University of Waterloo

PHYS460B

The Analogue Computer

Experiment #17

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Abstract

For this lab we investigated the analogue computer looking specifically at six circuits that all solved various problems. The first circuit was a summing circuit, we had two input voltages and the output would be the sum of the two circuits. When the data was analyzed we found that it fit well with the theory presented. The next Circuit that we we looked at was the differentiating circuit where we found that it fit well with the theory. We then looked at the integrating circuit, here we found that the output voltage and the calculated voltage were not the same and we attribute it to the slew rate of the op-amp. The next circuit dealt with exponential functions and on this on one we determined that were we found that for some of the data that we obtained the results did not fare well with the theory. Then we made a circuit that would handle damped harmonic motion we were not able to gain a good insight into this circuit due to the data obtained; and finally we looked at forced damped harmonic motion were we found the $Q_{actual} = 1.838$ while the $Qtheory = 4.01$.

1 Introduction

An analog computer is a computer which is used to process analog data. They are particularly useful when you need to measure data directly without converting it to code or number representation first. Analogue computers were used in scientific and industrial applications when there digital counterparts lacked in performance. The main component of the analogue computers that we will be using in this lab is the Operational Amplifier (Op-Amp), which is the essential component of the analogue computer.It is what allows us to solve the various problems we are looking at with relatively simple circuits. The six we are looking at in this lab are, Summer, Differentiator, Integrator, Exponential, Damped Harmonic Oscillator and finally the Force Damped Harmonic Oscillator.

2 Theoretical Background

We wish to solve equations by using analogue components, and the heart of many of these systems is an operational amplifier which amplifies the signals inputted into it, but it is highly used in analogue components due to its versatility. It has three key properties when setup as an inverting op-amp as seen in Figure 1 are seen to be observed and are essential to its operation. Noting that $Gain = G = \frac{V_{out}}{V_{in}}$ They are:

1. Very High Gain ($V_{in} = V_{out}/G = 0$)
2. Very High Input Impedance ($I_{in} = V_{in}/R_{in}$)
3. Very Low Output Impedance ($V_{out} = GV_{in} + I_{out}V_{out} \approx GV_{in}$)

We can easily define the gain in the non-inverting op-amp as:

$$\frac{V_{ground} - V_{out}}{R_f} = \frac{V_{in} - V_{ground}}{R_{in}}$$

Noting $V_{ground} = 0$:

$$-\frac{V_{out}}{R_f} = \frac{V_{in}}{R_{in}}$$
$$\implies Gain = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

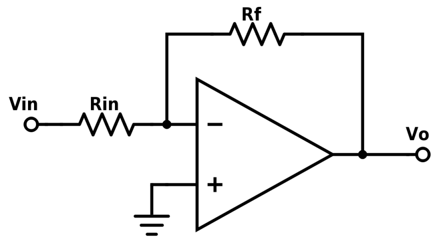


Figure 1: The circuit diagram used for the basic inverting op-amp

2.1 Summing

To sum a signal with an op-amp we can input two signals into the V_+ input, this can be shown in property 1 as we have a virtual ground at V_+, this means that:

$$\begin{aligned} I_{out} &= I_{in} \\ I_{out} &= I_1 + I_2 \\ \frac{V_{out}}{R_f} &= -\frac{V_1}{R_1} - \frac{V_2}{R_2} \\ &\boxed{V_{out} = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2\right]} \end{aligned} \tag{1}$$

Equation 1 allows us to see that we will be able to directly add the voltage signals together, but there will be a scaling factor for each of the inputted signals which depends on the resistor which is placed in the path of the signal.

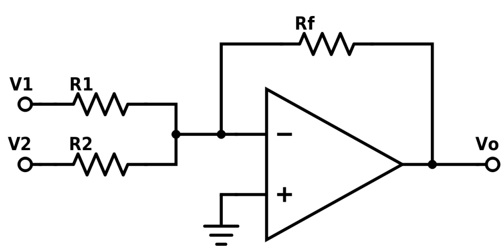


Figure 2: The circuit diagram used for summing signals from two inputs together

2.2 Differentiation

In general this op-amp has a RC network setup across the inputs which allows for the time dependant component to become significant. We can start analyzing the network:

$$Q = CV_{in}$$
$$\implies \frac{dQ}{dt} = C\frac{dV_{in}}{dt}$$

Noting $I_{feedback} = -V_{out}/R_f = I_{in}$ since we have a node at the input terminal:

$$\implies C\frac{dV_{in}}{dt} = \frac{dQ}{dt} = I_{in} = -\frac{V_{out}}{R_f}$$
$$\implies \boxed{V_{out} = -R_f C\frac{dV_{in}}{dt}} \tag{2}$$

Therefore we can now see that the output voltage of the Differentiator will be directly related to the derivative of the V_{in}, with a scaling factor related to the two RC components.

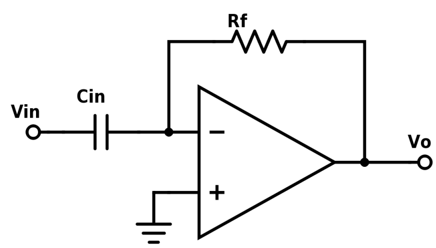


Figure 3: The circuit diagram used for Differentiating a signal

2.3 Integration

We now take a look at a system in which the feedback component is a capacitor, this feedback capacitor charges up as a function of it's RC time constant and hence the V_c which is induced across the capacitor slowly increases and this creates a linearly increasing ramp output voltage if the input is constant. However this only occurs until the capacitor is fully charged. At this point the capacitor will become essentially an infinite resistor and this will act as if R_f of an inverting op-amp is infinite. This means that gain will also be infinite.

We note that the Figure 4 has a C_{in} this is a component which is used to take out any DC current which may be introduced. Another component of the circuit diagram is the switch, which is used to create a zero gain network which will allow for the capacitor to discharge.

$$Q = CV_{in}$$
$$\implies \frac{dQ}{dt} = C\frac{dV_{cap}}{dt} = C\frac{d(V_{in} + V_{out})}{dt}$$

Noting $V_{in} = V_{ground} = 0$:

$$-\frac{1}{C}\frac{dQ}{dt} = \frac{dV_{out}}{dt}$$

From principle 2. We can say that input impedance of the op-amp is infinite and the nodal equation at the inverting input terminal is given as:

$$\begin{aligned} I_{in} = \frac{dQ}{dt} &= \frac{V_{in}}{R_{in}} \\ &= C\frac{dV_{out}}{dt} \\ \implies \frac{dV_{out}}{dt} &= \frac{-1}{R_{in}C}V_{in} \end{aligned} \tag{3}$$

$$\boxed{V_{out} = -\left(\frac{1}{RC}\right)\int V_{in}dt} \tag{4}$$

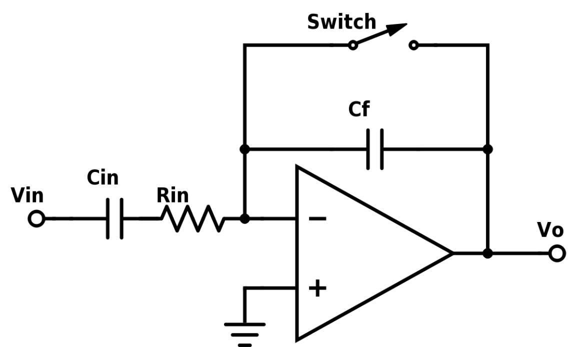


Figure 4: The circuit diagram used for integrating a signal

2.4 Exponential

We see that the exponential circuit above is directly related to the integration circuit. If we can ignore the potentimeter then we are able to isolate the integration section of the circuit. This can let us investigate the circuit starting with the basis of the integration circuit as in Equation 3.

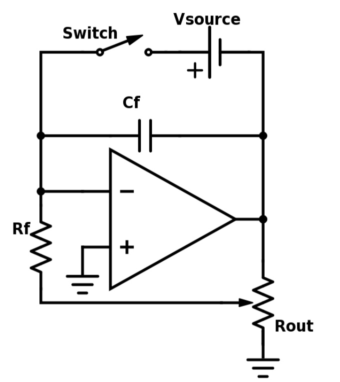


Figure 5: The circuit diagram used for creating the solution to the exponential equation

Letting $R_{pot} = 0$:

$$\frac{dV_{out}}{dt} = \frac{1}{R_{in}C} V_{in}$$

But we note that we will feed back our voltage back into itself so $V_{out} = V_{in}$:

$$\frac{dV_{out}}{dt} = \frac{1}{R_{in}C} V_{out}$$
$$\implies V = V_o e^{-t/RC}$$

However when we add in the R_{pot} the circuit becomes less simple to analyze and attempts were made to solve this circuit diagram. Taking the R_{pot} to be in series with the R_{feed} and simply summing them was the most obvious solution and lead to a solution of the form $V = V_o e^{-1/(R_{feed}+R_{pot})C}$. However this solution is not accurate as it alters the current flow and ignores the Voltage source. Another attempt to solve the circuit based off of current flow lead to an overall current of $I = \frac{V}{R_{pot}+R_{feed}}$ leads to a re-input voltage of:

$$\begin{aligned}
V_{in} &= V_{out} - R_{pot} \times \frac{V_{out}}{R_{pot} + R_{feed}} \\
&= V_{out} \left(1 - \frac{R_{pot}}{R_{pot} + R_{feed}}\right) \\
&= V_{out} \frac{R_{pot} + R_{feed} - R_{pot}}{R_{pot} + R_{feed}} \\
&= V_{out} \frac{R_{feed}}{R_{pot} + R_{feed}}
\end{aligned}$$

Inputting this into Equation 3:

$$\frac{d}{dt}V_{out} = V_{out}\frac{-1}{R_{in}C}\frac{R_{feed}}{R_{pot}+R_{feed}}$$

This leads to a solution of the form:

$$V_{out} = V_o e^{-\alpha t}\ ,\ \alpha = \frac{R_{feed}}{(R_{pot}+R_{feed})R_{in}C} \tag{5}$$

However as we later see in Section 4.4 that these solutions do not work in this form. This is most likely due to an error made in this derivation based off of the treatment of the voltage source, and how the capacitor is treated. We will see that in the analysis section an experimentally verified form is discovered, with little theoretical basis.

We do note however that on the input of the circuit we have $\frac{dV}{dt} = -\alpha V$ which are equivalent due to the feedback mechanisms described before. The derivative term exists form because of integrator built into the circuit and the alpha term is built into the circuit as how much feedback from the output of the integrator goes back into the input. We are unable to derive why this occurs, but we note that it will be directly related to the potentimeter's resistance as this is the term which alters the V_{in} from V_{out} as seen above.

2.5 Damped Harmonic Motion

As we can see in Figure 6 we are able to split up the sections of the Harmonic oscillator into several sections, each of which is described in a previous section. We note that the wave solution will be created through the feedback based in the exponential as described in the previous exponential section, and the integrator will increase the degree of it, so instead of a single exponential decay from the first order differential it will be a second order differential, which with the inverter leads to a wave solution, and with the damping will become our damped harmonic oscillator.

