3 presents the error between the FEM simulation and the analytical results of planar inductor model in terms of inductance value. The error of our model is less than 1% between FEM and analytical results compared with 11% for conventional model described by equation 1.

TABLE 3. ERROR BETWEEN SIMULATION RESULTS AND ANALYTICAL RESULTS.

Method	Error between FEM and new model (%)	Error between FEM and Equation (x) (%)
Sample#1	0.2	10.8
Sample#2	0.17	10.4
Sample#3	1.14	6.22
Sample#4	1.56	6.64

High frequency copper losses have a significant influence on the modeling and design of planar inductors. Therefore, skin and proximity effects are the two most important high frequency effects related to high frequency copper losses in planar inductors. Based on the Finite Element Method (FEM) simulations, Figure 4 shows the current distribution in conductors for different frequencies for planar inductor. As shown in this Figure, when frequency increased, the total current will flow through outer surface of conductors and therefore the copper losses are increased [12].

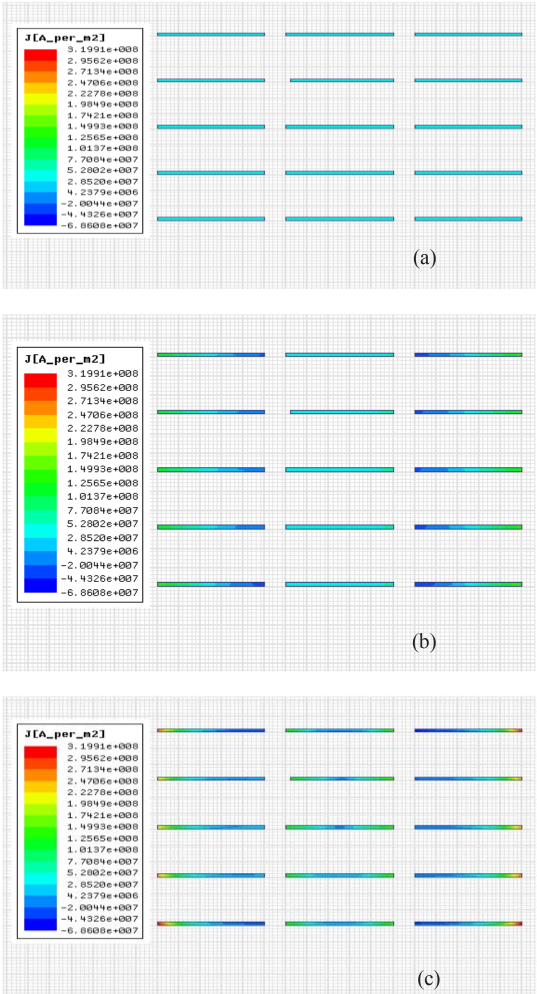


Fig 4. Current distribution in planar inductors for different frequencies: (a)50Hz, (b)100kHz, .(c) 1MHz

Table 4 illustrates the error between FEM simulation and analytical calculation results for resistance planar inductors at 100 kHz; this is the expected operating frequency of power converters application. The maximum error in resistance calculation is found to be around 22%. This error is introduced by approximation in the original article in which Dowell provides the basis of the analytical method 1D [13].

Table 5 shows a comparative study between simulation results based on 3D finite element method and analytical calculation results of the parasitic capacitance between layers of the planar inductors samples. Error between the simulation results and analytical results are given in Table 5. The maximum error in capacitance is found to be around 16%.

TABLE 4. SIMULATION AND ANALYTICAL CALCULATION RESULTS FOR PLANAR INDUCTORS: RESISTANCE VALUE@100KHZ

Method	FEM Resistance Value (mΩ)	Analytical Resistance Value (mΩ)	Error (%)
Sample#1	380.48	305.7	19.6541
Sample#2	626.28	577.7	7.7569
Sample#3	284.27	220.6	22.3977
Sample#4	485.88	397.7	18.1485

TABLE 5. ERROR BETWEEN SIMULATION RESULTS AND ANALYTICAL RESULTS OF PARASITIC CAPACITANCE.

