Quiz 3

นายอัยยะ บุ้งทอง 623021039-4 section 2

① Operation $\wedge$ Operation $-$

$G = \{0, 1\}$

Semigroup

① $ab \in G$

② $(ab)c = a(bc)$

Group

③ $e \in G$

④ inverse

$\boxed{1.1}$ Operation $\wedge$

$\wedge$	0	1
0	0	0
1	0	1

$a, b, c \in G.$

① $ab \in G$ ✓

② $(a \wedge b) \wedge c = (a \wedge b) \wedge c$ (by associativity of $\wedge$)

③ $e = 1$ = identity

× ④ $0 \wedge 0^{-1} = 0^{-1} \wedge 0 = 1$

0^{-1} does not exist.

$\boxed{1.2}$ Operation $-$

$0 - 1 \notin G.$

operation	Identity	Semigroup	Group
$\wedge$	1	✓	×
$-$	×	×	×

Ring

×

Semigroup

1. $ab \in G$
2. $(ab)c = a(bc)$

Group

3. $e \in G$
4. inverse

	Operation	Operation
(2) $P(x)$, $\lvert x \rvert \geq 2$	$\cup$	$-$

$a, b, c \in P(x)$

2.1 Operation $\cup$.

1. $a \cup b \in P(x)$
2. $(a \cup b) \cup c = a \cup (b \cup c)$ (สมบัติของ $\cup$)
3. $\phi \in \cup$, $\phi \cup a = a \cup \phi = a$

× 4. ถ้า $a \neq \phi$

$a^{-1} \notin P(x)$

2.2 Operation $-$

1. $a - b \in P(X)$

× 2. $(a - b) - c \neq a - (b - c)$

ให้ $a = \{1, 2, 3\}$

$b = \{2, 3\}$

$c = \{3\}$

$(a - b) - c = \{1\}$

$a - (b - c) = \{1, 3\}$

operation	Identity	Semigroup	Group
$\cup$	ϕ	✓	×
$-$	×	×	×

Ring

×

	Semigroup
	① $ab \in G$
	② $(ab)c = a(bc)$
Group	③ $e \in G$
	④ inverse

3) $P(X)$ Operation $\cup$ Operation $\cap$

3.1 operation $\cup$
ข้อ 2.

3.2 operation $\cap$
$A, B, C \in P(X)$

① $A \cap B \in P(X)$
$x \in A \cap B$, $x \in X$.
$A \cap B \subseteq X$.
$A \cap B \in P(X)$ by definition of $P(X)$.

② $(A \cap B) \cap C = A \cap (B \cap C)$ (สมบัติของ $\cap$)

③ $X \cap A = A \cap X = A$
$X = e$

× ④ $\emptyset \cap A = A \cap \emptyset = \emptyset$
$(\emptyset)^{-1} \notin P(X)$

operation	Identity	Semigroup	Group
$\cup$	✓	✓	×
$\cap$	✓	✓	×

Ring
×

Operation Operation

4) $G = \{0, 1\}$ $\vee$ $\wedge$

$\boxed{4.1}$ Operation $\vee$

$\vee$	0	1
0	0	1
1	1	1

$a, b, c \in G$.

1. $a \vee b \in G$.
2. $(a \vee b) \vee c = a \vee (b \vee c)$ (สมบัติของ $\vee$)
3. $0 \vee a = a \vee 0 = a$

   $0 = e$

× 4. $1 \vee a = 1$

   $(1)^{-1} \notin G$.

4.2 Operation $\vee$

ข้อ 1

Semigroup

1. $ab \in G$
2. $(ab)c = a(bc)$

Group

3. $e \in G$
4. inverse

operation	Identity	Semigroup	Group
$\wedge$	✓	✓	×
$\vee$	✓	✓	×

Ring

×

	Operation	Operation
5) E	$+$	$\times$

set of even

Semigroup

① $ab \in G$

② $(ab)c = a(bc)$

Group

③ $e \in G$

④ inverse

Let $a, b, c \in E$.

$\boxed{5.1}$ Operation $+$.

① $a+b = 2k + 2m = 2(k+m) \in E$, for some $k, m \in \mathbb{Z}$.

② $(a+b)+c = a+(b+c)$ (สมบัติของจำนวนจริง)

③ $0 + a = a + 0 = a$

$0 = e$

Ring R.

$a, b, c \in R$

(A) $(R, +)$ is an abelian group.

(B) $ab \in R$. [semigroup]

(C) $(ab)c = a(bc)$

(D) $a(b+c) = ab+bc$ [distributive law]

(E) $(a+b)c = ac+bc$

④ ให้ $a > 0$,

$a + (-a) = (-a) + a = 0$

$a^{-1} = (-a)$

$(-a)^{-1} = a$

⑤ $a+b = b+a$, $(E, +)$ is abelian.

$\boxed{5.2}$ Operation $\times$

① $ab = 2k2m = 2(k2m) \in E$. for some $m, k \in \mathbb{Z}$.

② $(ab)c = (2k2m)2n = 2k2m2n = 2k(2m2n) = a(bc)$

[สมบัติของ $\mathbb{Z}$] $k, m, n \in \mathbb{Z}$.

× ③ $1 \notin E$

× ④ $(2)^{-1} = \frac{1}{2} \notin E$.

(D) $a, b, c \in E$, ดังนั้น $a, b, c \in \mathbb{Z}$.

$a(b+c) = ab + ac$ (สมบัติของ $\mathbb{Z}$)

(E) $(a+b)c = ac + bc$ (สมบัติของ $\mathbb{Z}$)

operation	Identity	Semigroup	Group	commutative
$+$	✓	✓	✓	✓
$\times$	✗	✓	✗	

E is a ring.