二维几何

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目录

1.几何公式

1.1 三角形

1. 半周长 P=(a+b+c)/2
2. 面积 S=aHa/2=absin(C)/2=sqrt(P(P-a)(P-b)(P-c))
3. 中线 Ma=sqrt(2(b^2+c^2)-a^2)/2=sqrt(b^2+c^2+2bccos(A))/2
4. 角平分线 Ta=sqrt(bc((b+c)^2-a^2))/(b+c)=2bccos(A/2)/(b+c)
5. 高线 Ha=bsin(C)=csin(B)=sqrt(b^2-((a^2+b^2-c^2)/(2a))^2)
6. 内切圆半径 r=S/P=asin(B/2)sin(C/2)/sin((B+C)/2)
   =4Rsin(A/2)sin(B/2)sin(C/2)=sqrt((P-a)(P-b)(P-c)/P)
   =Ptan(A/2)tan(B/2)tan(C/2)
7. 外接圆半径 R=abc/(4S)=a/(2sin(A))=b/(2sin(B))=c/(2sin(C))

1.2 四边形

D1,D2 为对角线,M 对角线中点连线,A 为对角线夹角

1. a^2+b^2+c^2+d^2=D1^2+D2^2+4M^2
2. S=D1D2sin(A)/2

(以下对圆的内接四边形)

3. ac+bd=D1D2
4. S=sqrt((P-a)(P-b)(P-c)(P-d)),P 为半周长

1.3 正 n 边形

R 为外接圆半径,r 为内切圆半径

1. 中心角 A=2PI/n
2. 内角 C=(n-2)PI/n
3. 边长 a=2sqrt(R^2-r^2)=2Rsin(A/2)=2rtan(A/2)
4. 面积 S=nar/2=nr^2tan(A/2)=nR^2sin(A)/2=na^2/(4tan(A/2))

1.4 圆

1. 弧长 l=rA

2. 弦长 a=2sqrt(2hr-h^2)=2rsin(A/2)
3. 弓形高 h=r-sqrt(r^2-a^2/4)=r(1-cos(A/2))=atan(A/4)/2
4. 扇形面积 S1=rl/2=r^2A/2
5. 弓形面积 S2=(rl-a(r-h))/2=r^2(A-sin(A))/2

1.5 棱柱

1. 体积 V=Ah,A 为底面积,h 为高
2. 侧面积 S=lp,l 为棱长,p 为直截面周长
3. 全面积 T=S+2A

1.6 棱锥

1. 体积 V=Ah/3,A 为底面积,h 为高
(以下对正棱锥)
2. 侧面积 S=lp/2,l 为斜高,p 为底面周长
3. 全面积 T=S+A

1.7 棱台

1. 体积 V=(A1+A2+sqrt(A1A2))h/3,A1.A2 为上下底面积,h 为高
(以下为正棱台)
2. 侧面积 S=(p1+p2)l/2,p1.p2 为上下底面周长,l 为斜高
3. 全面积 T=S+A1+A2

1.8 圆柱

1. 侧面积 S=2PIrh
2. 全面积 T=2PIr(h+r)
3. 体积 V=PIr^2h

1.9 圆锥

1. 母线 l=sqrt(h^2+r^2)
2. 侧面积 S=PIrl
3. 全面积 T=PIr(l+r)
4. 体积 V=PIr^2h/3

1.10 圆台

1. 母线 l=sqrt(h^2+(r1-r2)^2)
2. 侧面积 S=PI(r1+r2)l
3. 全面积 T=PIr1(l+r1)+PIr2(l+r2)
4. 体积 V=PI(r1^2+r2^2+r1r2)h/3

1.11 球

1. 全面积 T=4PIr^2 2.
体积 V=4PIr^3/3

1.12 球台

1. 侧面积 S=2PIrh
2. 全面积 T=PI(2rh+r1^2+r2^2)
3. 体积 V=PIh(3(r1^2+r2^2)+h^2)/6

1.13 球扇形

1. 全面积 T=PIr(2h+r0),h 为球冠高,r0 为球冠底面半径
2. 体积 V=2PIr^2h/3

1.14 欧拉四面体公式

```
double solve(double a,double b,double c,double d,double e,double f)
{return
sqrt((4*a*a*b*b*c*c-a*a*(b*b+c*c-e*e)*(b*b+c*c-e*e)-b*b*(c*c+a*a-f*f)*(c*c+a*a-f*f)-c*c*(a*a+b*b-d*d)*(a*a+b*b-d*d)+(a*a+b*b-d*d)*(b*b+c*c-e*e)*(c*c+a*a-f*f)))/12;}
```

2.点与常用函数

2.1 点定义

```
int sgn(double x)
{
	if(fabs(x)<eps) return 0;
	else return x>0?1:-1;
}
//点定义
struct Point
{
	double x,y;
	Point(){}
	Point(double a,double b)
	{
		x=a;
		y=b;
	}
	void input()
	{
		scanf("%lf%lf",&x,&y);
	}
};
typedef Point Vector;
```

2.2 + − * / < ==

```
Vector operator +(Vector a,Vector b){return Vector(a.x+b.x,a.y+b.y);}
Vector operator -(Vector a,Vector b){return Vector(a.x-b.x,a.y-b.y);}
Vector operator *(Vector a,double p){return Vector(a.x*p,a.y*p);}
Vector operator /(Vector a,double p){return Vector(a.x/p,a.y/p);}
bool operator <(Point a,Point b){return a.x<b.x||(a.x==b.x&&a.y<b.y);}
bool operator ==(Point a,Point b){return sgn(a.x-b.x)==0&&sgn(a.y-b.y)==0;}
```

