ME455

Computer Homework | Part I

Tolga Çatak - 2018405144

Bogazici University
Istanbul, Turkey

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Table of Contents

List of Figures

List of Tables

Question 1 - Part A

For the first part of the question, firstly, the Mach number of the flow just before the shock (M_x) is found by using the information of the shock location. Since the area profile and the shock position is known, the critical area and the area of the shock is known. By using Equation 1, M_x is calculated.

$$\frac{A_{\text{shock}}}{A_\star} - \frac{1}{M_x}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_x^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0 \tag{1}$$

Then, with the known value of M_x, the Mach number of the flow just after the shock is found using Equation 2 and Equation 4.

$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1} \tag{2}$$

$$M_y = \sqrt{M_y^2} \tag{3}$$

The ratio of the pressure before and after the shock, P_x and P_y respectively, is found using Equation 5.

$$\frac{P_y}{P_x} = \frac{2\gamma M_x^2 - (\gamma-1)}{\gamma+1} \tag{4}$$

After M_x and M_y are calculated, the Mach number at the inlet (M_1) and the outlet (M_2) are found using the isentropic relations. The ratio of the inlet area to the critical area and the ratio of the shock area to the critical area are known. Combining these two ratios, we get the following equations:

$$\frac{A_{\text{inlet}}}{A_{\text{shock}}} - \frac{1}{M_1}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{M_x}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_x^2\right)\right)^{\frac{2(\gamma-1)}{\gamma+1}} = 0 \tag{5}$$

$$\frac{A_{\text{outlet}}}{A_{\text{shock}}} - \frac{1}{M_2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{M_y}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_y^2\right)\right)^{\frac{2(\gamma-1)}{\gamma+1}} = 0 \tag{6}$$

where

$$\frac{A_{\text{outlet}}}{A_{\text{shock}}} = \frac{A_{\text{outlet}}}{A^{\star}} \frac{A^{\star}}{A_{\text{shock}}} \tag{7}$$

Since Equation 1, Equation 6, and Equation 7 are all nonlinear equations, they are solved by using built-in fzero function in MATLAB. After all the Mach values are found, the the stagnation pressure ratio across the shock and the exit stagnation pressure to exit pressure ratios can be calculated as in Equation 8 and Equation 9.

$$\frac{P_{0y}}{P_{0x}} = \frac{\left(\frac{\gamma+1}{2} M_x^2\right)^{\frac{\gamma}{\gamma-1}} \left(1 + \frac{\gamma-1}{2} M_x^2\right)^{\frac{\gamma}{1-\gamma}}}{\left(\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}} \tag{8}$$

$$\frac{P_2}{P_{2,0}} = \left(1 + \frac{1}{2}(\gamma - 1) M_2^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{9}$$

Finally, the ratio of duct exit pressure to duct inlet stagnation pressure that will result in a standing normal shock at x=0.4m and the corresponding stagnation pressure loss are calculated with the following equation:

$$\frac{P_2}{P_{01}} = \frac{P_2}{P_{02}} \frac{P_{02}}{P_{01}} \tag{10}$$

where

$$\frac{P_{02}}{P_{01}} = \frac{P_{0y}}{P_{0x}} \tag{11}$$

since the stagnation pressure is constant due to the isentropic flow.

The results are found as

$$\frac{\text{Exit Pressure}}{\text{Inlet Stagnation Pressure}} = \frac{P_2}{P_{01}} = 0.4635 \tag{12}$$

$$\text{Stagnation Pressure Loss} = (1 - \frac{P_{02}}{P_{01}})100 = 49.4939\% \tag{13}$$

Question 1 - Part B

In the second part of the question, an iterative process is developed using the equations given in the first part.

The corresponding locations for the given pressure ratios are given in Table 1.

Table 1: Pressure ratio vs. the location the shock.

Pressure Ratio	**Location of the Shock (m)**
0.4	0.4454
0.6	0.3146
0.8	0.2029

Additionally, the continuous variation of the location of the shock with changing pressure ratio can be seen in Figure 1 below, where points correspond to the values given in Table 1.