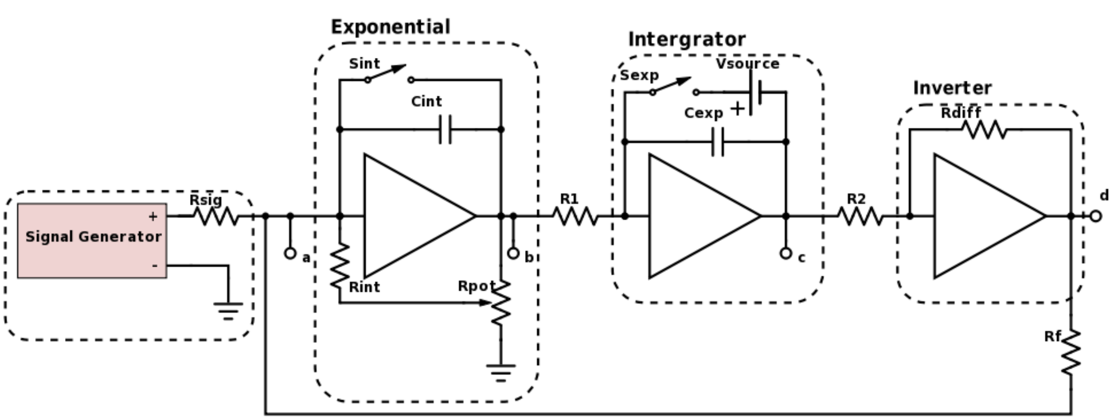


Figure 6: The circuit diagram which is used for making the solution to the Damped Harmonic Oscillator, the forced section is also included

We also note that we can pick up on sections of the solution at different points throughout the ramp as labelled. These are:

$$a = \frac{d^2V}{dt^2}$$
$$b = -\frac{dV}{dt} \times \frac{1}{RC}$$
$$c = \frac{V}{(RC)^2}$$
$$d = -\frac{V}{(RC)^2}$$

2.6 Forced Damped Harmonic Motion

We note that if we attempt to drive our harmonic oscillator with another wave we will be able to make it into a forced damped harmonic oscillator, this leads to the known standing wave solution and will result in different cases of dampening based off of the damping factor added in.

3 Experimental Background & Procedure

3.1 Apparatus

In this experiment we used a rack containing a number of op amps as the main equipment. It was here that we built all of our circuits. As well to build the circuits we had an assortment of wires, plugins and resistors and capacitors to build the circuits, and two function generators where provided. We also had an oscilloscope where we read all of our values.

3.1.1 Op-Amp Rack

This was a rack that we built all the circuits on. It was a board that contained several different op-amps as well as connection points that allowed for us to build all the circuits that we needed.

3.1.2 Function Generator

This was what we used to produce the different sine, square and triangular waves that we used throughout the experiment.

3.1.3 Oscilloscope

This was what we used to read the results of the voltages that we were meant to measure.

3.1.4 Wires/Capacitors/Resistors

We had an assortment of wires, capacitors, resistors that we used to build the circuit. The resistors had to be around $100k\Omega$ and the capacitors had to be around $0.1\mu F$. The values that were measure are listed in Table 1

3.2 Experimental Procedure

For this experiment we had to build six different op-amp circuits and test to make sure that they work as expected. The first step was to collect the resistors and the capacitors that we needed to build the circuits and measure there values. Once we had obtained and recorded the values the next step was to build the first circuit, this being the Summing.

All analysis is done through custom python software which can be found at:
`https://github.com/tokiyoshi/460_E17_Analogue_Computer`

3.2.1 Summing

For this we needed to record two input voltages and an out put voltage that should be the sum of the two input voltages. After several trials with different waves (sine, triangular, square) and some in and out of phase. The results were saved for analysis

3.2.2 Differentiating

The next circuit being the differentiating circuit. In this circuit we had one input voltage and on output voltage. The output voltage being the differentiated voltage for the input. We applied a sine, triangular and square wave to this circuit and verified it worked.The results were saved for analysis

3.2.3 Integration

Then we built the integration circuit. Again we had one input and one output and for this circuit we verified that the circuit was indeed integrating the input voltage. We first had to apply a square wave, then a sine and a triangular wave. The next thing for this was that we had to apply a DC voltage of +3V to the input to observe the voltage ramp. For this we had to add a clock to the circuit in place of the switch. The clock was a square wave going between +6V and 0V. This clock was used for the last three parts of the lab.The results were saved for analysis

3.2.4 Exponential Functions

Then we had to built this circuit; for this part we had to measure two voltages based on the given circuit and from there we had to compare them with the solution to the equation that we were given. Again we had several trials of this and the results were saved to be analyzed.

3.2.5 Damped Harmonic Motion

In this section we built the circuit provided and from that we measured the voltage at four points, to verify them against the equations that were given. We did several trials for this section and the results were saved for analysis.

3.2.6 Forced Damped Harmonic Motion

For this part we had to slightly modify the previous circuit and from that measure the amplitude in the steady state. The results were saved for analysis

3.2.7 Components

Here we will present the table of all passive electrical components used throughout this lab.

Component	Value
R_1	$103.78 \pm .05\ \Omega$
R_2	$110.60 \pm .05\ \Omega$
R_3	$104.24 \pm .05\ \Omega$
R_4	$99.15 \pm .05\ \Omega$
R_5	$99.29 \pm .05\ \Omega$
R_6	$99.24 \pm .05\ \Omega$
C_1	$102.54 \pm .05$ nF
C_2	$97.90 \pm .05$ nF
C_3	$97.05 \pm .05$ nF
R_{DC}	$2.28 \pm .01$ V

Table 1: The values our components are stated here.

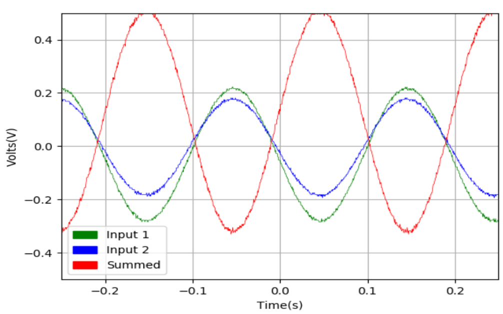
4 Analysis

4.1 Summing

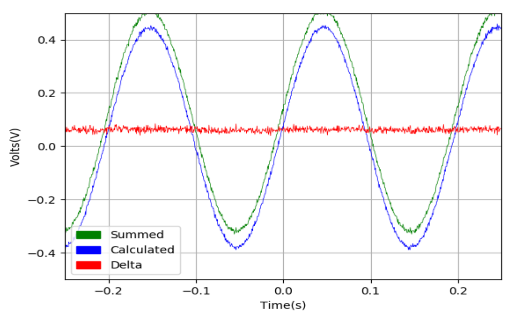
Initially we take the circuit built in Figure 2 and we then inputted signals into the summer to see how well the summed signals compared to the theoretical signals which we expected to get. We graph the inputted signals and the outputted data on the same graph, and then we numerically sum the data and compare the theoretical signal to the outputted data by taking a difference of the two values. This gives us our second graph shown on the right.

In Equation 1 it can be seen that the the Voltages are scaled by a factor, this factor was calculated by referencing Table 1 and noting that $R_1 = R_1$, $R_2 = R_2$ and $R_f = R_3$. These gains are nearly 1 in both cases unity, and are included for completeness, but we saw that they were small enough to not noticeably change the shape of the data.

Here we will show multiple graphs, starting with sine waves at various frequencies:

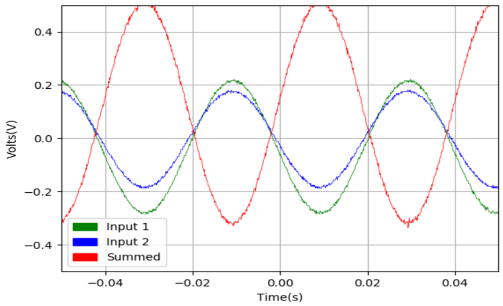


(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)

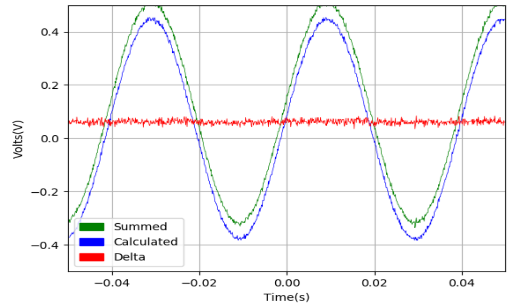


(b) The summed raw data (V), calculated signal -(v_1+v_2) & difference between them (delta).

Figure 7: Sine waves Summed with 0 phase difference, 5Hz signal



(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)

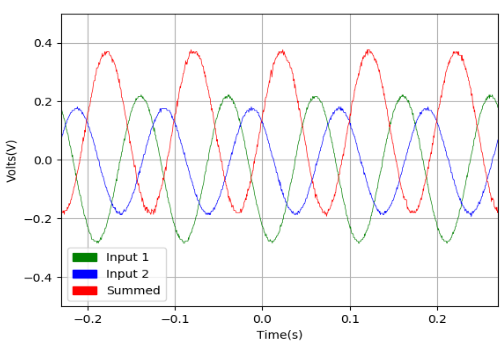


(b) The summed raw data (V), calculated signal -($v_1 + v_2$)& difference between them (delta).

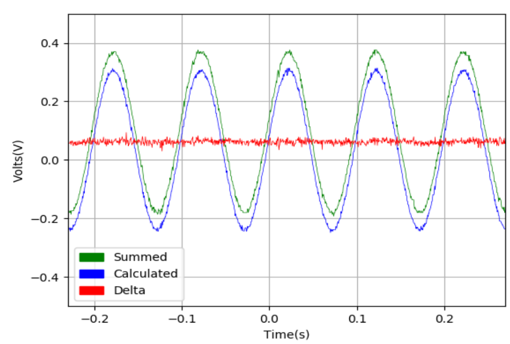
Figure 8: Sine waves Summed with 0 phase difference, 25Hz signal

From Figures 7 & 22 we can see that the difference between 25Hz and 5Hz signals show no clear sign of difference between the two, if there is any significant difference it is dominated by the noise of the signal. We do notice that there is a DC difference between the two signals, which is caused by the digital DC offset which was set by ourselves in the lab to differentiate the signals visually. However this does not explain all of the signal as these DC offsets lead to a 83.5mV difference from zero, but we see a consistent 61 $\pm$2 mV signal found from the delta. This implies that there is a small DC element which is being added into the summed signal.