2.3 常用函数

2.3.1 两个向量夹角 TwoVectorAngle

```
double TwoVectorAngle(Vector a,Vector b){return acos(dot(a,b)/length(a)/length(b));}
```

2.3.2 向量的模 length

```
double length(Vector a){return sqrt(dot(a,a));}
```

2.3.3 点积 dot

```
double dot(Vector a,Vector b){return a.x*b.x+a.y*b.y;}
```

2.3.4 叉积 cross

```
double cross(Vector a,Vector b){return a.x*b.y-a.y*b.x;}
```

2.3.5 两点距离 dist

```
double dist(Point a,Point b){return sqrt(dot(a-b,a-b));}
```

2.3.6 线段 ab 中点 midseg

```
Point midseg(Point a,Point b){return (a+b)/2;}
```

2.3.7 垂直法向量 Normal

```
Vector Normal(Vector x){return Point(-x.y,x.x)/length(x);}
```

2.3.8 向量翻转 Rotate

```
Vector Rotate(Vector a,double rad){return Vector(a.x*cos(rad)-a.y*sin(rad),a.x*sin(rad)+a.y*cos(rad));}
```

2.3.9 三个点求三角形面积 calarea

```
double calarea(Point c,Point b,Point a){return cross(b-a,c-a);}
```

2.3.10 单位向量 vecunit

```
Vector vecunit(Vector x){return x/length(x);}
```

2.3.11 向量的极角 VectorAngle

```
double VectorAngle(Vector v){return atan2(v.y,v.x);}
```

2.3.12 向量叉积意义

若 PxQ>0,则 P 在 Q 的顺时针方向。
若 PxQ<0,则 P 在 Q 的逆时针方向。
若 PxQ=0,则 P 与 Q 共线,但可能同向也可能反向。

3.直线与线段

3.1 直线的定义

```
//有向直线,它的左边就是对应的半平面
struct Line
{
    Point p;//直线上任意一点
    Vector v;
    double ang;//极角,即从 x 正半轴旋转到向量 v 所需要的角（弧度）
    double a,b,c;//直线一般式
    Line(){}
    Line(Point a,Vector b)//点斜式
    {
        p=a;
        v=b;
        ang=atan2(v.y,v.x);
        LineGeneralEquation();
    }
    void twopoint(Point a,Point b)//两点式
    {
        p=a;
        v=b-a;
        ang=atan2(v.y,v.x);
        LineGeneralEquation();
    }
    Point getpoint(double a)
    {
        return p+(v*a);
    }
    void LineGeneralEquation()//计算一般式的 a,b,c
    {
        Point p1,p2;
        p1=p;
        p2=p+v;
        a=p2.y-p1.y;
        b=p1.x-p2.x;
        c=-a*p1.x-b*p1.y;
    }
};
```

3.2 线段

3.2.1 点 p 在线段上 OnSeg

```
//线段包含端点时改成<=
bool OnSeg(Point p,Point p1,Point p2)
{
    return sgn(cross(p1-p,p2-p))==0&&sgn(dot(p1-p,p2-p))<0;
}
```

3.2.2 点 p 在线段范围内 PointInRange

```
bool PointInRange(Point p,Point a,Point b)
{
    return dot(p-a,b-a)>0&&dot(p-b,a-b)>0;//pab<90    pba<90
}
```

3.2.3 点到线段距离 DisPointToSeg

```
double DisPointToSeg(Point p,Point a,Point b)
{
    if(a==b) return length(p-a);
    Vector v1,v2,v3;
    v1=b-a;
    v2=p-a;
    v3=p-b;
    if(sgn(dot(v1,v2))<0) return length(v2);
    else if(sgn(dot(v1,v3))>0) return length(v3);
    else return fabs(cross(v1,v2))/length(v1);
}
```

3.2.4 两线段最近距离 DisSegToSeg

```
double DisSegToSeg(Point a,Point b,Point c,Point d)
{
    return min(min(DisPointToSeg(a,c,d),DisPointToSeg(b,c,d))
    ,min(DisPointToSeg(c,a,b),DisPointToSeg(d,a,b)));
}
```

3.2.5 线段相交判定 JudgeSegInter

```
bool JudgeSegInter(Point a,Point b,Point c,Point d)
{
	double t1,t2,t3,t4;
	t1=cross(b-a,c-a);
	t2=cross(b-a,d-a);
	t3=cross(d-c,a-c);
	t4=cross(d-c,b-c);
	return sgn(t1)*sgn(t2)<0&&sgn(t3)*sgn(t4)<0;
}
```

3.3 直线

3.3.1 判断直线 ab 和线段 cd 是否相交 SegInterLine

```
bool SegInterLine(Point a,Point b,Point c,Point d)
{
	return sgn(cross((c-b),(a-b)))*sgn(cross((d-b),(a-b)))<=0;
}
```

3.3.2 点到直线距离 DisPointToLine

```
double DisPointToLine(Point p, Point a, Point b)
{
	Vector v1,v2;
	v1=b-a;
	v2=p-a;
	return fabs(cross(v1,v2))/length(v1);//如果不取绝对值,得到的是有向距离
}
```

3.3.3 判断两条直线位置关系(重合/平行/相交) JudgeLineInter

```
int JudgeLineInter(Line a,Line b)
{
	if(sgn(cross(a.v,b.v))==0)
	{
		if(sgn(cross(a.p-b.p,b.v))==0) return 0;//重合
		else return 1;//平行
	}
	else return 2;//有交点
}
```