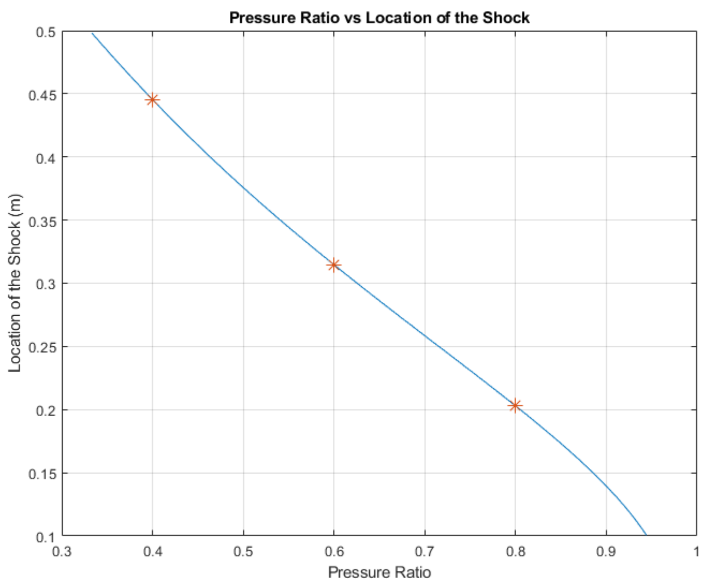

Figure 1: Pressure ratio vs. the location the shock.

Question 2

For the second question, the equation for the Fanno flow given below is used.

$$l^{\star} = l_1 + \frac{D}{f}\left(\frac{1}{\gamma}\frac{1-M_1^2}{M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2}\right)\right) \tag{14}$$

To find the required length for the flow to reach the given Mach numbers, this equation is iterated various Mach numbers using two different γ values for air and helium. The results are given in Table 2 and Table 3, respectively.

Table 2: Inlet mach number vs. the required duct length for air.

Inlet Mach Number	**Reqired Duct Length (m)**
0.1	669.2156
0.2	145.3327
0.3	52.9925
0.4	23.0849
0.5	10.6906
0.6	4.9082
0.7	2.0814
0.8	0.7229
0.9	0.1451
1.0	0

Table 3: Inlet mach number vs. the required duct length for helium.

Inlet Mach Number	**Reqired Duct Length (m)**
0.1	559.3183
0.2	120.4184
0.3	43.4581
0.4	18.7206
0.5	8.5686
0.6	3.8872
0.7	1.6288
0.8	0.5590
0.9	0.1109
1.0	0

The change of the required duct length to accelerate the flow to a Mach number of unity with given parameters can be also plotted on a graph and the change can be shown continuously as in Figure 2 and Figure 3, where points correspond to the values given in Table 2 and Table 3, respectively.

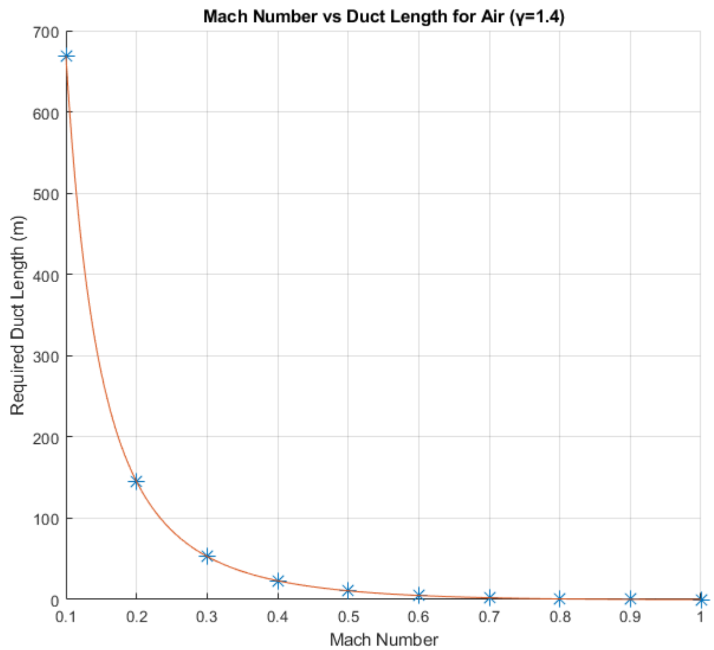

Figure 2: Inlet mach number vs. the required duct length for air.