We now take a look into phase differences, we focus first on the sine waves again as a test case:



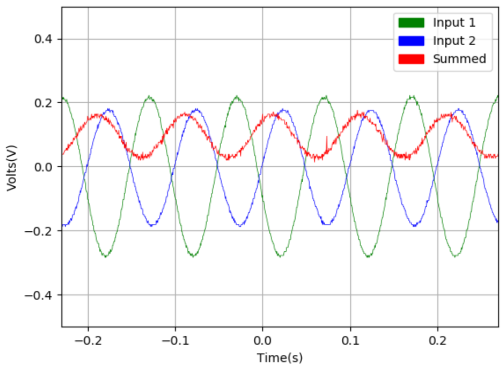
(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)



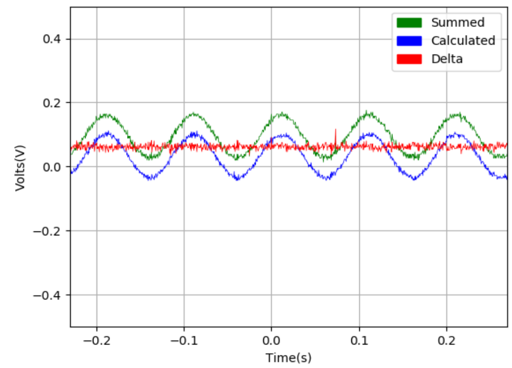
(b) The summed raw data (V), calculated signal -($v_1 + v_2$)& difference between them (delta).

Figure 9: Sine waves Summed with 90° phase difference, 10Hz signal

We see that the phase difference appears to make no visible difference to the amount of difference between the calculated and found signal. This is concluded by observing the difference in size and shape of the delta signal between Figures 9 & 10. However as a percentage of signal it is important to note that deconstructive interference lead to the case where the noise in the delta signal is significant compared to the signal (Figure 13).



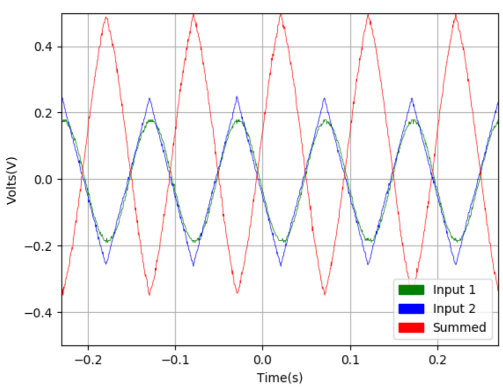
(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)



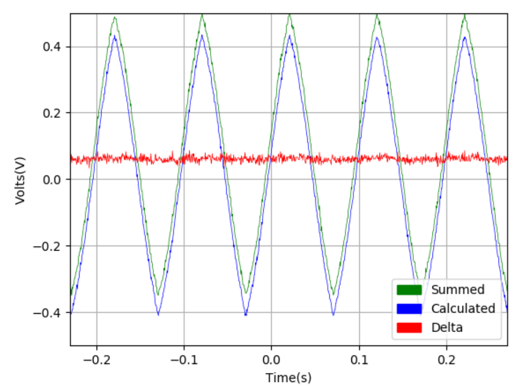
(b) The summed raw data (V), calculated signal -($v_1 + v_2$)& difference between them (delta).

Figure 10: Sine waves Summed with 170° phase difference, 10Hz signal

At this point we also note that many other combinations of signals were completed and are presented in Appendix A. These were also done with varying phase factors, but these simply followed the pattern of the points stated above. We show three examples here, the first two are shown in Figures 31 & 12 shows that the addition of non-sine wave functions works similarly to summations seen of sine waves, even with odd phase differences.



(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)



(b) The summed raw data (V), calculated signal -($v_1 + v_2$)& difference between them (delta).

Figure 11: Sine wave summed with a triangular wave, 10Hz signal

Then we take a look at Figure 13. This leads to our signal due to deconstructive interference, but the noise still dominates any significant issues that we are able to see in the data. The variance of each of the deltas is seen to be constant, leading us to conclude that the different setups did not cause significant differences in output.

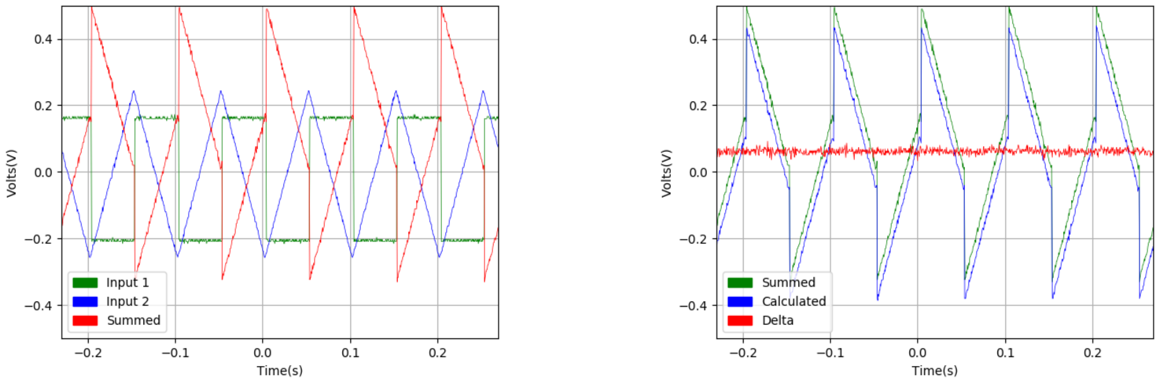
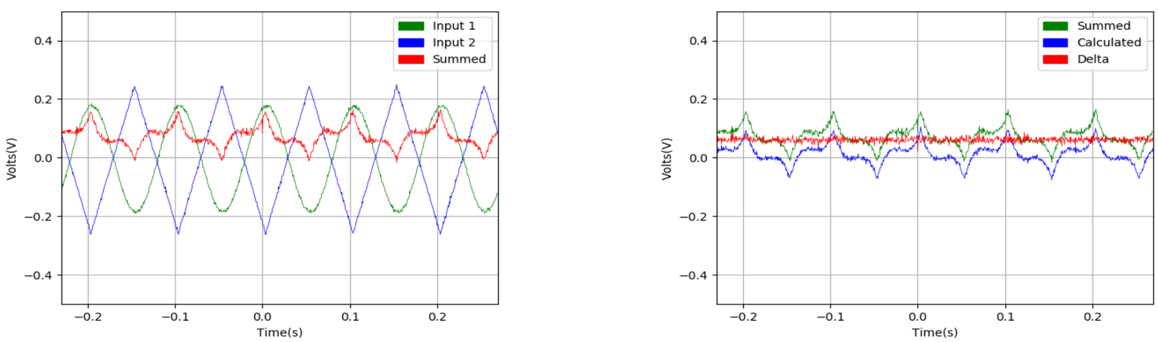


Figure 12: Square summed with a triangular, 10Hz signal, off phase by $\sim 1/2$ wave



(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)

(b) The summed raw data (V), calculated signal -(v_1+v_2)& difference between them (delta).

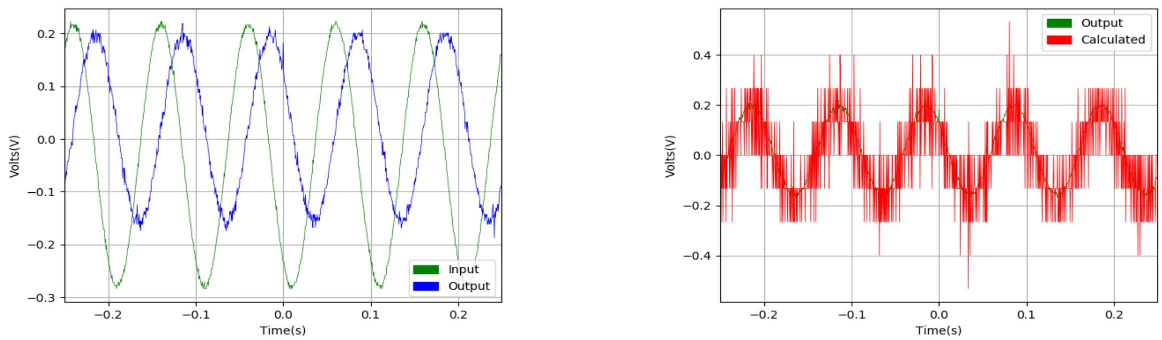
Figure 13: Sine summed with a triangular, 10Hz signal, off by $\sim 1/2$ a wave

4.2 Differentiation

With the circuit built in Figure 35 the inputted signals are taken, and the output is also taken then each of those signals are graphed against one another. Then a numerical differentiation is completed using custom Python software, which is simply calculated by finding the difference between the last point and current point. These are then appropriately scaled according to the known constant factors in Equation 2 based off of the reference values in Table 1. This scaling factor is -$R_1 \times C_1$.

We first present the data found for the sine wave:

We see that in Figure 14a we have a signal which looks to be differentiated, and we see that it creates a sine wave with a phase difference. This phase difference appears to be 90°, which is consistent with how an inverted cos wave would appear, which is in fact what the differentiated signal should be, due to the negative scaling factor from the inverting op-amp.



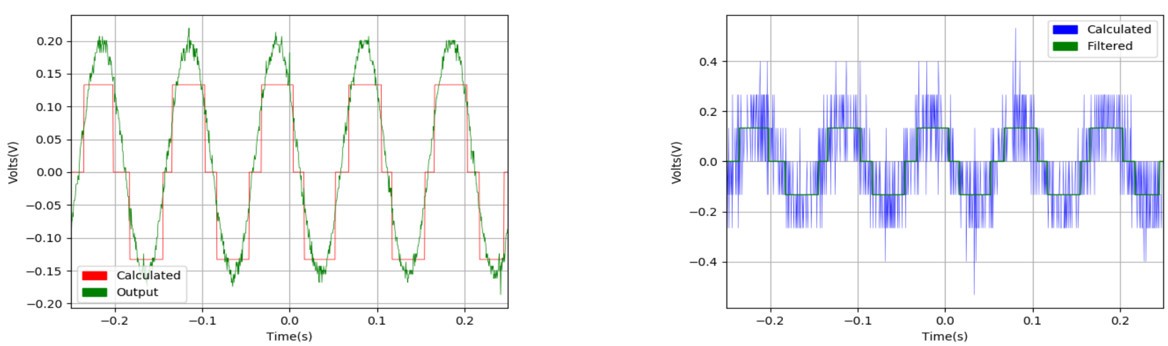
(a) The raw data from the oscilloscope. Inputted and Differentiated signal

(b) The summed raw data (V), and the numerically differentiated data.

Figure 14: Differentiated sine wave with numerical differentiation compared to the analogue differentiation

Figure 14b is striking, as we see that it is highly noisy, and if detail is paid to the values it can be seen that the points only lie on a “quantized” levels. These levels are likely due to the ADC which is built into the oscilloscope itself, meaning that the numerical differentiation will be limited by these levels as well.