3.3.4 求两直线交点 PointOfLineInter

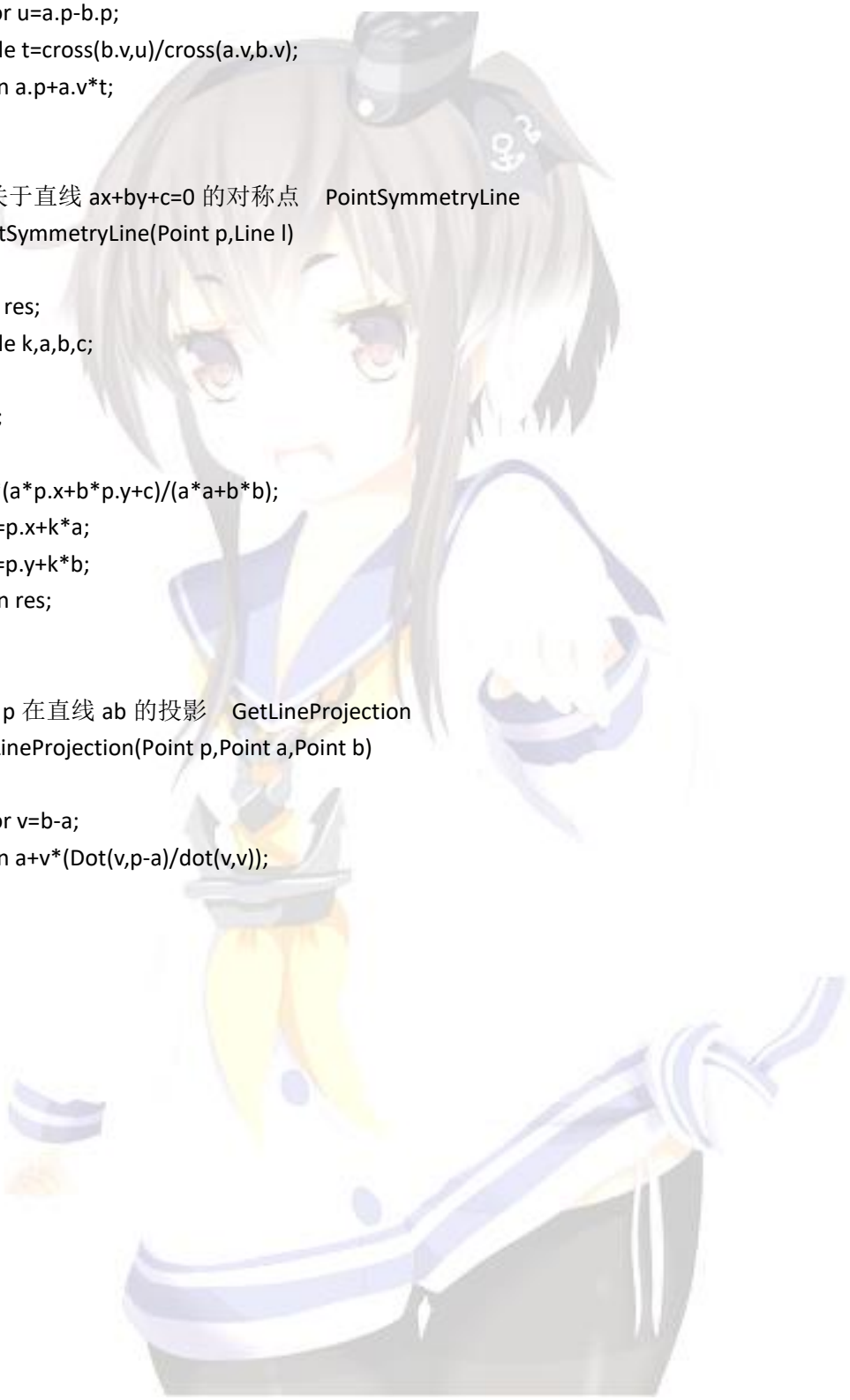
```
Point PointOfLineInter(Line a,Line b)
{
    Vector u=a.p-b.p;
    double t=cross(b.v,u)/cross(a.v,b.v);
    return a.p+a.v*t;
}
```

3.3.5 点关于直线 ax+by+c=0 的对称点 PointSymmetryLine

```
Point PointSymmetryLine(Point p,Line l)
{
    Point res;
    double k,a,b,c;
    a=l.a;
    b=l.b;
    c=l.c;
    k=-2*(a*p.x+b*p.y+c)/(a*a+b*b);
    res.x=p.x+k*a;
    res.y=p.y+k*b;
    return res;
}
```

3.3.6 点 p 在直线 ab 的投影 GetLineProjection

```
Point GetLineProjection(Point p,Point a,Point b)
{
    Vector v=b-a;
    return a+v*(Dot(v,p-a)/dot(v,v));
}
```



