University of Waterloo

PHYS460A

# Waves and Pulses in Cables

Experiment #7

*Authors:*
Ben MacLellan
20510640

Michael Hamel
20519421

Waterloo, ON

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**Abstract**

Coaxial cables are one of the most common cables used for transmitting signals, used for over a century as the backbone of networking and for accurate information transfer. In this lab we investigate the transmittance of electrical waves down the cables and some of their properties. By exploring standing waves and pulsed waves in cables and we can look at how cable terminations affect signals and determine various characteristics of the cable. In this experiment we determine the characteristic impedance of a specific coaxial cable at $76.4 \pm 2.0\ \Omega$. We also calculate the the cable's inductance per unit length as $452.93 \pm 11.89$ nH m$^{-1}$ and capacitance per unit length as $77.597 \pm 2.037$ pF m$^{-1}$. The dielectric used in the cable was then determined to be polyethylene with a relative permittivity of $\epsilon_r = 2.2840 \pm 0.0358$.

# 1 Introduction

Coaxial cables are the gold standard cable for transmitting electrical signals. First invented for telegraph transmission in 1880 by Oliver Heaviside, it has since been used for mass deployment of our major information age, specifically used for television and the internet. However it is also in every lab since it is can carry signals with the highest accuracy, but the coaxial cable also has inbuilt characteristics which significantly affect the signals as they propagate so the purpose of this experiment is to explore and measure some of these characteristics.

A coaxial cable is named as such because it has layers of conductors throughout its axis. There are four distinct layers: the center is a conductive wire, then a dielectric layer around it, followed by another conductive layer, and finally a insulating layer for protection. These layers can be seen in a cross-section of a wire as shown in Figure 1.

**Figure 1:** Coaxial cable cross-section demonstrating the four layers [5].

Generally, alternating current (AC) electricity is transmitted through the cables, so it is important to consider the characteristics of these AC waves as the propagate through the wave-guide, as well as measure and derive characteristics of each cable.

In this paper we will first explore the theory of a coaxial cable as a wave-guide, and then experimentally measure several properties of the cable. They are the: relative permittivity of the dielectric, speed of electrical transmission, capacitance, inductance, impedance. We observe other trends that are seen in the cable mainly due to cable terminations.

# 2 Theoretical Background

A natural method of transmitting electricity from point A to point B is to simply use a conducting wire to connect the two points. However when this simple wire is exposed to outside effects (eg. magnetic) they can significantly affect the original signal. This is especially important if there are two electrical lines within close proximity, which often can occur when transmitting a signal both to and from a location. These effects can be minimized by using a twisted pair of cables, but at high frequencies there will be significant losses due to radiation into the environment or other electrical components. [3]

The solution to this problem is to *shield* the cables by wrapping it in a conductive material. This creates a Faraday cage against external magnetic fields. This shielding in coaxial cables is combined with a precisely sized dielectric material to control its electrical properties.

Hence, we now have the coaxial cable modelled as an inner wire surrounded by a conductive tube at a regulated distance away. Modelling it as two conducting cylinders (as shown in Figure 2a), the two conductors can also be considered as parallel wires along the wire's axis, as represented in Figure 2b.


**(a)** A coaxial transmission line. [3]

**(b)** A coaxial transmission line along its radial axis [3]

**Figure 2:** The coaxial cable representations

We can denote the two currents and voltages as separated at points $x$ and $x + \Delta x$ are (as seen in Figure 2b). In an AC circuit $V(t) = L\frac{dI(t)}{dt}$, where $I$ is current and $L$ is the inductance, so therefore we can write that:

$$\begin{aligned} V(x) + \Delta V &= V(x + \Delta x) \\ \Delta V &= V(x + \Delta x) - V(x) \\ \Delta V &= -L\Delta x\frac{dI}{dt} \end{aligned}$$

Taking, $\Delta x \to 0$. Lets us state that the changing current gives us the gradient of voltage with a proportionality relative to the Inductance:

$$\frac{\partial V}{\partial x} = -L\frac{dI}{dt} \tag{1}$$

Next we can also look at the capacitance between the two wires, and between $x$ and $x + \Delta x$ is a charge $q$ which follow the relationship:

$$q = \frac{q}{V} V \Delta x$$
$$q = CV\Delta x$$

Therefore with respect to time it is:

$$\Delta I = C\Delta x \frac{\partial V}{\partial x}$$

Taking, $\Delta x \to 0$ Lets us state:

$$\frac{\partial I}{\partial x} - C\frac{dV}{dt} \tag{2}$$

Then we can combine these two equations (which are the basic equations of a transmission line) to obtain:

$$\frac{\partial^2 V}{\partial x^2} = CL\frac{\partial^2 V}{\partial t^2} \tag{3}$$

Or we can obtain:

$$\frac{\partial^2 I}{\partial x^2} = CL\frac{\partial^2 I}{\partial t^2} \tag{4}$$

We can then see that clearly the wave equation will solve this equation, meaning that:

$$V = V_1 e^{i(\omega t - kx)} + V_2 e^{i(\omega t + kx)} \tag{5}$$
$$I = I_1 e^{i(\omega t - kx)} + I_2 e^{i(\omega t + kx)} \tag{6}$$