Method	FEM Capacitance Value (pF)	Analytical Capacitance Value (pF)	Error (%)
Sample#1	10.2	9.3	8.82
Sample#2	10.1	10.4	2.97
Sample#3	19.0	15.8	16.8
Sample#4	19.2	16.8	12.5

V. CONCLUSIONS

In this work, a new analytical model for planar inductors with magnetic ferrite core has been developed. First the authors detail a new approach to calculate with accurate the inductance value model. Second, this model includes the eddy current effect in the conductor and the parasitic capacitance effects. The authors discuss the validation the new magnetic planar inductor. Large planar inductors samples are designed, modeled and validated using FEM simulation. The analytical model results are compared with FEM simulation data, and excellent agreements are obtained.

ACKNOWLEDGMENT

This work was supported by the Tunisian Ministry of High Education and Research under Grant LSE-ENIT LR 11ES15.

REFERENCES

[1] Andersen, T.M.; Zingerli, C.M.; Krismer, F.; Kolar, J.W.; Ningning Wang; Mathuna, C.O., "Modeling and Pareto Optimization of Microfabricated Inductors for Power Supply on Chip," IEEE Trans. Power Electron., vol. 28, no. 9, 2013..

[2] S. Stalf. Printed Inductors in RF Consumer Applications. IEEE Trans. Consum. Electron. 47, 3 (2001).

[3] R. Meere, T. O'Donnell, H. Bergveld, N. Wang, and S. O'Mathuna, "Analysis of microinductor performance in a 20–100 MHz DC/DC converter," IEEE Trans. Power Electron., vol. 24, no. 9, pp. 2212–2218, Sep. 2009.

[4] Ignacio Lope, Claudio Carretero, Jesus Acero, Rafael Alonso, and Jose M. Burdio, "Frequency-Dependent Resistance of Planar Coils in Printed Circuit Board With Litz Structure," IEEE Transactions on Power Magnetics, vol. 50, no. 12, 2014.

[5] J. M¨uhlethaler, J. W. Kolar, and A. Ecklebe," A Novel Approach for 3D Air Gap Reluctance Calculations," International Power Electronics Conference (ICPE- ASIA 2011).

[6] A. Balakrishnan, W. T. Joines, and T. G. Wilson, "Air-gap reluctance and inductance calculations for magnetic circuits using a schwarzchristoffel transformation," IEEE Transactions on Power Electronics, vol. 12, pp. 654–663, July 1997.

[7] A. Ammouri, W. Ben Salah, S. Khachroumi, T. Ben Salah, F. Kourda, H. Morel,,"Development of a physically-based planar inductors VHDL-AMS model for integrated power converter design,"Eur. Phys. J. Appl. Phys. 66, 20901 (2014).

[8] P. Dowell, "Effects of eddy currents in transformer windings," Proc. Inst. Electr. Eng., vol. 113, no. 8, pp. 1387–1394, Aug. 1966.

[9] J. Ferreira, "Improved analytical modeling of conductive losses in magnetic components," IEEE Trans. Power Electron., vol. 9, no. 1, pp. 127–131, Jan. 1994.

[10] Z. Ouyang, O. C. Thomsen and M. A. E. Andersen, "Optimal design and tradeoff analysis of planar transformer in high-power dc–dc converters," IEEE Trans. on Industrial Electronics, vol. 59, no. 7, pp. 2800 - 2810, July 2012.

[11] Peter Wallmeier, N. Frohleke, H. Grotstollen, "Automated optimization of high frequency inductors," IEEE 1998.

[12] Riccardo Pittini, Zhe Zhang, and Michael A.E. Andersen," High current planar transformer for very high efficiency isolated boost dc-dc converters," International Power Electronics Conference (IPEC-Hiroshima 2014 - ECCE-ASIA), 2014.

[13] Ziwei Ouyang, and Michael A. E. Andersen, "Overview of Planar Magnetic Technology—Fundamental Properties," IEEE Trans. Power Electron., vol. 29, no. 9, 2014.