3.4 半平面交 hpi

```
bool OnLeft(Line l,Point p)
{
	return cross(l.v,p-l.p)>0; //点 p 在有向直线 L 的左边(线上不算)
}
bool hpicmp(Line a,Line b)
{
	return a.ang<b.ang;
}
vector<Point> hpi(vector<Line> l)
{
	int n,i;
	n=l.size();
	sort(l.begin(),l.end(),hpicmp);//按极角排序
	int first,last;		// 双端队列的第一个元素和最后一个元素的下标
	vector<Point> p(n);		// p[i]为 q[i]和 q[i+1]的交点
	vector<Line> q(n);		// 双端队列
	vector<Point> ans;		// 结果
	q[first=last=0]=l[0];	// 双端队列初始化为只有一个半平面 L[0]
	for(i=1;i<n;i++)
	{
		while(first<last&&!OnLeft(l[i],p[last-1])) last--;
		while(first<last&&!OnLeft(l[i],p[first])) first++;
		q[++last]=l[i];
		if(fabs(cross(q[last].v,q[last-1].v))<eps)
		{// 两向量平行且同向，取内侧的一个
			last--;
			if(OnLeft(q[last],l[i].p)) q[last]=l[i];
		}
		if(first<last) p[last-1]=PointOfLineInter(q[last-1],q[last]);
	}
	while(first<last&&!OnLeft(q[first],p[last-1])) last--; //删除无用平面
	if(last-first<=1) return ans; //空集
	p[last]=PointOfLineInter(q[last],q[first]); //计算首尾两个半平面的交点
	for(i=first;i<=last;i++)
	{
		ans.push_back(p[i]);// 从 deque 复制到输出中
	}
	return ans;
}
```

4.三角形

4.1 内心(内切圆请见圆部分) InCenter

```
Point InCenter(Point a,Point b,Point c)
{
	Line u,v;
	double m,n;
	u.a=a;
	m=atan2(b.y-a.y,b.x-a.x);
	n=atan2(c.y-a.y,c.x-a.x);
	u.b.x=u.a.x+cos((m+n)/2);
	u.b.y=u.a.y+sin((m+n)/2);
	v.a=b;
	m=atan2(a.y-b.y,a.x-b.x);
	n=atan2(c.y-b.y,c.x-b.x);
	v.b.x=v.a.x+cos((m+n)/2);
	v.b.y=v.a.y+sin((m+n)/2);
	return PointOfLineInter(u,v);
}
```

4.2 外心(外接圆请见圆部分) CircumCircle

```
Point CircumCircle(Point a,Point b,Point c)
{
	Point res;
	double a1=b.x-a.x,b1=b.y-a.y,c1=(a1*a1+b1*b1)/2;
	double a2=c.x-a.x,b2=c.y-a.y,c2=(a2*a2+b2*b2)/2;
	double d=a1*b2-a2*b1;
	return Point(a.x+(c1*b2-c2*b1)/d,a.y+(a1*c2-a2*c1)/d);
}
```

4.3 垂心 OrthoCenter

```
Point OrthoCenter(Point a,Point b,Point c)
{
	double c1,c2,d;
	Point p1,p2;
	p1=c-b;
	c1=0;
	p2=c-a;
	c2=dot(b-a,p2);
	d=cross(p1,p2);
	return Point(a.x+(c1*p2.y-c2*p1.y)/d,a.y+(p1.x*c2-p2.x*c1)/d);
}
```

5.多边形

5.1 凸包 graham

```
// 如果不希望在凸包的边上有输入点，把两个 <= 改成 <
// 注意：输入点集会被修改
vector<Point> graham(vector<Point> p)
{
	int n,m,k,i;
	sort(p.begin(),p.end());//要用到 operator <
	p.erase(unique(p.begin(),p.end()),p.end());//要用到 operator ==
	n=p.size();
	m=0;
	vector<Point> res(n+1);
	for(i=0;i<n;i++)
	{
		while(m>1&&cross(res[m-1]-res[m-2],p[i]-res[m-2])<=0) m--;
		res[m++]=p[i];
	}
	k=m;
	for(i=n-2;i>=0;i--)
	{
		while(m>k&&cross(res[m-1]-res[m-2],p[i]-res[m-2])<=0) m--;
		res[m++]=p[i];
	}
	if(n>1) m--;
	res.resize(m);
	return res;
}
```

5.2 判断点是否在多边形内 JudgePointInPolygon

```
int JudgePointInPolygon(Point p,vector<Point> poly)
{
	int cnt,n,k,d1,d2;
	cnt=0;
	n=poly.size();
	for(int i=0;i<n;i++)
	{
		if(OnSeg(p,poly[i],poly[(i+1)%n])) return -1;//在边上
		k=sgn(cross(poly[(i+1)%n]-poly[i],p-poly[i]));
		d1=sgn(poly[i].y-p.y);
		d2=sgn(poly[(i+1)%n].y-p.y);
		if(k>0&&d1<=0&&d2>0) cnt++;
		if(k<0&&d2<=0&&d1>0) cnt--;
	}
	if(cnt) return 1;//内部
	else return 0;//外部
}
```

5.3 多边形重心（点集逆时针给出） bcenter

```
Point bcenter(vector<Point> p)
{
	int i,j,n;
	Point t,ans;
	double tp,s,tpx,tpy;
	s=t.x=t.y=0;
	n=p.size();
	for(i=0;i<n;i++)
	{
		if(i+1==n) j=0;
		else j=i+1;
		tp=cross(p[i],p[j]);
		s+=tp/2;
		t=t+(p[i]+p[j])*tp;
	}
	ans=t/(6*s);
	return ans;
}
```