Therefore by inputting Equation 5 into Equation 3 or Equation 6 into 4 we can obtain that:

$$\left(\frac{k}{\omega}\right)^2 = LC$$

Which means that:

$$\left|\frac{\omega}{k}\right| = \frac{1}{\sqrt{LC}} \tag{7}$$

But noting that we know wave velocity $v = \frac{\omega}{k}$

$$v = \frac{\omega}{k} = \pm\frac{1}{\sqrt{LC}} \tag{8}$$

Inserting the waves in Equations 5 & 6 into our original Current to Voltage Relation in Equation 1 :

$$\begin{aligned}
\frac{\partial}{\partial x}\left(V_1 e^{i(\omega t-kx)} + V_2 e^{i(\omega t+kx)}\right) &= -L\frac{\partial}{\partial t}\left(I_1 e^{i(\omega t-kx)} + I_2 e^{i(\omega t+kx)}\right) \\
ik\left(-V_1 e^{i(\omega t-kx)} + V_2 e^{i(\omega t+kx)}\right) &= -Li\omega\left(I_1 e^{i(\omega t-kx)} + I_2 e^{i(\omega t+kx}\right) \\
\frac{k}{\omega}V_1 e^{i(\omega t-kx)} - \frac{k}{\omega}V_2 e^{i(\omega t+kx)} &= L\left(I_1 e^{i(\omega t-kx)} + I_2 e^{i(\omega t+kx)}\right)
\end{aligned}$$

But noting that $\pm\frac{k}{\omega} = \sqrt{LC}$ can let us say:

$$\begin{aligned}
\sqrt{LC}\left(V_1 e^{i(\omega t-kx)} + V_2 e^{i(\omega t+kx)}\right) &= L\left(I_1 e^{i(\omega t-kx)} + I_2 e^{i(\omega t+kx)}\right) \\
V &= \frac{L}{\sqrt{LC}}I
\end{aligned}$$

And by definition, the impedance $Z = \frac{V}{I}$:

$$\boxed{Z_o = \frac{V}{I} = \sqrt{\frac{L}{C}}} \tag{9}$$

This quantity $Z_o$ is known as the characteristic impedance, meaning that the cable acts with an impedance of $Z_o$ if it is of infinite length. We note if characteristic impedance was to be matched by a resistor at the end of the cable, then it would absorb all of the power which hit it. This means the finite length effectively acts as if it was infinite. This type of cable termination is called **matched** and will not reflect any signals at cable termination.

We can also look at two other types of cable terminations. The first case is if the cable has nothing at the end of the cable. This is known as **open** and means that the current will be 0 by definition (it will be a node) and the voltage will be maximal (anti-node). We will denote the distance to the end of the cable as $x_{end}$. This means we can re-write the wave equation with this boundary condition, stating with current:

$$I_{end} = 0 = I_1 e^{i(\omega t - kx)} + I_2 e^{i(\omega t + kx)}$$

But we note that the wave is simply being reflected off of the end of the cable which means $I_1 = I_2 = I_o$. But $I_o$ is only an amplitude so we will define the wave as:

$$\begin{aligned} 0 &= I_o(\pm e^{i(\omega t - kx_{end})} \pm e^{i(\omega t + kx_{end})}) \\ 0 &= I_o e^{i\omega t}(\pm e^{-ikx_{end}} \pm e^{+ikx_{end}}) \\ 0 &= I_o(\pm e^{-ikx_{end}} \pm e^{+ikx_{end}}) \end{aligned} \tag{10}$$

Noting $sin(0) = 0$ we can then note the solution of:

$$0 = I_o(e^{-ikx_{end}} - e^{+ikx_{end}})$$

Which means that the general solution is:

$$\boxed{I = I_o(e^{i(\omega t - k(x - x_{end}))} - e^{i(\omega t + k(x - x_{end}))})}$$

Similarly by noting the solution of $\cos(0) = 1$ and changing the signage at equation 10 appropriately we can state:

$$\boxed{V = V_o(e^{i(\omega t - k(x - x_{end}))} + e^{i(\omega t + k(x - x_{end}))})}$$

We also wish to see what the impedance will look like at x = 0, which will give us:

$$\begin{aligned} Z_{in} &= \frac{V_o(e^{i(\omega t - k(0 - x_{end}))} + e^{i(\omega t + k(0 - x_{end}))})}{I_o(e^{i(\omega t - k(0 - x_{end}))} - e^{i(\omega t + k(0 - x_{end}))})} \\ &= Z_o \frac{e^{i(\omega t)} e^{-ikx_{end}} - e^{+ikx_{end}}}{e^{i(\omega t)} e^{-ikx_{end}} + e^{+ikx_{end}}} \\ &= Z_o \frac{2\cos(kx_{end})}{2i\sin(kx_{end})} \\ &= -iZ_o \frac{\cos(kx_{end})}{\sin(kx_{end})} \\ &\boxed{Z_{in} = -iZ_o \cot(kx_{end})} \end{aligned} \tag{11}$$