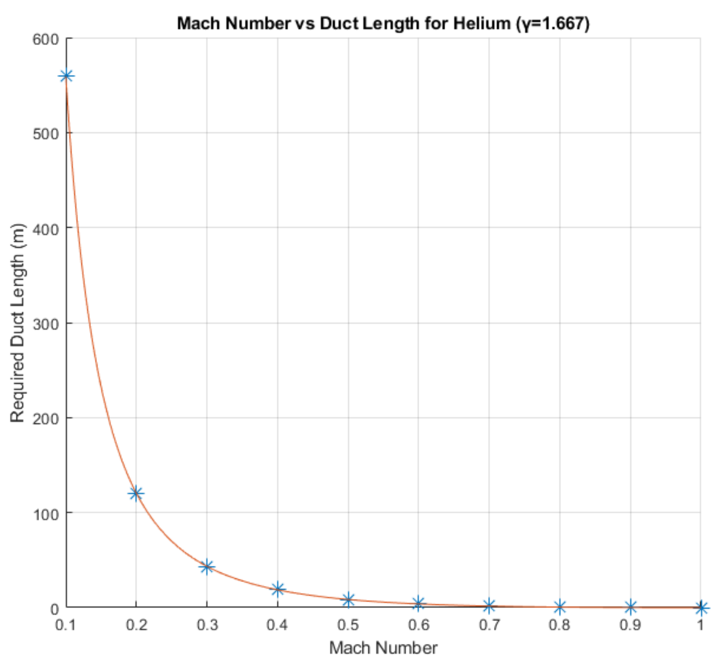

Figure 3: Inlet mach number vs. the required duct length for air.

The results show that as the Mach number increases, the required duct length decreases. This is expected since as the fluid moves faster, it requires less distance to reach the Mach number of unity. Additionally, it can be observed that for both air and helium, the duct length decreases more rapidly with smaller Mach values.

It is also evident that the value of the ratio of specific heats (γ) has a significant effect on the required duct length. The required duct length for helium is consistently smaller than that of air, which is due to the higher value of γ for helium. This indicates that a higher value of γ results in a greater increase in temperature and pressure, which leads to a more rapid acceleration of the fluid. This is also because a smaller γ implies a greater degree of compressibility of the fluid, meaning that the fluid is more easily compressed and therefore requires a longer duct length to achieve a given change in Mach number and maintain a steady flow. In contrast, a larger γ indicates a lower degree of compressibility and the fluid can be accelerated more easily, requiring a shorter duct length to achieve the same Mach number.

It is important to note that the required duct length is zero at a Mach number of unity, which is expected since the fluid has already achieved the desired speed.

All the equations used in this report are taken from the course textbook [1].

References

[1] Munson Bruce R.; Okiishi Theodore H.; Huebsch Wade W.; Rothmayer Alric P. *Fundamentals of Fluid Mechanics*. John Wiley Sons, Inc., 2013.