5.4 最近点对

```
//返回距离
Point p1[MAX],temp[MAX];
bool cmpxy(Point a,Point b)
{
	if(a.x!=b.x) return a.x<b.x;
	return a.y<b.y;
}
bool cmpy(Point a,Point b){return a.y<b.y;}
double ClosestPair(int l,int r)
{
	int mid,i,j,k;
	double d,d1,d2,d3;
	d=INF;
	if(l==r) return d;
	if(l+1==r) return dist(p1[l],p1[r]);
	mid=(l+r)>>1;
	d1=ClosestPair(l,mid);
	d2=ClosestPair(mid+1,r);
	d=min(d1,d2);
	k=0;
	for(i=l;i<=r;i++)
	{
		if(fabs(p1[mid].x-p1[i].x)<=d) temp[k++]=p1[i];
	}
	sort(temp,temp+k,cmpy);
	for(i=0;i<k;i++)
	{
		for(j=i+1;j<k&&temp[j].y-temp[i].y<d;j++)
		{
			d3=dist(temp[i],temp[j]);
			if(d>d3) d=d3;
		}
	}
	return d;
}
int main()
{
	//.....................
	sort(p1,p1+n,cmpxy);
	ans=ClosestPair(0,n-1);
	//.....................
}
```

5.5 模拟退火求费马点

```
double getres(Point t,Point *p,int n)
{
	int i;
	double res=0;
	for(i=0;i<n;i++)
	{
		res+=dist(t,p[i]);
	}
	return res;
}
pair<Point,double> SA(Point *p,int n)
{
	Point s;
	double T=100,t,ds,dz,ans;
	int i,j,k,flag,dir[5][2]={0,0,0,1,0,-1,1,0,-1,0};
	s=Point(0,0);
	for(i=0;i<n;i++)
	{
		s.x+=p[i].x;
		s.y+=p[i].y;
	}
	s.x/=n;
	s.y/=n;
	ans=0;
	for(k=1;k<=100000;k++)
	{
		t=T/k;//精度控制
		for(i=0;i<5;i++)
		{
			Point z;
			z.x=s.x+dir[i][0]*t;
			z.y=s.y+dir[i][1]*t;
			dz=getres(z,p,n);
			if(dz<ans)
			{
				ans=dz;
				s=z;
			}
		}
	}
	pair<Point,double> res;
	res=make_pair(s,ans);
```

```
    return res;
}
```

5.6 最小圆覆盖

```
double r;
Point center;
void CircumCircle(Point a,Point b,Point c) //三角形外心
{
    Point res;
    double a1=b.x-a.x,b1=b.y-a.y,c1=(a1*a1+b1*b1)/2;
    double a2=c.x-a.x,b2=c.y-a.y,c2=(a2*a2+b2*b2)/2;
    double d=a1*b2-a2*b1;
    res=Point(a.x+(c1*b2-c2*b1)/d,a.y+(a1*c2-a2*c1)/d);
    center=res;
}
void MinDisWith2Point(Point pi,Point pj,int m,vector<Point> p)
{
    int k;
    center=MidSeg(pi,pj);
    r=dist(pi,pj)/2;
    for(k=0;k<m;k++)
    {
        if(dist(center,p[k])<=r) continue;
        double t=cross((pi-pj),(p[k]-pj));
        if(t!=0)
        {
            CircumCircle(pi,pj,p[k]);
            r=dist(center,pi);
        }
        else
        {
            double d1,d2,d3;
            d1=dist(pi,pj);
            d2=dist(pi,p[k]);
            d3=dist(pj,p[k]);
            if(d1>=d2&&d1>=d3)
            {
                center=MidSeg(pi,pj);
                r=dist(pi,pj)/2;
            }
            else if(d2>=d1&&d2>=d3)
            {
                center=MidSeg(pi,p[k]);
                r=dist(pi,p[k])/2;
```

```
                }
                else
                {
                    center=MidSeg(pj,p[k]);
                    r=dist(pj,p[k])/2;
                }
            }
        }
}
void MinDisWithPoint(Point pi,int m,vector<Point> p)
{
    center=MidSeg(pi,p[0]);
    r=dist(pi,p[0])/2;
    for(int j=1;j<m;j++)
    {
        if(dist(center, p[j])<=r) continue;
        else MinDisWith2Point(pi,p[j],j,p);
    }
}
int main()
{
    //.................
    //n 点的个数
    vector<Point> p;
    if(n==1)
    {
        printf("%.2lf %.2lf 0.00\n",p[0].x,p[0].y);
        continue;
    }
    r=dist(p[0],p[1])/2;
    center=MidSeg(p[0],p[1]);
    for(i=2;i<n;i++)
    {
        if(dist(center,p[i])<=r) continue;
        else MinDisWithPoint(p[i],i,p);
    }
    printf("%.2lf %.2lf %.2lf\n",center.x,center.y,r);
    //.................
}
```

5.7 旋转卡壳

5.7.1 平面最远点对(凸包直径) RC_FarthestPair

```
//先求凸包
//最远点对距离的平方
double dist2(Point a,Point b){return dot(a-b,a-b);}//距离的平方
double RC_FarthestPair(vector<Point> p)
{
    int n,i,j,k;
    n=p.size();
    if(n==1) return 0;
    if(n==2) return dist2(p[0],p[1]);
    double temp,res=0;
    k=1;
    for(j=1;j<n;j++)//定点
    {
        for(i=0;i<n;i++)//旋转
        {
            while(cross(p[i]-p[(i+j)%n],p[(k+1)%n]-p[k])<0) k=(k+1)%n;
            temp=max(dist2(p[j],p[i]),dist2(p[j],p[k]));
            res=max(res,temp);
        }
    }
    return res;
}
```