We then look at the final case where there is a bridge made at the end of the cable, or it is **shorted**. This means that current will be maximal (it will be an anti-node) and the voltage will be 0 (node). We can represent these waves as the opposite of the open cables stating with equation 10 and building to get:

$$\boxed{I = I_o(e^{i(\omega t - k(x-x_{end}))} + e^{i(\omega t + k(x-x_{end}))})}$$
$$\boxed{V = V_o(e^{i(\omega t - k(x-x_{end}))} - e^{i(\omega t + k(x-x_{end}))})}$$

Following a similar procedure as we saw in the open cable:

$$\begin{aligned} Z_{in} &= Z_o \frac{2i\sin(kx_{end})}{2\cos(kx_{end})} \\ &\boxed{Z_{in} = iZ_o\tan(kx_{end})} \end{aligned} \tag{12}$$

We will note that during this derivation we have assumed a lossless cable, hence in real situations we will get systematic losses known as attenuation of the signal.

Finally we will take a look again at Figure 2a. Our goal here is to calculate values of $C$ and $L$. Regarding the cylindrical shell of thickness $dr$ and length $\ell$ and after noting that the magnetic energy will be an integral of $\epsilon_o c^2 B^2/2$ we are able to state that energy ($U$) is:

$$\begin{aligned} U &= \int_a^b r\,dr \int_0^\theta d\theta \int_0^\ell dz\ \frac{\epsilon_o c^2 B^2}{2} \\ &= \int_a^b r\,dr\ 2\pi \frac{\epsilon_o c^2 B^2}{2} \end{aligned}$$

Noting that $B = \frac{I}{2\pi\epsilon_o c^2 r}$ for a cylindrical conductor:

$$\begin{aligned} U &= \epsilon_o c^2 \frac{1}{2} \int_a^b dr\ \ell \left(\frac{I}{2\pi\epsilon_o c^2 r}\right)^2 \ell 2\pi r \\ &= \frac{1}{2}\int_a^b dr\ \frac{I^2\ell}{2\pi\epsilon_o c^2 r} \\ &= \frac{I^2\ell}{4\pi\epsilon_o c^2}\int_a^b \frac{dr}{r} \\ U &= \frac{I^2\ell}{4\pi\epsilon_o c^2}\ \ln\left(\frac{b}{a}\right) \end{aligned}$$

Noting that the energy stored in an inductor is defined as $U = \frac{1}{2}LI^2$ and noting $\epsilon_r$ for permitivity of a non-vacuum:

$$\boxed{L = \frac{\ell\ \ln(b/a)}{2\pi\epsilon_o\epsilon_r c^2}} \tag{13}$$

Noting the velocity of electricity in a vacuum is c by definition and using equation 8 gives us:

$$
\begin{aligned}
c &= \frac{1}{\sqrt{LC}} \\
\frac{1}{c^2} &= \frac{\ell \quad \ln{(b/a)}}{2\pi\epsilon_o\epsilon_r c^2}C \\
\Aboxed{C &= \frac{\ell \; 2\pi\epsilon_o\epsilon_r}{\ln{(b/a)}}}
\end{aligned}
\tag{14}
$$

Therefore by using the calculated values of L and C in equation 9 and noting $c = 1/\sqrt{\mu_o\mu_r\epsilon_o\epsilon_r}$ we get:

$$
\boxed{Z_o = \frac{\ln{(b/a)}}{2\pi}\sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}}}
\tag{15}
$$

# 3 Experimental Background & Procedure

## 3.1 Apparatus

To determine properties of wave propagation in the cables, first a function generator is used to generate the voltage\current waves while a oscilloscope is used to observe them. Multiple standard coaxial cables (same manufacturer and model) are used, with lengths of 60m and 9m (cable lengths of 18 m produced connecting two of these).

As seen in the theory section, we are interested in cable termination so two pieces of semi-specialized equipment are used to create the termination boundary conditions. First is a cable shorter which creates a electrical bridge between the inner and outer section of the coaxial cable. This creates our *shorted* cable termination. To create a *matched* cable termination the cable is shorted, but a variable resistor is included in the path, which allows control over the impedance.

Since the function generator mentioned before will have an internal resistance, this means that the generating side of the circuit will have a impedance equal to this internal resistance. The output impedance for our signal generator used was 50 $\Omega$. However this cable termination is not ideal, so a Buffer is placed between the input signal and the cable to allow for modulation of this input impedance.

## 3.2 Experimental Procedure

There are multiple goals of this experiment – investigating and characterizing various cables properties, including: the dielectric material, different cable terminations, speed of propagation, characteristic impedance, and capacitance and inductance per unit length.

Two methods are used to characterize these properties. The first uses standing waves in the cable to measure the various properties. Initially, measurements of the conductor diameters are taken to calculate characteristic capacitance and inductance (once the dielectric constant is known). A single tone sine wave is input to the cable and standing waves (current and voltage) at resonance for frequencies ranging from 1-10 MHz are measured and used to calculate impedance. The resonance frequencies and corresponding impedance can be used for calculating the speed of the wave and the dielectric constant. A higher resolution of data is needed around the resonance to ensure accuracy and a better fit. The shorted and open cable terminations will provide different solutions and hence they are both studied.