Here we present the results of filtering which will clean up our data:



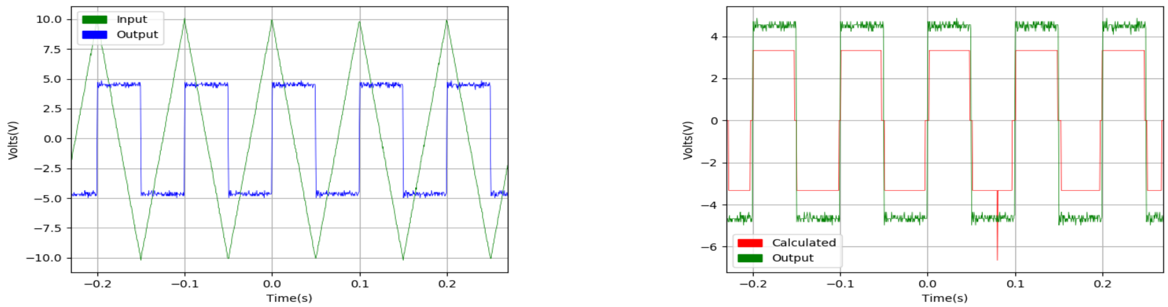
(a) The raw data output of the oscilloscope compared with the filtered numerical result.

(b) The numerical differentiation compared with the filtered numerical result.

Figure 15: Filtering of the sine wave to show the numerical differentiation.

Here we have shown the effect of filtering. This filtering is median filtering which simply takes the median value which is found in the window. Hence there is no interpolation, which means that we can clearly see the “quantized” levels created by the ADC. We can also see that it generally follows the trend of the expected graph. I will note that the median filter is of 41 points which relates to a 20.5 ms window, and the waves appear to be in phase, although it is difficult to establish as the filtering obscures the peaks.

We will next look at differentiating the triangular wave in Figure 18. We see in this that the triangular wave differentiated appears to be a square wave, this is what we would expect. We see that the amplitude of the numerically differentiated signal is also below that of the actual signal which is not surprising due to the issues discussed earlier.

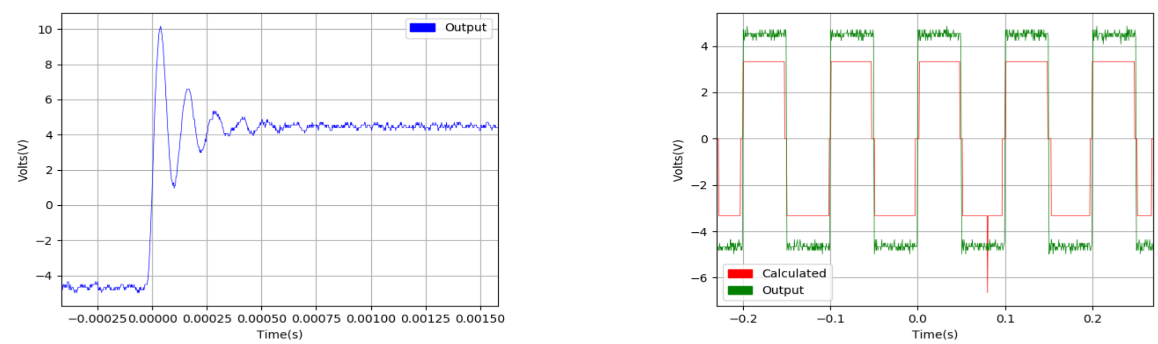


(a) The raw data input and output of the scope

(b) The filtered numerical differentiation compared with the output signal.

Figure 16: Differentiated triangular wave signal.

From visual analysis we are also able to pick up another detail of the analogue computer. For the triangular wave this is shown in the Figure 17a. We are able to see the RC bounce which occurs because of the analogue circuitry overshooting equilibrium and then stabilizing down to it. We note that this occurs over about 1 ms, which is 10% of the RC time constant which is 10.64 ms.

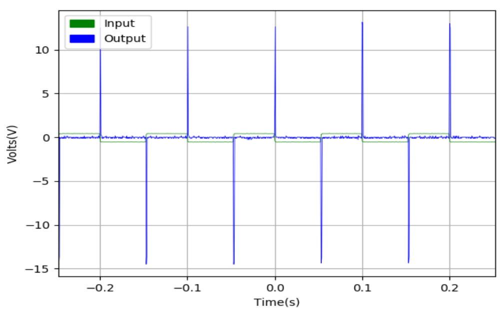


(a) The raw data output of the triangular wave differentiation, showing RC bounce.

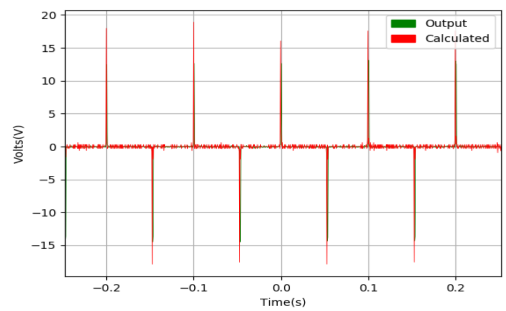
(b) The raw data output of the square wave differentiation compared to the calculated

Figure 17: Analogue effects seen in the circuitry.

We now take a look at the square wave input. We see that the square differentiated signal creates a large dirac delta peak, which is again as expected. We do not median filter this data as median filtering would remove the sharp peak. In the unfiltered data we see that the peak is bigger in the calculated data then it is in the real data. When looking at this effect at a higher resolution we see Figure 17b. This is why we see the peak as lower and wider than the real delta peak. We see that this analogue effect of the capacitor discharging is causing this.



(a) The raw data input and output of the scope



(b) The numerical differentiation compared with the output signal.

Figure 18: Differentiated square wave signal.

In the Appendix B we present the results of the DC coupling. We see in Figures 41 and 42 that we are unable to differentiate between the two situations. This is caused by the capacitor on the input of the differentiation circuit which means that the DC vs AC coupling will have no difference as the capacitor will remove any DC component to the signal anyways.

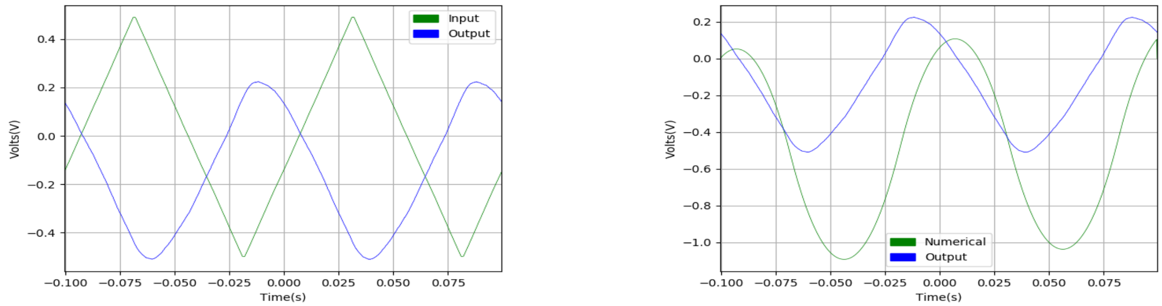
4.3 Integration

For this part of the lab we take the circuit built in Figure 4 and we then inputted signals into the integrator, to see that the signals were integrated. We then verified that the output signal was what we expected before we saved the data.

In Equation 4 it can be seen that the the Voltages are scaled by a factor, this factor was calculated by referencing Table 1 and noting that $R_2 = R_2$ $C_3 = C_3$ and $C_b = R_2$.

Here we will show multiple graphs frequencies:

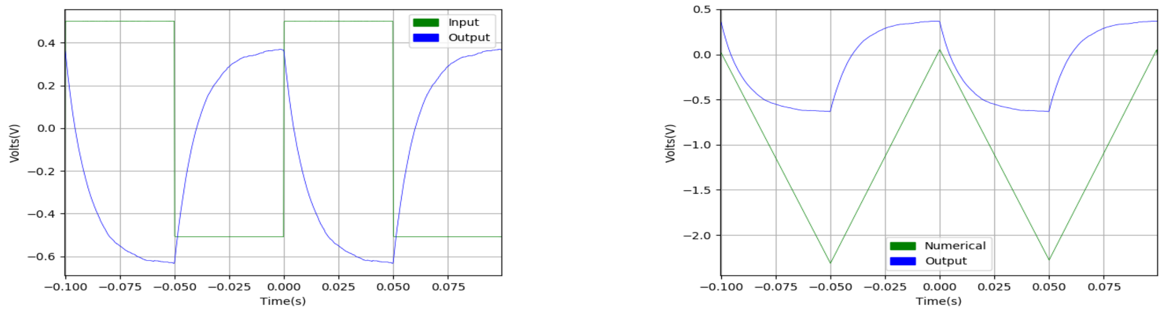
Here we see that the calculated integrated wave and the raw output that was taken from the oscilloscope do not match very well, though the standard shape of each is kept. Th reason for this discrepancy may be because of the slew rate. The slew rate in an op-amp is the rate in change in the output voltage that is a direct result of the change in the step of the input voltage. This would lead to the distortion in the plots that we see.



(a) The raw data from the oscilloscope. Inputs (V1) and the Integrated Signal (V0)

(b) The integrated raw data (V), calculated signal -($\frac{-1}{RC}\int dV_i dt$).

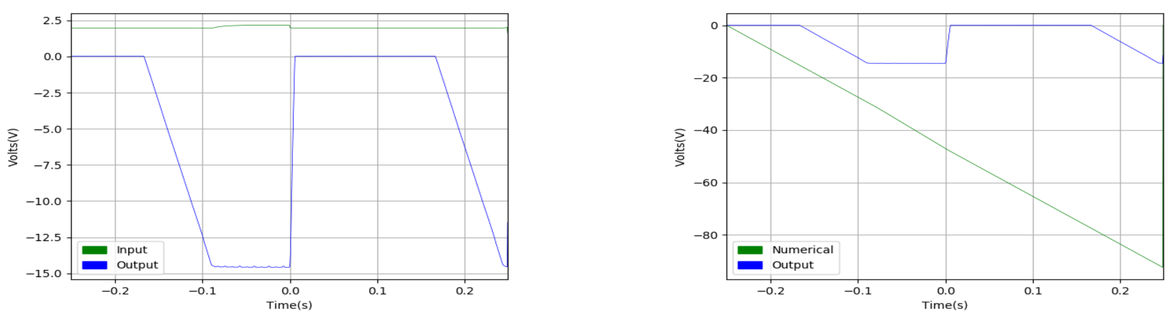
Figure 19: Triangular waves



(a) The raw data from the oscilloscope. Inputs (V1) and the Integrated Signal (V0)

(b) The integrated raw data (V), calculated signal -($\frac{-1}{RC}\int dV_i dt$).