5.7.2 点集最大三角形 RC_Triangle

```
//先求凸包
//返回三角形面积
double RC_Triangle(vector<Point> p)
{
    int n,i,j,k;
    n=p.size();
    if(n<3) return 0;
    double temp,res=0;
    k=1;
    for(j=1;j<n;j++)//定点
    {
        for(i=0;i<n;i++)//旋转
        {
            while(cross(p[i]-p[(i+j)%n],p[(k+1)%n]-p[k])<0) k=(k+1)%n;
            temp=max(cross(p[(i+j)%n]-p[i],p[k]-p[i]),cross(p[(i+j)%n]-p[i],p[(k+1)%n]-p[i]));
            res=max(res,temp);
```

```
            }
        }
        return res/2;
}
```

5.7.3 两凸包最近距离 RC_Dis

```
double RC_Dis(vector<Point> p,vector<Point> q)
{
        int i,j,k,n,m;
        n=p.size();
        m=q.size();
        j=k=0;
        for(i=0;i<n;i++)
        {
            if(p[i].y-p[k].y<-eps) k=i;
        }
        for(i=0;i<m;i++)
        {
            if(q[i].y-q[j].y>eps) j=i;
        }
        double temp,ans=1e9;
        for(i=0;i<n;i++)
        {
            while((temp=cross(p[(k+1)%n]-p[k],q[(j+1)%m]-p[k])-cross(p[(k+1)%n]-p[k],q[j]-p[k]))>e
        ps) j=(j+1)%m;
            if(temp<-eps) ans=min(ans,DisPointToSeg(q[j],p[k],p[(k+1)%n]));
            else ans=min(ans,DisSegToSeg(p[k],p[(k+1)%n],q[j],q[(j+1)%m]));
            k=(k+1)%n;
        }
        return ans;
}
```

6.圆

6.1 圆的定义

```
struct Circle
{
	Point c;
	double r;
	Circle(){}
	Circle(Point a,double b)
	{
		c=a;
		r=b;
	}
	Point getpoint(double a) //根据圆心角求点坐标
	{
		return Point(c.x+cos(a)*r,c.y+sin(a)*r);
	}
};
```

6.2 点 P 到圆的切线 getTangents

```
//切点存在 v 数组
int getTangents(Point p, Circle c, Vector* v)
{
	Vector u=c.c-p;
	double d=length(u);
	if(d<c.r) return 0;
	else if(sgn(d-c.r)==0)
	{
		v[0]=Rotate(u,PI/2);
		return 1;
	}
	else
	{
		double ang=asin(c.r/d);
		v[0]=Rotate(u,-ang);
		v[1]=Rotate(u,+ang);
		return 2;
	}
}
```

6.3 求 a 点到 b 点(逆时针)在的圆上的圆弧长度 ArcLen

```
double ArcLen(Circle c,Point a,Point b)
{
	double ang1,ang2;
	Vector v1,v2;
	v1=a-Point(c.c.x,c.c.y);
	v2=b-Point(c.c.x,c.c.y);
	ang1=atan2(v1.y,v1.x);
	ang2=atan2(v2.y,v2.x);
	if(ang2<ang1) ang2+=2*PI;
	return c.r*(ang2-ang1);
}
```

6.4 两圆的公切线 CommonTangents

```
//两圆的公切线, -1 表示无穷条切线
//返回切线条数 切点保存在 a b 数组
int CommonTangents(Circle c1,Circle c2, Point* a, Point* b)
{
	int cnt=0;
	if(c1.r<c2.r)
	{
		swap(c1,c2);
		swap(a,b);
	}
	int d=(c1.c.x-c2.c.x)*(c1.c.x-c2.c.x)+(c1.c.y-c2.c.y)*(c1.c.y-c2.c.y);
	int rd=c1.r-c2.r;
	int rs=c1.r+c2.r;
	if(d<rd*rd) return 0; //内含
	double base=atan2(c2.c.y-c1.c.y,c2.c.x-c1.c.x);
	if(d==0&&sgn(c1.r-c2.r)==0) return -1;//无线多条切线
	if(d==rd*rd)//内切, 1 条切线
	{
		a[cnt]=c1.getpoint(base);
		b[cnt]=c2.getpoint(base);
		cnt++;
		return 1;
	}
	//有外公切线
	double ang=acos((c1.r-c2.r)/sqrt(d));
	a[cnt]=c1.getpoint(base+ang);
	b[cnt]=c2.getpoint(base+ang);
```

```
    cnt++;
    a[cnt]=c1.getpoint(base-ang);
    b[cnt]=c2.getpoint(base-ang);
    cnt++;
    if(d==rs*rs)//一条内公切线
    {
        a[cnt]=c1.getpoint(base);
        b[cnt]=c2.getpoint(PI+base);
        cnt++;
    }
    else if(d>rs*rs)//两条内公切线
    {
        double ang=acos((c1.r+c2.r)/sqrt(d));
        a[cnt]=c1.getpoint(base+ang);
        b[cnt]=c2.getpoint(PI+base+ang);
        cnt++;
        a[cnt]=c1.getpoint(base-ang);
        b[cnt]=c2.getpoint(PI+base-ang);
        cnt++;
    }
    return cnt;
}
```

6.5 三角形外接圆 CircumscribedCircle

```
Circle CircumscribedCircle(Point p1,Point p2,Point p3)
{
    double bx=p2.x-p1.x,by=p2.y-p1.y;
    double cx=p3.x-p1.x,cy=p3.y-p1.y;
    double d=2*(bx*cy-by*cx);
    double ex=(cy*(bx*bx+by*by)-by*(cx*cx+cy*cy))/d+p1.x;
    double ey=(bx*(cx*cx+cy*cy)-cx*(bx*bx+by*by))/d+p1.y;
    Point p=Point(ex,ey);
    return Circle(p,length(p1-p));
}
```