The next method uses pulsed signals to investigate the same properties via a different procedure and investigate the cable further. A single pulse will allow us to see the effects of the cable terminations in a more visual manner. The matched cable termination can be determined at this point by finding the point at which the reflected wave does not get reflected and instead is absorbed by the resistor. The pulsed input will then allow for qualitative study of the terminations, and allow for verification based off of the alternative measure velocity of the wave by measuring the time to reflect from one end of the cable to the other. The length of the cable can also be altered to look at how this affects the the wave propagation.

### 3.3 Data Analysis

All data was analyzed using custom Matlab (Mathworks Ltd) scripts.

# 4 Analysis

## 4.1 Determination of $L$ and $C$

The diameters of the inner and outer cylinders were measured using a manual micrometer five times and averaged to remove random errors. The mean values for the inner cylinder diameter, $a$, and outer cylinder diameter, $b$, are;

$$a = 0.59 \pm 0.01 \text{ mm}, \qquad b = 4.83 \pm 0.18 \text{ mm}$$

with uncertainty values as the standard deviation. For the larger cylinder, the volume is far more malleable than the inner cylinder, which introduces greater error than the inner cylinder – as when the micrometer is tightened, it can compress the material slightly – adding a bias below the true diameter.

From Equations 13 and 14, we can compute the inductance and conductance per unit length up to a scale factor from the dielectric material used in the wire.

$$L = \frac{\ln(b/a)}{2\pi\epsilon_o\epsilon_r c^2} = (0.334 \pm 0.013)\mu_0\mu_r \text{ H m}^{-1}$$
$$C = \frac{2\pi\epsilon_0\epsilon_0}{\ln(b/a)} = \frac{\epsilon_0\epsilon_r}{0.334 \pm 0.013} \text{ F m}^{-1}$$

where $\epsilon_0 = 8.85 \times 10^{-12}$ Fm$^{-1}$ is the permittivity of free space, $\mu_0 = 1.2566 \times 10^{-6}$ Hm$^{-1}$ is the permeability of free space, $\epsilon_r$ is the relative permittivity of the dielectric separating the coaxial cylinders, and $\mu_r$ is the relative permeability of the dielectric (which we assume is approximately $\mu_r = 1$ for future calculations).

## 4.2 RF Input

The acquired data points for both the open and short-circuited setups were fit using a custom function to determine the parameters. From Equation 11 and 12 and that we are examining only positive values we can define the fitting function to be,

$$|Z_{\text{in}}| = A|\tan(B(f + C))| + Df \tag{16}$$

Where $A, B, C, D$ are the fitting coefficients and their respective units are determined through dimensional analysis. The linear term $Df$ is added to account for an experimental drift of increasing impedance at the resonance frequencies. This drift could be explained from attenuation of the signal through the wire.

| **Fit Coefficient** | **Open Termination** | **Shorted Termination** |
|---|---|---|
| A | $46.03 \pm 2.01$ $\Omega$ | $43.5 \pm 6.06$ $\Omega$ |
| B | $1.909 \pm 0.010$ $\text{MHz}^{-1}$ | $1.902 \pm 0.011$ $\text{MHz}^{-1}$ |
| C | $-2.440 \pm 0.17$ MHz | $-1.606 \pm 0.023$ MHz |
| D | $4.142 \pm 0.268$ $\Omega\ \text{MHz}^{-1}$ | $4.040 \pm 0.268$ $\Omega\ \text{MHz}^{-1}$ |

**Table 1:** Best fit coefficients using least squares method with the associated error the 95% confidence interval of the fitting function. Fitting was performed using a custom Matlab (Mathworks Ltd) script.