Figure 20: Square Waves

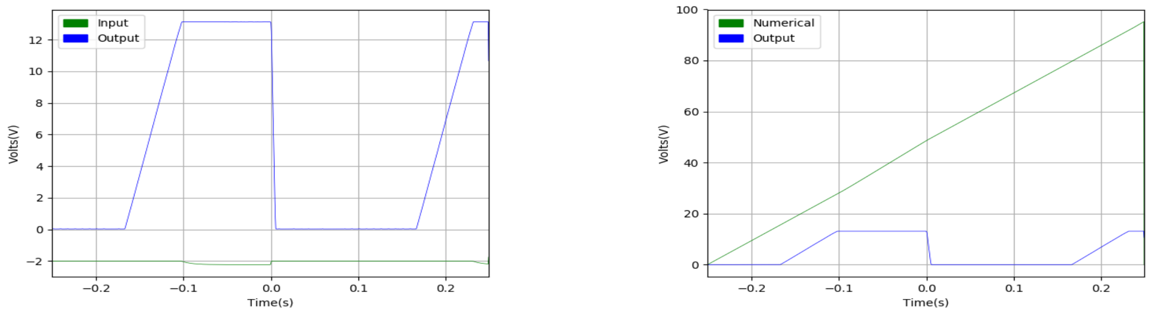


(a) The raw data from the oscilloscope. The input is constant and the Integrated Signal (V0)

(b) The integrated raw data (V), calculated signal -($\frac{-1}{RC}\int dV_i dt$).

Figure 21: A constant DC voltage was sent with the switch being controlled by a clock

The next part of this section we had to apply a DC voltage to the circuit to observe the voltage ramp at the output. We used a clock pulse in this to reset the voltage in place of the switch, that is why when you look at the output data you see that it comes in pulses. What we are looking for is the ramp that occurs before the new pulse. This is compared to the calculated ramp that we have, looking at the two graphs we see that the rate at which the two increase is very similar.



(a) The raw data from the oscilloscope. The input is constant and the Integrated Signal (V0)

(b) The integrated raw data (V), calculated signal -($\frac{-1}{RC} \int dV_i dt$).

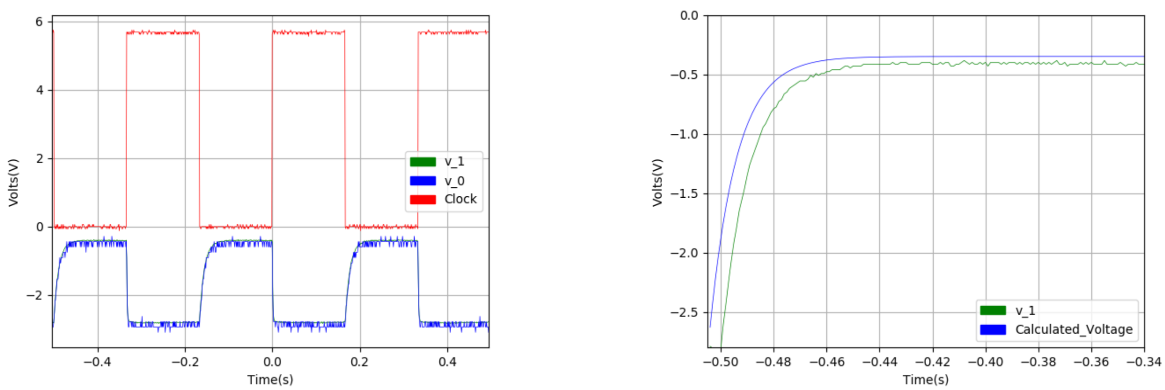
Figure 22: A constant DC voltage was sent with the switch being controlled by a clock

In the two plots above we see a positive and negative constant input. We see from he graph that when you flip the input voltage from positive to negative. Our Oscilloscope was DC coupled which means that which means the it allowed for both the AC and DC current to pass through. This means that there is no extra capacitor that filters the DC current.

4.4 Exponential

We now setup the circuit as it is seen in Figure 5. This circuit will provide an exponential decay in the inputted signal as the signal is slowly reduced over time by the pentameter's resistor. We initially note that the solution will be of the form of $V_{out} = V_{in} = -RCddtV_{out}$ therefore the solution to the Voltage will be: $V = V_o e^{-1/RC}$.

Therefore we graph the expected solution alongside the found signal, taking probes at before and after the op-amp. We show this here:



(a) The raw data from the oscilloscope showing the signal before and after the op-amp.

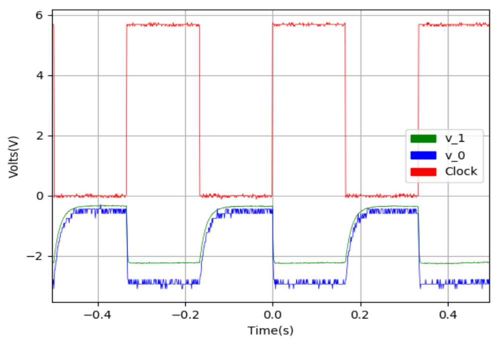
(b) The exponential graph along with our calculated signal.

Figure 23: The exponential solution of our circuit along with our expected signal.

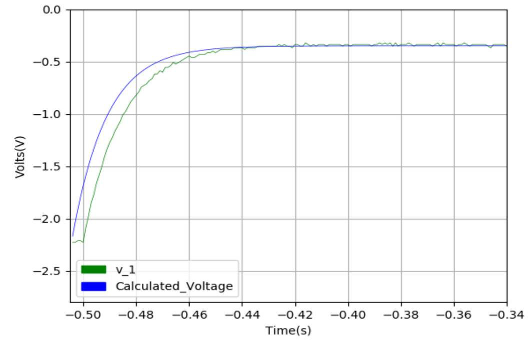
We see here that in Figure 23b we have a very similar signal between our calculated and derived signal, however our solution is vertically and horizontally shifted. The horizontal shift is necessary as we start our exponential signal at the start point where the clock allows for the circuit to begin the exponential decay. However the vertical shift has less basis, we believe that the circuit should decay to 0 V, but instead we see that the voltage stays at -0.35 Volts. This deviation from theory is odd, and cannot be explained by digital shifts. It is also possible that the oscilloscope and the rest of the analogue circuitboard are at different grounds, but we see that the clock signal is properly set to zero therefore it is unlikely that the op-amp is referencing an incorrect ground.

Moving on from this circuit we note that a solution of the form of $V = -(1-\beta)\, e^{\frac{-(1-\beta)}{RC}}$ where $\beta = \frac{R_pot}{R_o}$ where R_o is the maximal resistance of the resistor. This presents us with solutions for our graph with progress as:

We then present our solution and we can see that the correlation between our guessed solution and the experimentally found solution is strong. We see that the solutions for the lower ranged β (0.20-0.60, as presented in Figure 24 to 26)) correlate well.

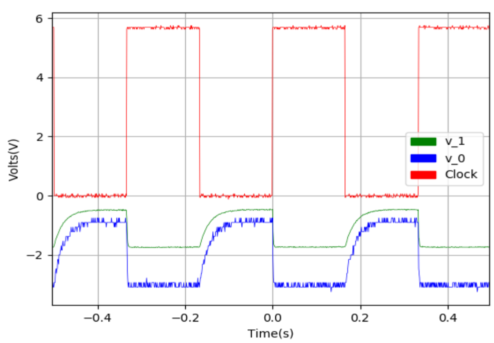


(a) The raw data from the oscilloscope showing the signal before and after the op-amp.

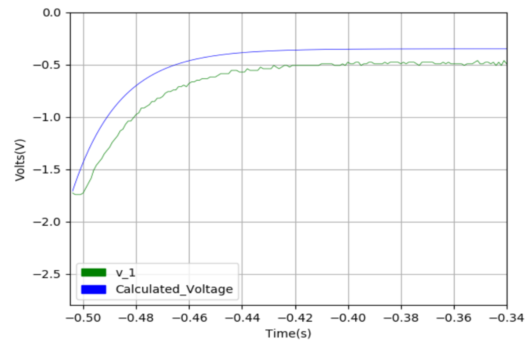


(b) The exponential graph along with our calculated signal.

Figure 24: The exponential solution of our circuit along with our expected signal for $\beta = 0.20$.

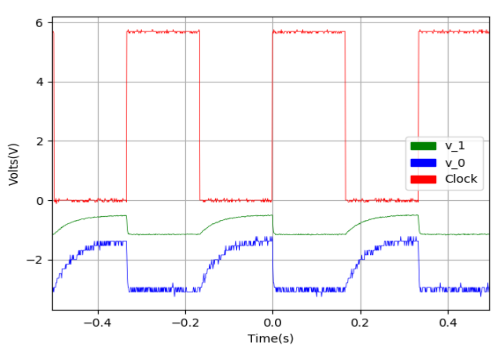


(a) The raw data from the oscilloscope showing the signal before and after the op-amp.

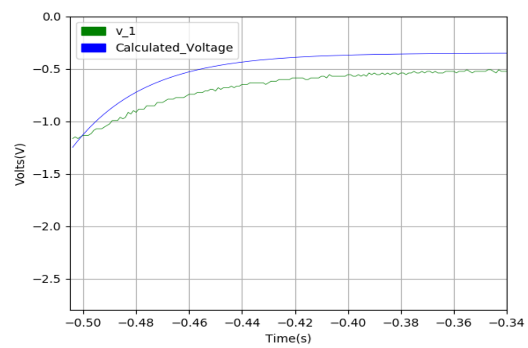


(b) The exponential graph along with our calculated signal.

Figure 25: The exponential solution of our circuit along with our expected signal for $\beta = 0.40$.

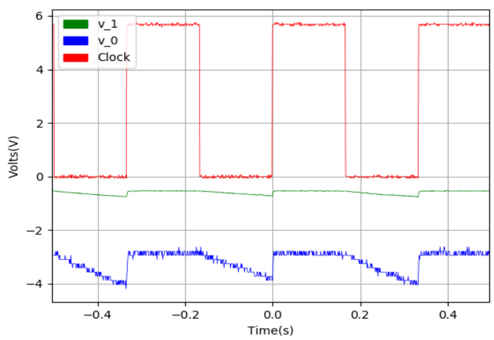


(a) The raw data from the oscilloscope showing the signal before and after the op-amp.

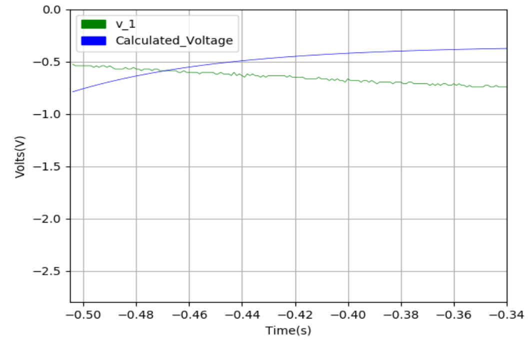


(b) The exponential graph along with our calculated signal.

Figure 26: The exponential solution of our circuit along with our expected signal for $\beta = 0.60$.

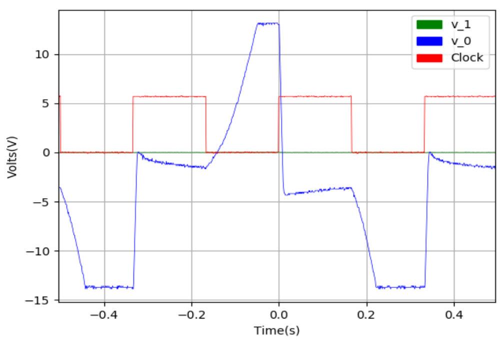


(a) The raw data from the oscilloscope showing the signal before and after the op-amp.