6.6 三角形内切圆 InscribedCircle

```
Circle InscribedCircle(Point p1,Point p2,Point p3)
{
    double a=length(p2-p3);
    double b=length(p3-p1);
    double c=length(p1-p2);
    Point p=(p1*a+p2*b+p3*c)/(a+b+c);
    return Circle(p,DisPointToLine(p,p1,p2));
```

```
}
```

6.7 线段与圆的交点 getSegCircleInter

```
int getSegCircleInter(Line l,Circle c,Point *sol)
{
	Vector nor=Normal(l.v);
	Line l1=Line(c.c,nor);
	Point p1=PointOfLineInter(l1,l);
	double d=length(p1-c.c);
	if(sgn(d-c.r)>0) return 0;
	Point p2=vecunit(l.v)*sqrt(c.r*c.r-d*d);
	int res=0;
	sol[res]=p1+p2;
	if(OnSeg(sol[res],l.p,l.getpoint(1))) res++;
	sol[res]=p1-p2;
	if(OnSeg(sol[res],l.p,l.getpoint(1))) res++;
	return res;
}
```

6.8 直线与圆交点 getLineCircleInter

```
//直线与圆交点 返回个数
int getLineCircleInter(Line l,Circle cc,Point *sol)
{
	double a,b,c,d,e,f,g,delta,t;
	a=l.v.x;
	b=l.p.x-cc.c.x;
	c=l.v.y;
	d=l.p.y-cc.c.y;
	e=a*a+c*c;
	f=2*(a*b+c*d);
	g=b*b+d*d-cc.r*cc.r;
	delta=f*f-4*e*g;
	if(sgn(delta)<0) return 0;
	if(sgn(delta)==0)
	{
		t=-f/(2*e);
		sol[0]=l.getpoint(t);
		return 1;
	}
	else
```

```
    {
        t=(-f-sqrt(delta))/(2*e);
        sol[0]=l.getpoint(t);
        t=(-f+sqrt(delta))/(2*e);
        sol[1]=l.getpoint(t);
        return 2;
    }
}
```

6.9 两圆交点 getCircleInter

```
//两圆交点 返回个数
int getCircleInter(Circle c1,Circle c2,Point *sol)
{
    double r1,r2,x1,y1,x2,y2,d;
    r1=c1.r;
    r2=c2.r;
    x1=c1.c.x;
    y1=c1.c.y;
    x2=c2.c.x;
    y2=c2.c.y;
    d=length(c1.c-c2.c);
    if(sgn(fabs(r1-r2)-d)>0) return -1;
    if(sgn(r1+r2-d)<0) return 0;
    double a,b,c,p,q,r;
    a=r1*(x1-x2)*2;
    b=r1*(y1-y2)*2;
    c=r2*r2-r1*r1-d*d;
    p=a*a+b*b;
    q=-a*c*2;
    r=c*c-b*b;
    double cosa,sina,cosb,sinb;
    if(sgn(d-(r1+r2))== 0||sgn(d-fabs(r1-r2))==0)
    {
        cosa=-q/p/2;
        sina=sqrt(1-cosa*cosa);
        Point p(x1+c1.r*cosa,y1+c1.r*sina);
        if(!OnCircle(p,c2)) p.y=y1-c1.r*sina;
        sol[0]=p;
        return 1;
    }
    double delta=sqrt(q*q-p*r*4);
    cosa=(delta-q)/p/2;
    cosb=(-delta-q)/p/2;
```

```
    sina=sqrt(1-cosa*cosa);
    sinb=sqrt(1-cosb*cosb);
    Point p1(x1+c1.r*cosa,y1+c1.r*sina);
    Point p2(x1+c1.r*cosb,y1+c1.r*sinb);
    if(!OnCircle(p1,c2)) p1.y=y1-c1.r*sina;
    if(!OnCircle(p2,c2)) p2.y=y1-c1.r*sinb;
    if(p1==p2) p1.y=y1-c1.r*sina;
    sol[0]=p1;
    sol[1]=p2;
    return 2;
}
```

6.10 判断点在圆内 InCircle

```
bool InCircle(Point x,Circle c){return sgn(c.r-length(c.c-x))>=0;}//在圆内(包括圆上)
```

7.面积问题

7.1 多边形面积 PolygonArea

```
double PolygonArea(Point *p,int n)
{
	double res=0;
	for(int i=0;i<n;i++)
	{
		res+=cross(p[i],p[(i+1)%n]);
	}
	return fabs(res/2);
}
```

7.2 凸多边形面积并 CPIA

```
//凸多边形面积并 ConvexPolygonInterArea
double CPIA(Point *a,Point *b,int na,int nb)
{
	Point p[MAX],q[MAX];
	int tn,sflag,eflag,i,j;
	a[na]=a[0];
	b[nb]=b[0];
	for(i=0;i<=nb;i++)
	{
		p[i]=b[i];
	}
	for(i=0;i<na&&nb>2;i++)
	{
		sflag=sgn(cross(a[i+1]-a[i],p[0]-a[i]));
		for(j=0,tn=0;j<nb;j++)
		{
			if(sflag>=0) q[tn++]=p[j];
			eflag=sgn(cross(a[i+1]-a[i],p[j+1]-a[i]));
			if((sflag^eflag)==-2)
q[tn++]=PointOfLineInter(Line(a[i],a[i+1]-a[i]),Line(p[j],p[j+1]-p[j]));
			sflag=eflag;
		}
		for(j=0;j<tn;j++)
		{
			p[j]=q[j];
		}
```

```
            nb=tn;
            p[nb]=p[0];
        }
        if(nb<3) return 0;
        return PolygonArea(p,nb);
}
```