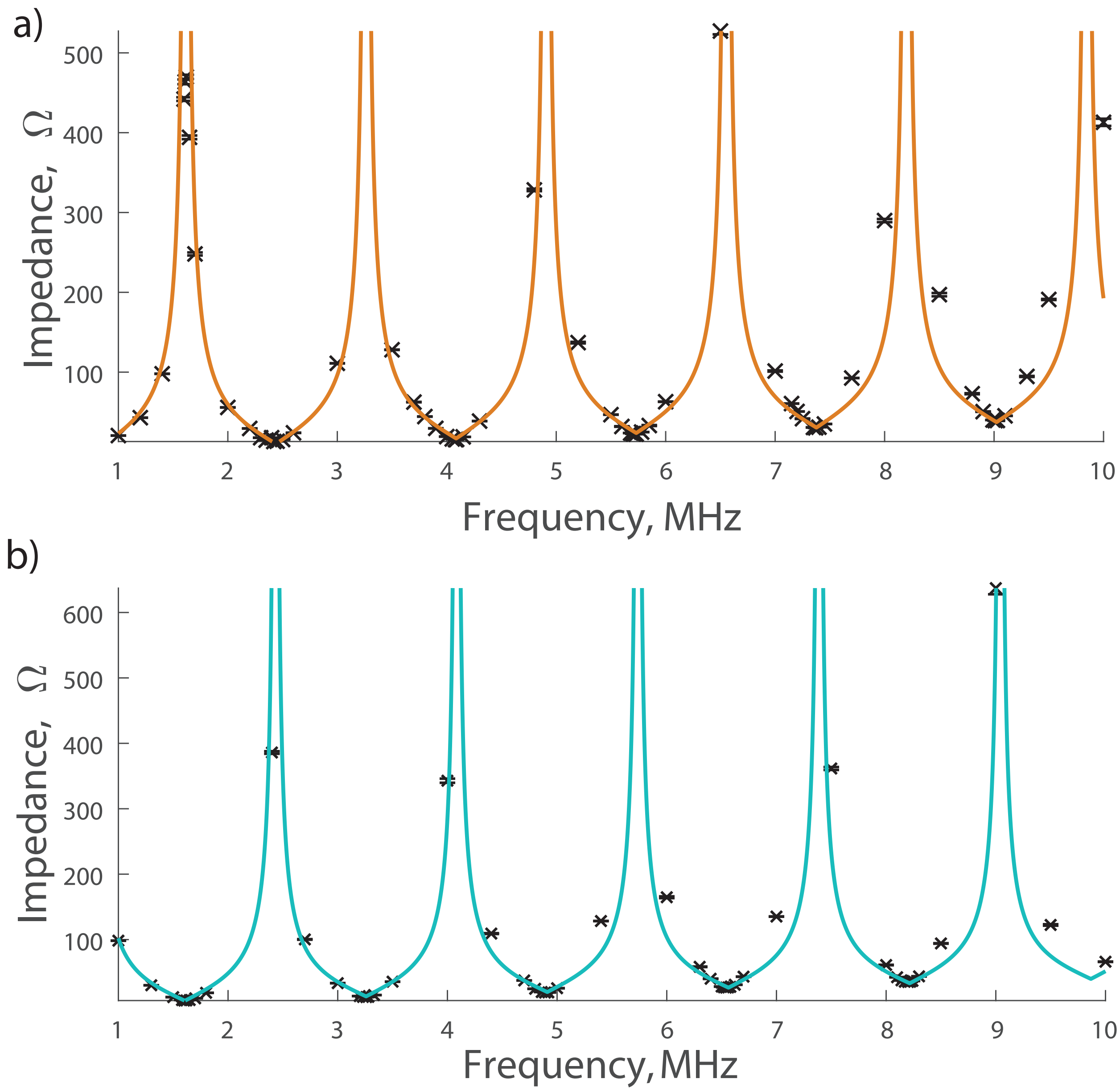


**Figure 3:** Input impedance as function of the frequency. Crosses represent measured impedance values with corresponding y-error bars, while the solid lines are fit best fit via least squares method for **a)** open-ended circuit and **b)** short-circuited setup

The frequency difference between minimum impedance points is calculated from the fitting parameter $B$ as;

$$\Delta f_{\text{open}} = \frac{\pi}{B_{\text{open}}} = 1.6457 \pm 0.0086 \text{ MHz} \tag{17}$$

$$\Delta f_{\text{closed}} = \frac{\pi}{B_{\text{closed}}} = 1.6517 \pm 0.0087 \text{ MHz} \tag{18}$$

The speed of the pulse can be computed with our calculation of the frequency spacing of the impedance minimums. With the knowledge that $v = \omega/k$ and assuming our impedance minimums are at $Z_{\text{min}} = 0$,

$$\begin{aligned} Z_{\text{in}} = 0 &= Z_0|\tan(kx_{\text{end}})| && (19)\\ &= \tan(k\ell) && (20)\\ \arctan(0) &= k\ell && (21)\\ n\pi &= \frac{\omega}{v}\ell && (22)\\ n\pi &= \frac{2\pi f}{v}\ell && (23)\\ v &= 2\Delta f\ell && (24) \end{aligned}$$

With the length of the cable at $\ell = 60.0 \pm 0.1$ m and the frequency separation from above,

$$v_{\text{open}} = (197.484 \pm 2.166) \times 10^6 \text{ ms}^{-1} \tag{25}$$

This value is calculated using the measured frequency spacing for the open cable. A similar analysis is done for the measured values of shorted cable,

$$v_{\text{closed}} = (198.204 \pm 2.190) \times 10^6 \text{ ms}^{-1} \tag{26}$$

The average of these two values is considered the measurement of the velocity.

$$\boxed{v = (197.8440 \pm 1.5486) \times 10^6 \text{ ms}^{-1}} \tag{27}$$

With the measured values of $L$ and $C$, we can determine the dielectric used in the cables.

$$\begin{aligned} LC &= \left(\frac{k}{\omega}\right)^2 = \frac{1}{v^2} && (28)\\ \mu_0\mu_r\epsilon_0\epsilon_r &= \frac{1}{v^2} && (29) \end{aligned}$$

Assuming that the relative permeability, $\mu_0$ is approximately 1,

$$\begin{aligned} \frac{\epsilon_r}{c^2} &= \frac{1}{v^2} && (30)\\ \Rightarrow \epsilon_r &= \frac{c^2}{v^2} && (31) \end{aligned}$$

So, the dielectric is computed with the velocity value from Equation 27,

$$\boxed{\epsilon_r = 2.2840 \pm 0.0358} \tag{32}$$

From a table of common dielectrics [4], we conclude that this material is likely polyethelene, with a listed dielectric of $\epsilon_r = 2.26$. However, more tests would need to be done to cross-check results with.