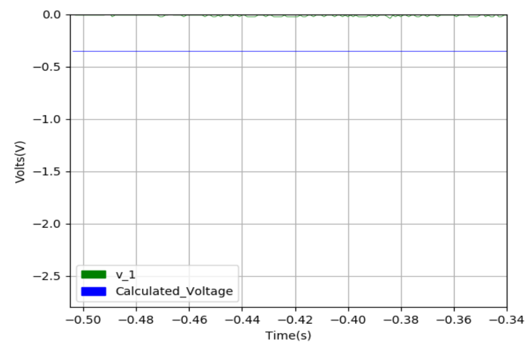


(b) The exponential graph along with our calculated signal.

Figure 27: The exponential solution of our circuit along with our expected signal for $\beta = 0.80$.



(a) The raw data from the oscilloscope showing the signal before and after the op-amp.



(b) The exponential graph along with our calculated signal.

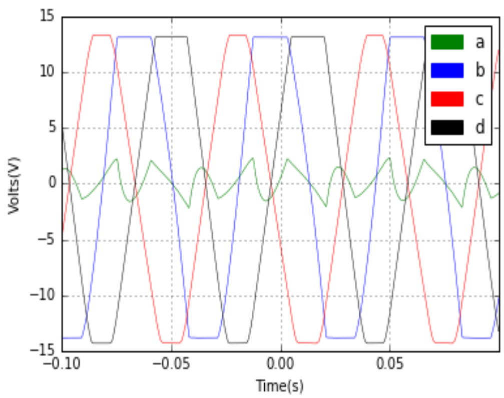
Figure 28: The exponential solution of our circuit along with our expected signal for $\beta = 1.00$.

However when we take a look at Figures 27 and 28 we see that the theory does not fare as well, however this is not unexpected as we are allowing very little of the original signal to be fed back into the input of the circuit again. This means that the signal is very low and the effect of ground noise and other imperfections in the op-amp and overall circuitry will become significant to the overall operating of the signal. To confirm this we see that the behaviour of the signals is relatively preserved in our limiting case (no feedback) of Figure 28b, even though the very low signal case Figure 27b has a different slope than is predicted by exponential decay.

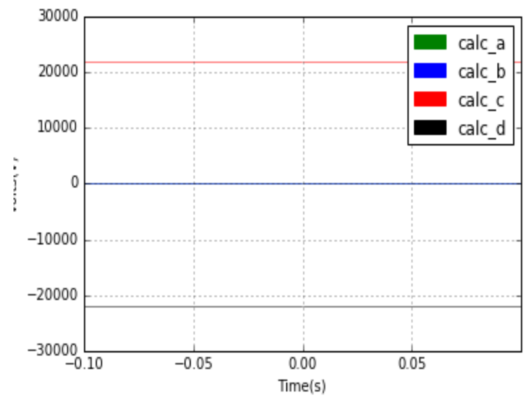
We also note that the signal which is being picked up as v_0 in Figure 28a is unexpected, but it is not surprising that in the limiting case of a signal that there may be odd signal behaviour in the amplified integrated signal. However this behaviour is not explainable by us, but seems to be related to the capacitor based off of the signal's curvature, also it looks to be hitting the op-amp's rail voltages.

4.5 Damped Harmonic Motion

For this part of the lab we essentially combined all of the previous circuits, to make a circuit that has the solutions to damped harmonic motion, that as the equation $\ddot{V} + \frac{b}{m}\dot{V} + \frac{k}{m}V = 0$ At specific parts of the circuit we would measure the voltage and the plots below show our results. Those points we marked as a, b, c, d, where there respective values were $\frac{d^2V}{dt^2}, \frac{-\frac{dV}{dt}}{RC}, \frac{V}{(RC)^2}, \frac{-V}{(RC)^2}$. The plots below detail the results that we found.

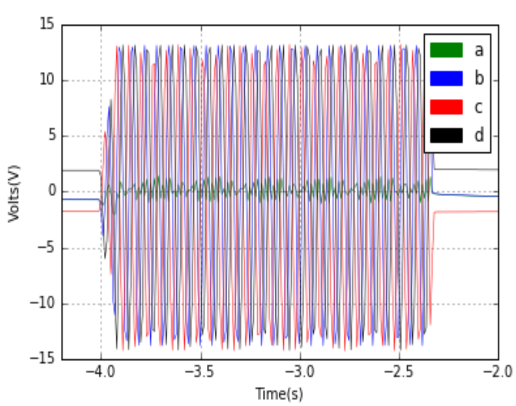


(a) The raw data from the oscilloscope, where a,b,c,d are all the voltages that are read

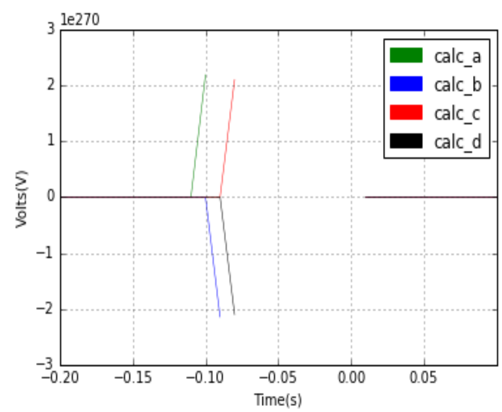


(b) The calculated values of a,b,c,d based on the corresponding equations

Figure 29: For this on $R = 0.00k\Omega$



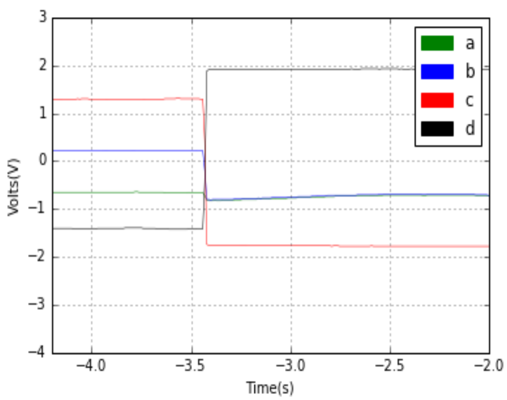
(a) The raw data from the oscilloscope, where a,b,c,d are all the voltages that are read



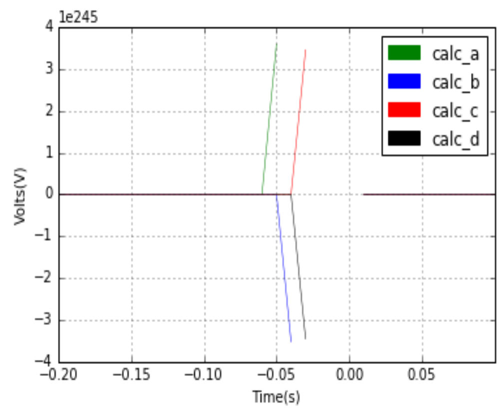
(b) The calculated values of a,b,c,d based on the corresponding equations

Figure 30: For this on $R = 9.91k\Omega$

It is evident from the graphs that the measured values do not match what we expect for a damp harmonic system. While completing the lab we looked at what could have caused this. The first thing that we looked at was to ensure that the circuit was properly built, which was the case. We then checked that each resistor and capacitor was of a correct value, that was not the cause, we kept investigating looking at the wires, the clock and were unable to determine the exact cause of the results that we obtained.



(a) The raw data from the oscilloscope, where a,b,c,d are all the voltages that are read

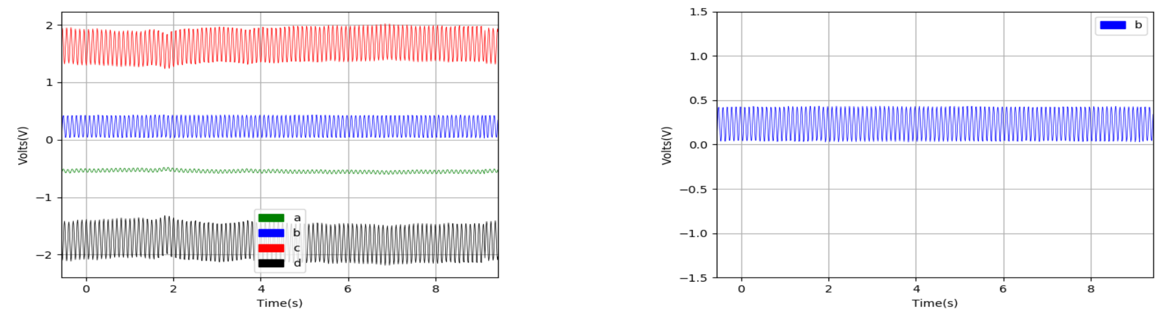


(b) The calculated values of a,b,c,d based on the corresponding equations

Figure 31: For this on $R = 3.37k\Omega$

4.6 Forced Damped Harmonic Motion

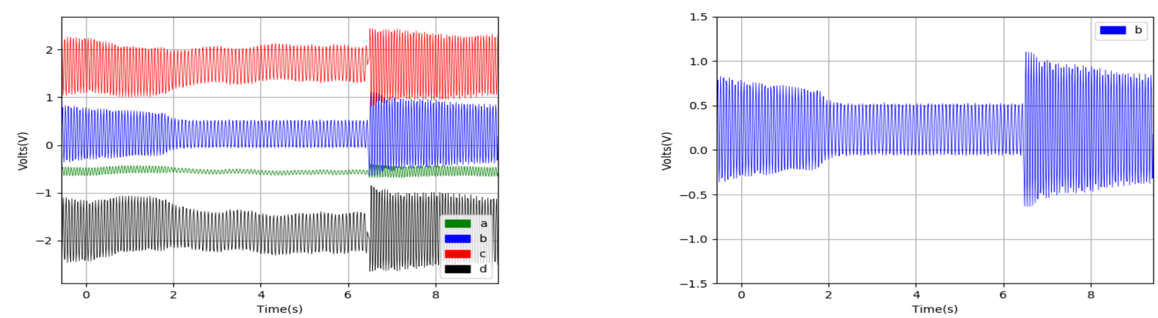
We can now add a signal generator which mean that the damped harmonic oscillator will reach a steady state. Therefore we have taken a ramp of the frequencies, and attempted to find the steady state amplitude for each. However due to issues with the Damped harmonic oscillator, we are unable to easily identify the exact section of steady state. We define the steady state to be reached at the middle of the graph (4-6 seconds) as before this we do not reach steady state in some of the graphs. This is seen best in Figure 33. Also we see the damped harmonic oscillator fails after a certain point in Figure 34, hence we attempt to capture the wave before this.



(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 32: The forced damped harmonic signal seen at 10Hz.