Appendix

The codes for the questions are given in this section.

```matlab
clc
clear
close all

% Define the parameters
gamma = 1.4;

% Define the area function
A = @(x) 0.1 + x^2;

% Calculate the area at x = 0 (throat), x = 0.4m (shock),
% x = 0.5m (inlet and outlet)
A_star = A(0);
A_shock = A(0.4);
A_outlet = A(0.5);
A_inlet = A(0.5);

% Make an initial guess for the function.
% (Should be bigger than 1 since it is before the shock.)
M0_shock = 2.5;

% Find Mach number before the shock (Mx)
fun = @(M) A_shock/A_star - (1/M) * ...
    (2/(gamma+1)*(1+(gamma-1)/2*M^2))^((gamma+1)/(2*(gamma-1)));
Mx = fzero(fun, M0_shock);

% Make an initial guess for the function.
% (Should be smaller than 1 since it is before the shock.)
M0_inlet = 0.5;

% Find Mach number at the inlet (M1)
fun = @(M) A_star/A_shock*A_inlet/A_star - ((1/M) * ...
    (2/(gamma+1)*(1+(gamma-1)/2*M^2))^((gamma+1)/(2*(gamma-1))))*((1/Mx)...
    * (2/(gamma+1)*(1+(gamma-1)/2*Mx^2))^((gamma+1)/(2*(gamma-1))))^-1;
M1 = fzero(fun, M0_inlet);

% Calculate Mach number after the shock (My) and pressure ratio P2/P1
My = ((Mx^2 + 2 / (gamma - 1)) / ((2*gamma/(gamma - 1)) * Mx^2 - 1))^0.5;
Py_Px = (2 * gamma * Mx^2 - (gamma - 1)) / (gamma + 1);

% Make an initial guess for the function.
% (Should be smaller than 1 since it is before the shock.)
M0_outlet = 0.5;

% Find Mach number at the outlet (M2)
fun = @(M) A_star/A_shock*A_outlet/A_star - ((1/M) * ...
    (2/(gamma+1)*(1+(gamma-1)/2*M^2))^((gamma+1)...
    /(2*(gamma-1))))*((1/My) * (2/(gamma+1)*(1+(gamma-1)/2*My^2))...
    ^((gamma+1)/(2*(gamma-1))))^-1;
M2 = fzero(fun, 0.5);

% Calculate the stagnation pressure ratio across the shock
p0y_p0x =(((gamma+1)*(Mx^2)/2)^(gamma/(gamma-1)))*(((1+(gamma-1)*...
    (Mx^2)/2))^(gamma/(1-gamma)))/((2*gamma/(gamma+1)*Mx^2 - ...
    (gamma-1)/(gamma+1))^(1/(gamma-1)));

% Since stagnation pressure for an isentropic flow is equal,
% p01=p0x and p02=p0y
p02_p01 = p0y_p0x;

% Calculate the exit pressure to exit stagnation pressure ratio
p2_p02 = (1 / (1+ (gamma-1)/2*M2^2)) ^ (gamma/(gamma-1));
```

```
% Combine the ratios to get the requested ratio
p2_p01 = p2_p02*p02_p01;
fprintf(['The ratio of duct exit pressure to duct inlet ' ...
    'stagnation pressure is p2/p01 = %.4f. \n'],p2_p01)

% Find the stagnation pressure loss
% format long
fprintf(['The stagnation pressure loss is (1-(p02_p01))' ...
    '*100 = %.4f%%. \n'],(1-(p02_p01))*100)
```

```
The ratio of duct exit pressure to duct inlet stagnation pressure is p2/p01 = 0.4635.
The stagnation pressure loss is (1-(p02_p01))*100 = 49.4939%.
```

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```
clc
clear
close all

% Define the parameters
gamma = 1.4;

% Define the area function
A = @(x) 0.1 + x^2;

% Calculate the area at x = 0 (throat),
% x = 0.4m (shock), x = 0.5m (inlet and outlet)
A_star = A(0);
A_outlet = A(0.5);
A_inlet = A(0.5);

% Define the function to find the Mach number before the shock
fun = @(M,A1_Astar) A1_Astar - (1/M) * ...
    (2/(gamma+1)*(1+(gamma-1)/2*M^2))^((gamma+1)/(2*(gamma-1)));

% Find Mach number at the outlet (M2)
fun2 = @(M,My,A_shock) A_star/A_shock*A_outlet/A_star - ...
    ((1/M) * (2/(gamma+1)*(1+(gamma-1)/2*M^2))^((gamma+1)...
    /(2*(gamma-1))))*((1/My) * (2/(gamma+1)*(1+(gamma-1)/2*My^2))...
    ^((gamma+1)/(2*(gamma-1))))^-1;

% Define the error
error = 1e-4;

% Define the pressure ratios to iterate
pressure_ratios = [0.4 0.6 0.8];

% Create a for loop for the iteration
for p2_p01 = pressure_ratios
    for position = 0.1:0.0001:0.5

        % Define the area of the shock
        A_shock = A(position);

        % Find the Mach number before the shock
        Mx = fzero(@(M) fun(M, A_shock/A_star), 2);

        % Find the Mach number after the shock
        My = ((Mx^2 + 2 / (gamma - 1)) / ...
            ((2*gamma/(gamma - 1)) * Mx^2 - 1))^0.5;