Calculating our characteristic impedance with this value of the dielectric constant (assuming relative permeability is $\mu_r = 1$),

$$Z_0 = \sqrt{\frac{L}{C}} \tag{33}$$

$$= \sqrt{(0.334 \pm 0.013)^2 \frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \tag{34}$$

$$= 83.3 \pm 9.7\ \Omega \tag{35}$$

We can also calculate the inductance and capacitance per unit length of the cable,

$$L = (4.197 \pm 0.163) \times 10^{-7}\ \mathrm{H\ \ m^{-1}} \tag{36}$$

$$C = (6.055 \pm 0.236) \times 10^{-11}\ \mathrm{F\ m^{-1}} \tag{37}$$

The phase relationship near resonance and off-resonance between the current and the voltage waveforms for open, shorted, and matched cables are tabulated in Table 2. This table summarizes which wave, either the current or voltage, is leading at frequencies just below, on, and just above resonance and off-resonance frequencies. We can see that for both the open and shorted cases, the leading wave switches at resonance and off-resonance frequencies – but they are opposite between open and shorted. The leading wave also flips between the open and shorted case, as we expect based on the change in our boundary condition of the cable termination. For the matched termination, the phase shift between voltage and current is constant over frequency, with the voltage always slightly leading over the current.

| | **Minimum Impedance** | | **Maximum Impedance** | |
|---|---|---|---|---|
| **Termination** | Frequency | Leading Wave | Frequency | Leading Wave |
| Open | $f \lesssim f_{\mathrm{min}}$ | Current | $f \lesssim f_{\mathrm{max}}$ | Voltage |
| | $f = f_{\mathrm{min}}$ | In-phase | $f = f_{\mathrm{max}}$ | In-phase |
| | $f \gtrsim f_{\mathrm{min}}$ | Voltage | $f \gtrsim f_{\mathrm{max}}$ | Current |
| Shorted | $f \lesssim f_{\mathrm{min}}$ | Voltage | $f \lesssim f_{\mathrm{max}}$ | Current |
| | $f = f_{\mathrm{min}}$ | In-phase | $f = f_{\mathrm{max}}$ | In-phase |
| | $f \gtrsim f_{\mathrm{min}}$ | Current | $f \gtrsim f_{\mathrm{max}}$ | Voltage |
| Matched | $f \lesssim f_{\mathrm{min}}$ | Voltage | $f \lesssim f_{\mathrm{max}}$ | Voltage |
| | $f = f_{\mathrm{min}}$ | Voltage | $f = f_{\mathrm{max}}$ | Voltage |
| | $f \gtrsim f_{\mathrm{min}}$ | Voltage | $f \gtrsim f_{\mathrm{max}}$ | Voltage |

**Table 2:** Phase relations between voltage and current for open, shorted and matched cables. $f_{\mathrm{min}}$ and $f_{\mathrm{max}}$ are the frequency values at which the minimum and maximum impedance occur, respectively. The leading wave is either the current or voltage signal which has a positive phase shift relative to the other.

Other analogous LCR circuits which give rise to such phase trends are **series resonance circuits** consisting of a single resistor, inductor, and capacitor wired in series with an AC power supply (see Figure 4). If the components are in this series order, then below resonance the voltage leads, at resonance the voltage and current are in phase, and above resonance the current signal leads.

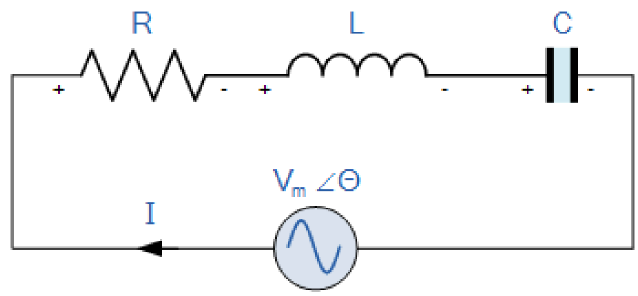


**Figure 4:** LCR circuit which provides similar phase trends between the voltage and current signals as a the coaxial cable demonstrates [2].