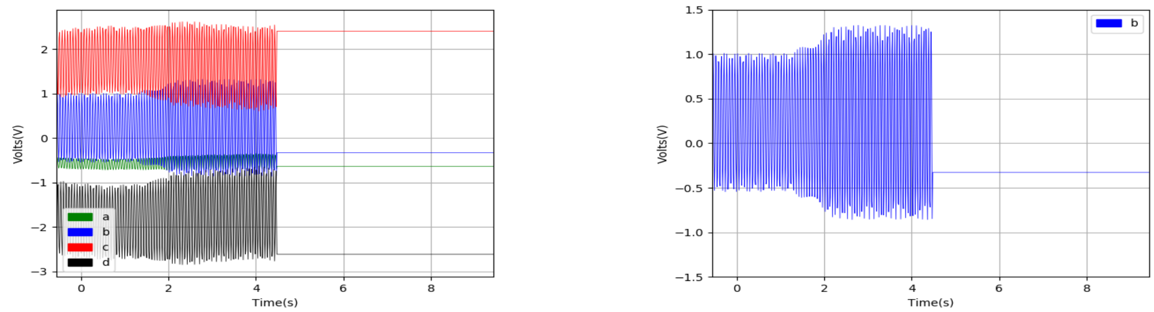


(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 33: The forced damped harmonic signal seen at 12Hz.

From each of these frequencies we get an amplitude of the signal from 4-6 seconds by finding the median values the overall function and finding the max and minimum values in the regions, by calculating the deviation from it, while ignoring outliers we can get a rough guess at the amplitude. All other graphs of the other frequencies are shown in Appendix C



(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 34: The forced damped harmonic signal seen at 18Hz.

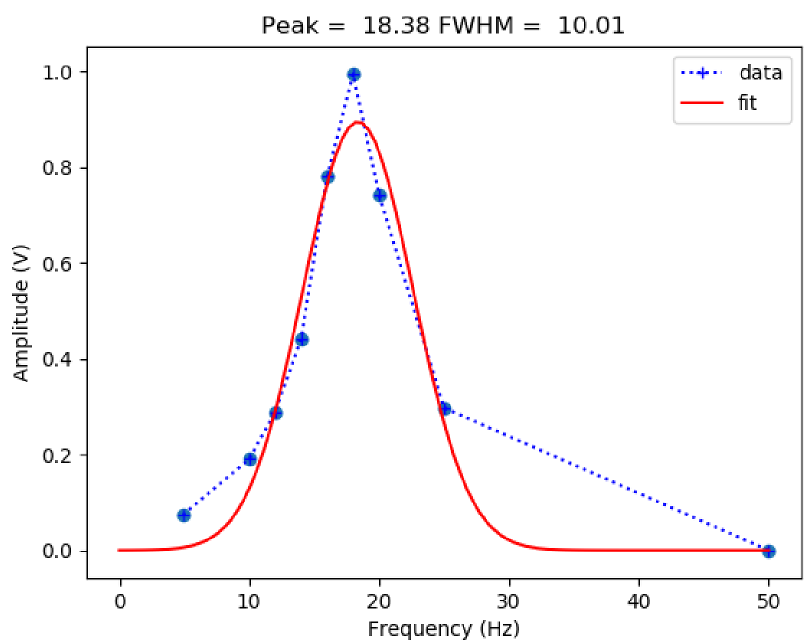


Figure 35: The frequency ramp to find the Quality Value

We use a 2.47 kΩ resistor in our system while we pass through the frequencies. This allows us to calculate the theoretical quality factor as $1/\beta$ and $\beta = \frac{2.47k\Omega}{9.91k\Omega}$ which $\implies Q_{theory} = 4.01$, but we find our $Q_{actual} = 1.838$ This implies that the damping is not functioning as expected, with damping occurring at a much higher rate than what would be expected. Since we can see that there are parts of the circuit that are non-ideal and are feeding into the damping of our signal, we believe that these unknown factors are causing our large deviation from theory.

5 Conclusion

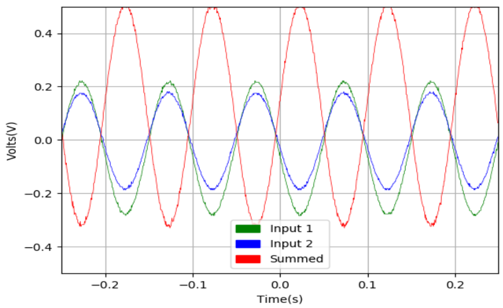
The purpose of this lab was to investigate the analogue computer through six different circuits.The first circuit was a summing circuit, we had two input voltages and the output would be the sum of the two circuits. When the data was analyzed we found that it fit well with the theory presented. Although due to our investigation with the summation circuit it is likely that the higher the frequency the less accurate a summed signal will be. It is recommended that a low frequency is set, we would recommend 5Hz based off our initial tests. The next circuit that we we looked at was the differentiating circuit, for this we had to filter the results and once we had done so we saw that the the results fit the theory. We then looked at the integrating circuit, here we found that the output voltage and the calculated voltage were not the same and we attribute it to the slew rate of the op-amp. The next circuit dealt with exponential functions and on this on one we determined that were we found that for some of the data that we obtained the results did not fare well with the theory. This however is not unexpected as we are allowing very little of the original signal to be fed back into the input of the circuit. Also we were unable to fully explain the unexpected signal that we obtained for Figure 28a, and thing that it may be due to the capacitor based off of the signals curvature. The data obtained for the damped harmonic motion did not allow us to properly gain insight into this circuit. The plots that we obtained did not follow the expected plots for damped harmonic motion. We were able to determine whand finallycause this but are unable to fully explain the discrepancy. we looked at forat did not ced damped harmonic motion were we found the $Q_{actual} = 1.838$ while the $Qtheory = 4.01$.

References

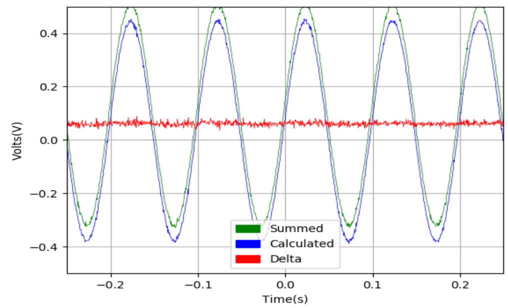
[1] G. Scholz, *Analogue Computer*. https://learn.uwaterloo.ca/d2l/le/content/345844/viewContent/1940294/View. Accessed: 2017-10-23.

Appendices

A Summation



(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)



(b) The summed raw data (V), calculated signal -($v_1 + v_2$)& difference between them (delta).

Figure 36: Sine waves Summed with 0 phase difference, 10Hz signal

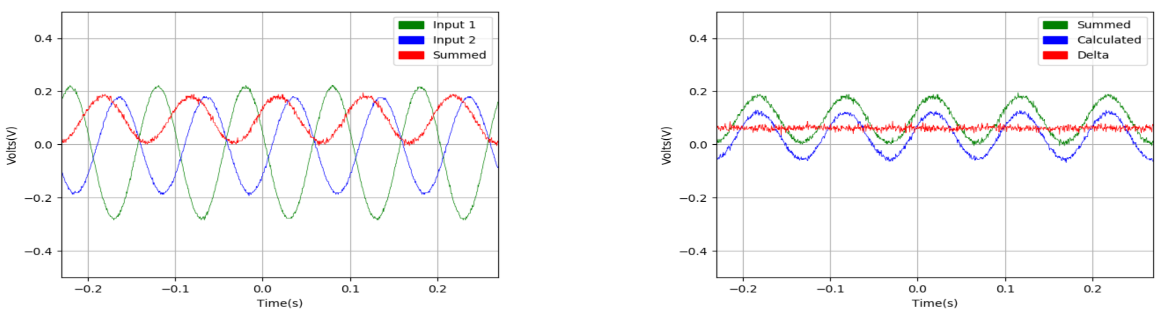


Figure 37: Sine waves Summed with 180° phase difference, 10Hz signal

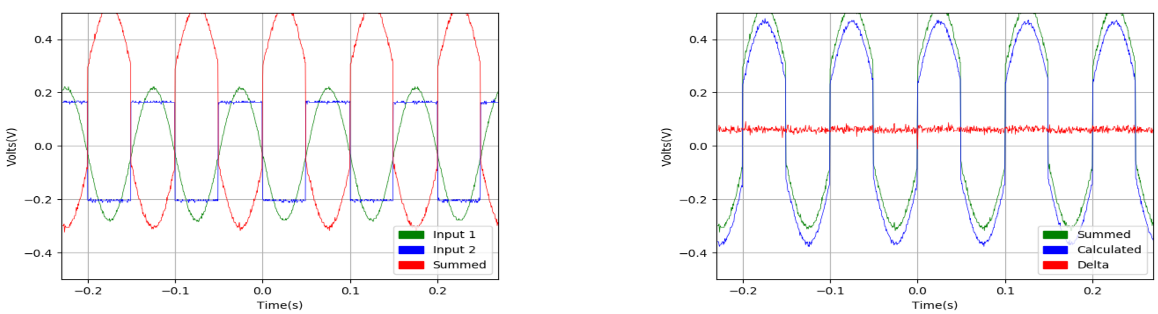
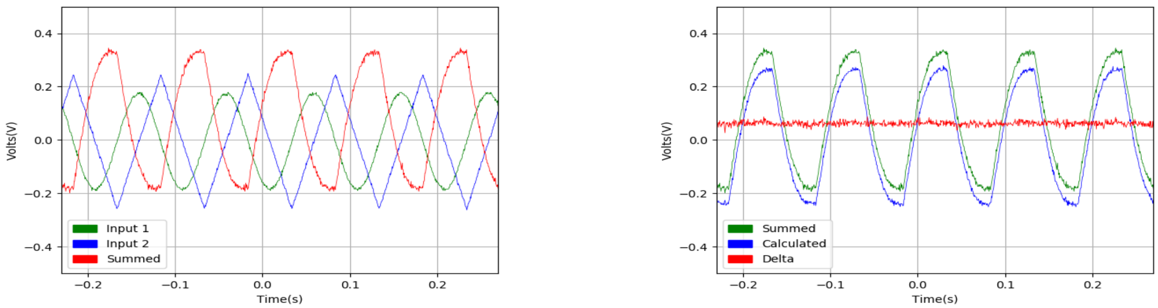


Figure 38: Sine wave summed with a square wave, 10Hz signal



(a) The raw data from the oscilloscope. Inputs (V1, V2) and the Summed Signal (V0)

(b) The summed raw data (V), calculated signal -($v_1 + v_2$)& difference between them (delta).