7.3 多边形面积并 SPIA

```
//多边形面积并   SimplePolygonInterArea
double SPIA(Point *a,Point *b,int na,int nb)
{
        int i,j,temp1,temp2;
        Point t1[4],t2[4];
        double res=0;
        a[na]=t1[0]=a[0];
        b[nb]=t2[0]=b[0];
        for(i=2;i<na;i++)
        {
            t1[1]=a[i-1];
            t1[2]=a[i];
            temp1=sgn(cross(t1[1]-t1[0],t1[2]-t1[0]));
            if(temp1<0) swap(t1[1],t1[2]);
            for(j=2;j<nb;j++)
            {
                t2[1]=b[j-1];
                t2[2]=b[j];
                temp2=sgn(cross(t2[1]-t2[0],t2[2]-t2[0]));
                if(temp2<0) swap(t2[1],t2[2]);
                res+=CPIA(t1,t2,3,3)*temp1*temp2;
            }
        }
        return res;
}
```

7.4 多边形与圆面积交 PolyCiclrArea

```
//多边形与圆面积交
Point PointOfLineInter(Line a,Line b)//线段交点
{
	Vector u=a.p-b.p;
	double t=cross(b.v,u)/cross(a.v,b.v);
	return a.p+a.v*t;
}
bool InCircle(Point x,Circle c){return sgn(c.r-length(c.c-x))>=0;}//在圆内(包括圆上)
int getSegCircleInter(Line l,Circle c,Point *sol)//线段与圆的交点
{
	Vector nor=Normal(l.v);
	Line l1=Line(c.c,nor);
	Point p1=PointOfLineInter(l1,l);
	double d=length(p1-c.c);
	if(sgn(d-c.r)>0) return 0;
	Point p2=vecunit(l.v)*sqrt(c.r*c.r-d*d);
	int res=0;
	sol[res]=p1+p2;
	if(OnSeg(sol[res],l.p,l.getpoint(1))) res++;
	sol[res]=p1-p2;
	if(OnSeg(sol[res],l.p,l.getpoint(1))) res++;
	return res;
}
double VectorAngle(Vector v){return atan2(v.y,v.x);}
double SegCircleArea(Circle c,Point a,Point b) //线段切割圆
{
	double a1=VectorAngle(a-c.c);
	double a2=VectorAngle(b-c.c);
	double da=fabs(a1-a2);
	if (sgn(da-PI)>0) da=PI*2.0-da;
	return sgn(cross(b-c.c,a-c.c))*da*c.r*c.r/2.0;
}
double PolyCiclrArea(Circle c,Point *p,int n)//多边形与圆面积交
{
	double res=0;
	Point sol[2];
	p[n]=p[0];
	for(int i=0;i<n;i++)
	{
		double t1,t2;
		int cnt=getSegCircleInter(Line(p[i],p[i+1]-p[i]),c,sol);
```

```
            if(cnt==0)
            {
                if(!InCircle(p[i],c)||!InCircle(p[i+1],c)) res+=SegCircleArea(c,p[i],p[i+1]);
                else res+=cross(p[i+1]-c.c,p[i]-c.c)/2.0;
            }
            if(cnt==1)
            {
                if(InCircle(p[i],c)&&!InCircle(p[i+1],c))
                {
                    res+=cross(sol[0]-c.c,p[i]-c.c)/2.0;
                    res+=SegCircleArea(c,sol[0],p[i+1]);
                }
                else
                {
                    res+=SegCircleArea(c,p[i],sol[0]);
                    res+=cross(p[i+1]-c.c,sol[0]-c.c)/2.0;
                }
            }
            if(cnt==2)
            {
                if((p[i]<p[i+1])^(sol[0]<sol[1])) swap(sol[0],sol[1]);
                res+=SegCircleArea(c,p[i],sol[0]);
                res+=cross(sol[1]-c.c,sol[0]-c.c)/2.0;
                res+=SegCircleArea(c,sol[1],p[i+1]);
            }
        }
        return fabs(res);
}
```

7.5 两圆面积交 CircleCircleArea

```
double CircleCircleArea(Circle c1,Circle c2)
{
    double d=dist(c1.c,c2.c);//计算圆心距
    if(sgn(d-(c1.r+c2.r))>=0||sgn(c1.r)==0||sgn(c2.r)==0) return 0;//相离、外切或有一圆半径为 0
    else if(sgn(d-fabs(c1.r-c2.r))<=0)//内切或内含
    {
        double r=min(c1.r,c2.r);
        return r*r*PI;
    }
    else//相交
    {
        double a,b,m,n,x,y;
        a=acos((d*d+c1.r*c1.r-c2.r*c2.r)/(2*d*c1.r));
        b=acos((d*d+c2.r*c2.r-c1.r*c1.r)/(2*d*c2.r));
        m=a*c1.r*c1.r;
        n=b*c2.r*c2.r;
        x=c1.r*c1.r*sin(a)*cos(a);
        y=c2.r*c2.r*sin(b)*cos(b);
        return m+n-(x+y);
    }
}
```

8.Pick 定理

Pick 定理：一个计算点阵中顶点在格点上的多边形面积公式：S=a+b/2-1，其中 a 表示多边形内部的点数，b 表示多边形边界上的点数，S 表示多边形的面积。

那么 a=(2S-b+2)/2

S 可以通过叉积求出

计算 b 的方法是：某一条边(x,y)上的整点的数量是 gcd(|x|,|y|)+1