For the open termination case, there will be a current node at the ends (and thus an impedance maximum) and the resonant frequencies are given by,

$$\frac{2n+1}{2}\pi = \frac{2\pi f \ell}{v} \tag{38}$$

And similarly for the closed termination case,

$$n\pi = \frac{2\pi f \ell}{v} \tag{39}$$

These theoretical resonance frequencies are tabulated in Tables 3 and 4, and we can see that our expected values for the resonance frequencies match closely with the experimental data points from the custom fit in Figure 3. The expected and measured resonant frequencies agree within one decimal place, but falling slightly outside the uncertainty bounds of the expected values.

| $n$ | $f(Z_{\text{min}})$, MHz | $Z_{\text{min}}$, $\Omega$ | **Theoretical** $f(Z_{\text{min}})$, MHz |
|---|---|---|---|
| 2 | 2.4401 | 10.1169 | $2.4731 \pm 0.0198$ |
| 3 | 4.0855 | 16.9295 | $4.1217 \pm 0.0330$ |
| 4 | 5.7309 | 23.7636 | $5.7704 \pm 0.0462$ |
| 5 | 7.3771 | 30.5845 | $7.4192 \pm 0.0594$ |
| 6 | 9.0225 | 37.3794 | $9.0679 \pm 0.0726$ |

**Table 3:** Open termination: Measured and expected resonance frequencies, along with corresponding experimental minimum impedance, for an open termination cable, 60 m long.

| $n$ | $f(Z_{\min})$, MHz | $Z_{\min}$, $\Omega$ | **Theoretical** $f(Z_{\min})$, MHz |
|---|---|---|---|
| 1 | 1.6067 | 6.5128 | 1.6487 $\pm$0.0132 |
| 2 | 3.2583 | 13.1762 | 3.2974 $\pm$0.0264 |
| 3 | 4.9109 | 19.8662 | 4.9461 $\pm$ 0.0396 |
| 4 | 6.5626 | 26.5203 | 6.5948 $\pm$ 0.0528 |
| 5 | 8.2151 | 33.2196 | 8.2435 $\pm$ 0.0660 |
| 6 | 9.8668 | 39.8644 | 9.8922 $\pm$ 0.0792 |

**Table 4:** Shorted termination: Measured and expected resonance frequencies, along with corresponding experimental minimum impedance, for an shorted termination cable, 60 m long.

## 4.3 Pulse Input

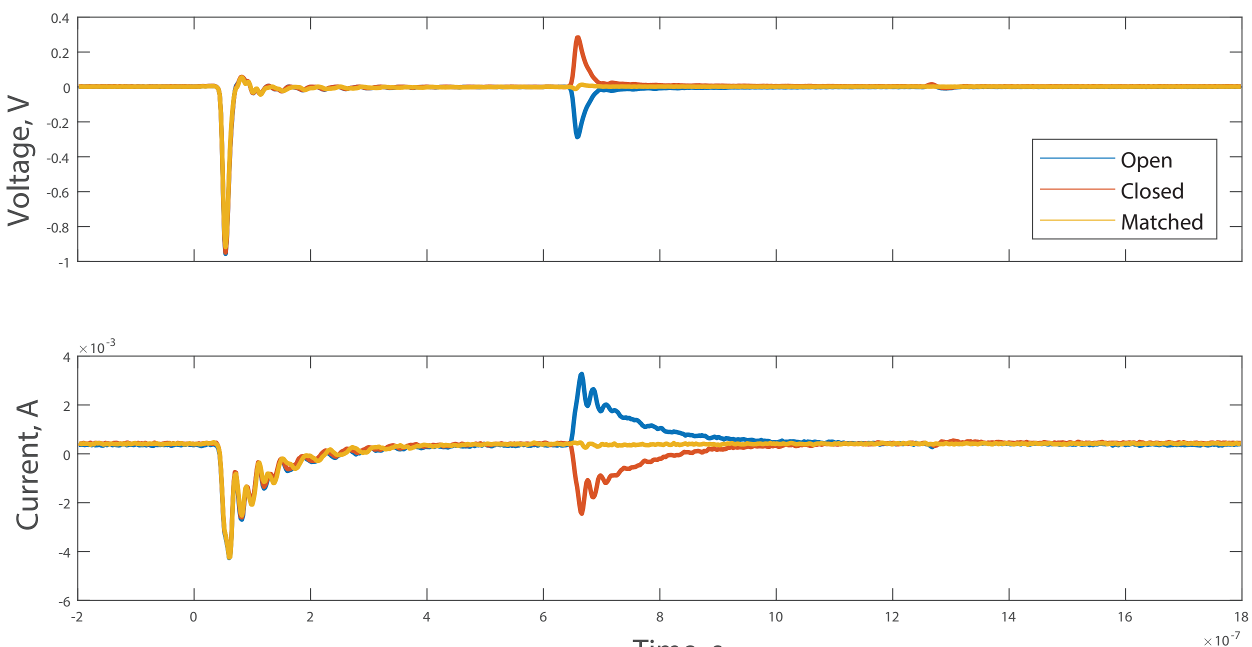


**Figure 5:** Voltage and current traces over time for three termination cases (open, shorted, and matched) with single pulses as input. The secondary peaks are the reflected pulses returning after one round-trip of the coaxial cable.

From Figure 5 we see that for the open termination, the reflected current pulse is inverted as the there is a current node at $x = x_{\text{end}} = \ell$, while the voltage pulse is not inverted. Conversely, for shorted termination the opposite is true and the current is not inverted while voltage is. When the load is matched, we do not have a reflected pulse as the matched impedance mimics an infinitely long cable where no reflections can be produced.