Figure 39: Sine summed with a triangular, 10Hz signal, off by $\sim 1/4$ a wave

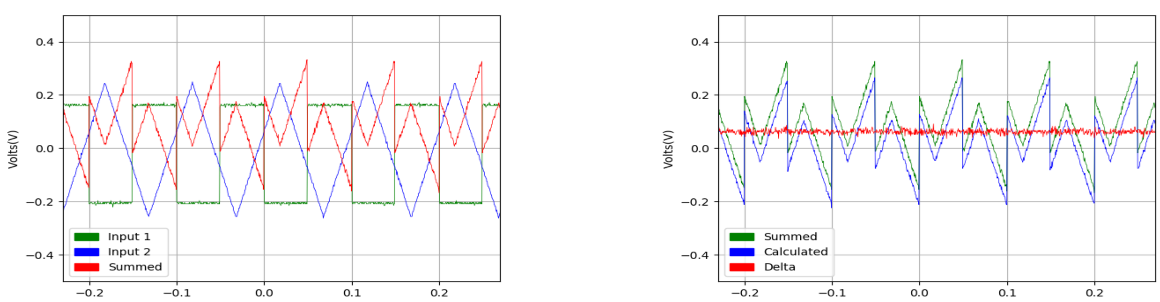
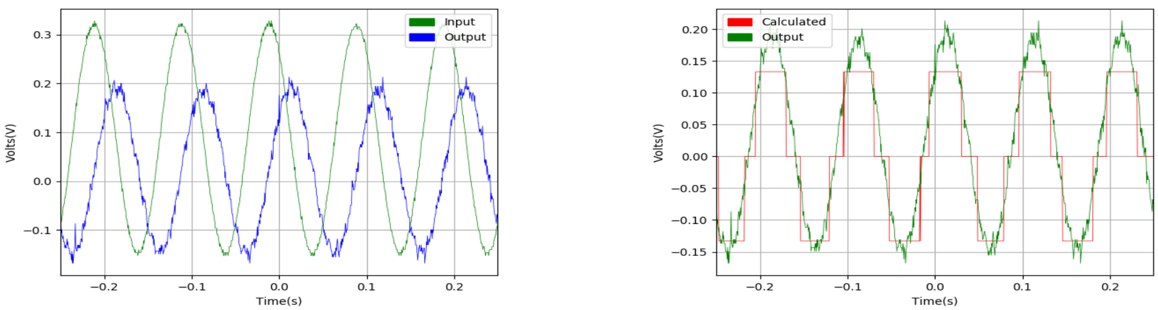


Figure 40: Square summed with a triangular, 10Hz signal off phase by $\sim 1/4$ wave

B Differentiation

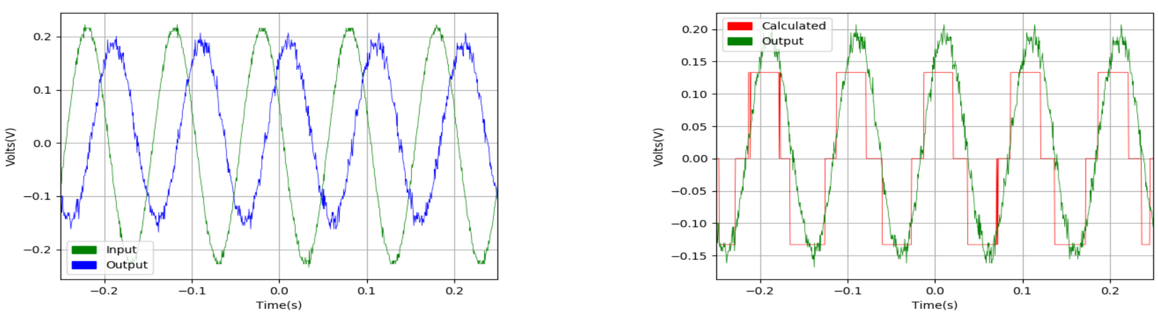
Here we will present the results of the DC vs AC coupling



(a) The raw data input and output of the scope

(b) The numerical differentiation compared with the output signal.

Figure 41: DC coupled with DC offset.



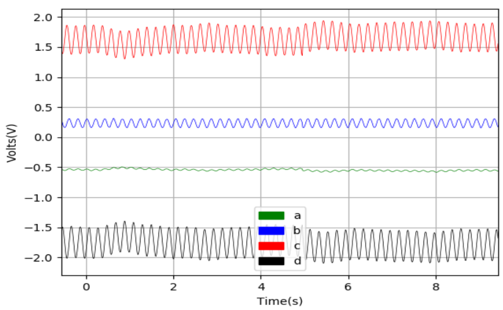
(a) The raw data input and output of the scope

(b) The numerical differentiation compared with the output signal.

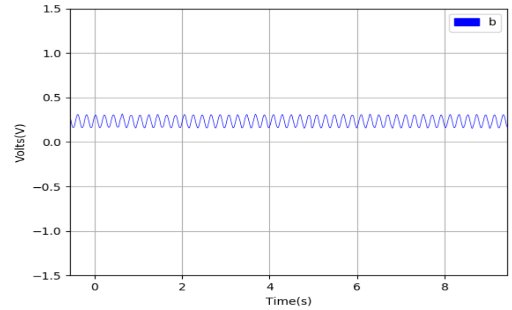
Figure 42: AC coupled with DC offset.

C Forced Damped Harmonic Motion

Here we will present the other results not shown in Section 4.6.

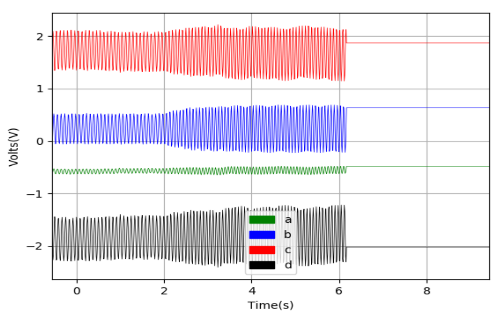


(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

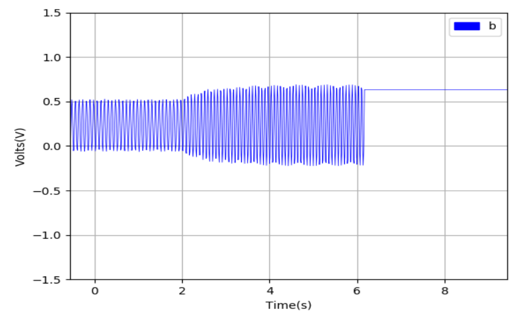


(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 43: The forced damped harmonic signal seen at 5Hz.

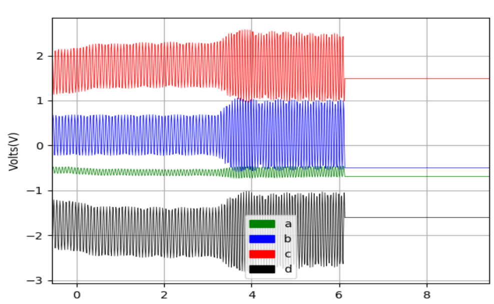


(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

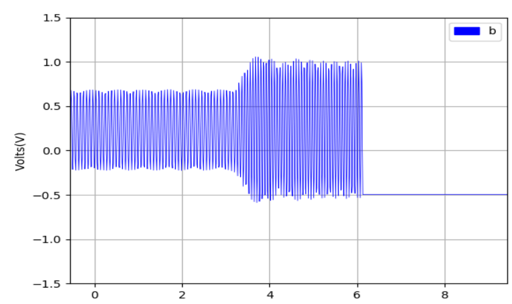


(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 44: The forced damped harmonic signal seen at 14Hz.

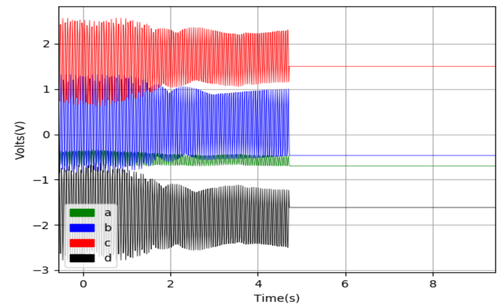


(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

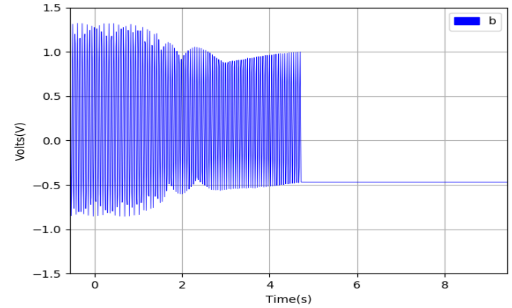


(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 45: The forced damped harmonic signal seen at 16Hz.

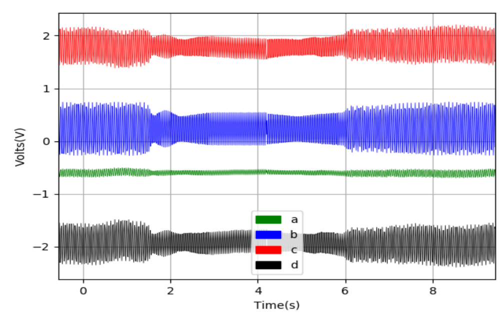


(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

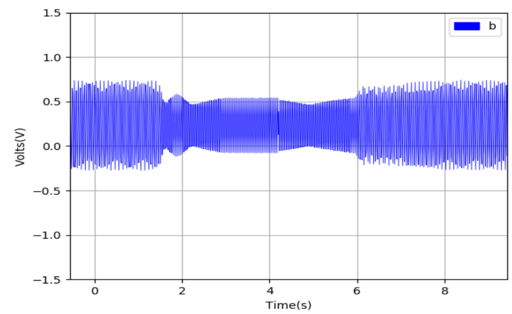


(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 46: The forced damped harmonic signal seen at 20Hz.

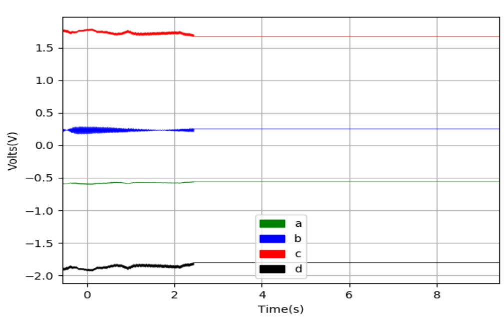


(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.

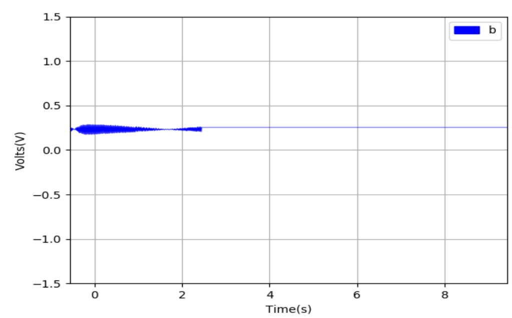


(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 47: The forced damped harmonic signal seen at 25Hz.



(a) The raw data from the oscilloscope showing a,b,c,d as in Section 4.5.



(b) The b output from the oscilloscope used to calculate the amplitude.

Figure 48: The forced damped harmonic signal seen at 50Hz.