The matched load resistance was measured at the point which minimized (removed) the reflection peak as viewed on an oscilloscope. The resistance uncertainty was determined through the measurement of the upper and lower bounds for where the reflected peak was still approximately zero to the naked eye.

$$Z_{\text{matched}} = 76.4 \pm 2.0\ \Omega \tag{40}$$

Comparing this to the value calculated in the earlier part, Equation 35, we see that this value is in the error bounds of the first calculation – but the uncertainty in this measurement is much lower as we have not propagated uncertainty through the calculation and this was a direct measurement. Thus, the value measured in this section, $Z_{\text{matched}} = 76.4 \pm 2.0\ \Omega$ is better suited for future calculations.

The time delay between the first peak and second peak is $\Delta t = 60.4\ \mu\text{s}$[1]. The velocity is then,

$$\begin{aligned} v &= \frac{\Delta x}{\Delta t} = \frac{2\ell}{\Delta t} && (41)\\ &= \frac{120.0 \pm 0.2}{60.4 \times 10^{-6}} && (42)\\ &= (198.68 \pm 0.33) \times 10^{6}\ \text{ms}^{-1} && (43) \end{aligned}$$

When we perform the same observation with 18 m and 9 m cables, the pulses are closer together in time, as expected. While no exact measurements were taken, the peaks are approximately 1/3 closer together in the 18 m cable case compared to the 60 m, and 1/6 closer for the 9 m cable – as we expect to see.

When the buffer impedance is set to greater than the cable impedance, we see a pulse train caused by multiple reflections of the original pulse at each end of the cable. The amplitude of these peaks decrease as the buffer impedance increases, as there is more attenuation of the signal during each round trip through the cable. Similarly, when the buffer impedance is set to less than the cable impedance, we again observe a similar pulse train with each pulse separated by the time to travel the length of the cable and back. Conversely, with the buffer impedance lower that the characteristic impedance the amplitude of each pulse increases as the buffer impedance is lowered, as there is less attenuation of the signal.

To calculate the values of $L$ and $C$,

$$Z_0 = \sqrt{\frac{L}{C}}, \qquad \left(\frac{k}{\omega}\right)^2 = \frac{1}{v^2} = LC \tag{44}$$

[1] The time resolution of the oscilloscope data is 2 ns, which we have omitted as it is negligible in the calculations. However, the uncertainty in the length of the cable is significant.

So, we can express the inductance and capacitance as,

$$Z_0 = \sqrt{L^2v^2} = Lv \;\Rightarrow\; \boxed{L = \frac{Z_0}{v}} \tag{45}$$

$$Z_0 = \sqrt{\frac{1}{C^2v^2}} = \frac{1}{Cv} \;\Rightarrow\; \boxed{C = \frac{1}{Z_0 v}} \tag{46}$$

$$\tag{47}$$

So,

$$L = \frac{Z_0}{v} = \frac{76.4 \pm 2.0}{(198.68 \pm 0.33) \times 10^6} \tag{48}$$

$$= (4.5293 \pm 0.1189) \times 10^{-7} \text{ H m}^{-1} \tag{49}$$

Similarly,

$$C = \frac{1}{Z_0 v} = \frac{1}{(76.4 \pm 2.0)(198.68 \pm 0.33) \times 10^6} \tag{50}$$

$$= (7.7597 \pm 0.2037) \times 10^{-11} \text{ F m}^{-1} \tag{51}$$

# 5 Conclusion

In this experiment we explored the transmission of electrical signals down a coaxial cable using two different methods: pulsed signals and standing wave resonances. The characteristic values of the coaxial cable were measured, including inductance and capacitance per unit length, the velocity of the travelling wave down the cable, the characteristic impedance, and the dielectric material used to insulate between the coaxial cylinders.

The capacitance and inductance per unit length were measured to be, $L = 419.7 \pm 16.3$, $C = 60.55 \pm 2.36$ pF m$^{-1}$ and $L = 452.93 \pm 11.89$ nH m$^{-1}$, $C = 77.597 \pm 2.037$ pF m$^{-1}$ using the standing wave and pulsed signal methods, respectively. The characteristic impedance was measured to be $Z_0 = 83.3 \pm 9.7\ \Omega$ and $Z_0 = 76.4 \pm 2.0\ \Omega$ with the same two methods, respectively. Using the standing wave method, the dielectric material separating the two coaxial cylinders was determined to be polyethylene with a relative permittivity of $\epsilon_r = 2.2840 \pm 0.0358$.

Major sources of error include the uncertainty in the length of cable, assumed to be 60 m with an uncertainty of $\pm 0.1$ m, observational error in determining the characteristic impedance when the reflected pulse amplitude reduces to approximately zero, and random errors while measuring the diameters of the inner and outer cylinders of the cable.

Between the two methods, the pulsed signal method is considered to be more accurate for calculating properties of the cables used, as values were measured directly and did not rely on theoretical models which ignore attenuation, and did not need to propagate errors through multiple calculations.

# References

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