        % Find the stagnation pressure ratio before and after the shock
        p0y_p0x = (((gamma+1)*(Mx^2)/2)^(gamma/(gamma-1)))...
            *(((1+(gamma-1)*(Mx^2)/2))^(gamma/(1-gamma)))/...
            ((2*gamma/(gamma+1)*Mx^2 - (gamma-1)/(gamma+1))...
            ^(1/(gamma-1)));

        % Find the stagnation pressure of the inlet and outlet
        p02_p01 = p0y_p0x;

        % Find the Mach number at the exit
        M2 = fzero(@(M) fun2(M,My,A_shock), 0.5);

        % Find the exit
        p2_p02 = (1 / (1+ (gamma-1)/2*M2^2)) ^ (gamma/(gamma-1));

        % Find the calculated pressure ratio with the given location
        p2_p01_calc = p2_p02*p02_p01;
```

```
        % Compare the calculated pressure ratio to the required ratio
        if abs(p2_p01_calc - p2_p01) < error
            fprintf('The location of the normal shock is x = %.4f m,'...
                , position);
            fprintf(' when the pressure ratio is p2/p01 = %.4f. \n'...
                ,p2_p01)

            % Break the loop if the difference is smaller ...
            % than the defined error
            break
        end
    end
end
```

```
The location of the normal shock is x = 0.4454 m, when the pressure ratio is p2/p01 = 0.4000.
The location of the normal shock is x = 0.3146 m, when the pressure ratio is p2/p01 = 0.6000.
The location of the normal shock is x = 0.2029 m, when the pressure ratio is p2/p01 = 0.8000.
```

```
clc
clear
close all

% Define the parameters
gamma_air = 1.4;
gamma_helium = 1.667;
f = 0.02;
D = 0.2;
M1 = 0.1;
l1 = 0;


% Calculate the required length for the Mach number of unity
l_star = @(M1,gamma) l1 + D / f * (1 / (gamma) * ...
    (1 - M1^2) / M1^2 + (gamma + 1) / (2 * gamma) * ...
    log(((gamma + 1) / 2 * M1^2) / (1 + (gamma - 1) / 2 * M1^2)));

% Define the range of the initial Mach number
M_values = [0.1:0.1:1];

% Iterate through the values for both air and helium
m=1;
for i = M_values
    l_air(m) = l_star(i,gamma_air);
    l_helium(m) = l_star(i,gamma_helium);
    m = m + 1 ;
end

% Repeat the same process for the continuous graph
M_range_cont = [0.1:0.001:1];

% Iterate through the values for both air and helium
n=1;
for j = M_range_cont
    l_air_cont(n) = l_star(j,gamma_air);
    l_helium_cont(n) = l_star(j,gamma_helium);
    n = n + 1;
end

% Plot the graphs

figure(1)
scatter(M_values,l_air,'*','SizeData', 100)
hold on
grid on
plot(M_range_cont,l_air_cont)
xlabel('Mach Number')
ylabel('Required Duct Length (m)')
title('Mach Number vs Duct Length for Air (γ=1.4)')


figure(2)
scatter(M_values,l_helium,'*','SizeData', 100)
hold on
grid on
plot(M_range_cont,l_helium_cont)
xlabel('Mach Number')
ylabel('Required Duct Length (m)')
title('Mach Number vs Duct Length for Helium (γ=1.667)')

% Display results
fprintf(['The required duct length to accelerate the flow from M = 0.1 '...
```

```
    'to M = 1.0 is l_air = %.4f m. \n'],l_air(1))

% Display results in a table
T = table(M_values', round(l_air',4), round(l_helium',4), ...
    'VariableNames', {'M_values', 'l_air', 'l_helium'});
disp(T)
```

```
The required duct length to accelerate the flow from M = 0.1 to M = 1.0 is l_air = 669.2156 m.
    M_values    l_air     l_helium
    ________    ______    ________

      0.1       669.22     559.32 
      0.2       145.33     120.42 
      0.3       52.992     43.458 
      0.4       23.085     18.721 
      0.5       10.691     8.5686 
      0.6       4.9082     3.8872 
      0.7       2.0814     1.6288 
      0.8       0.7229      0.559 
      0.9       0.1451     0.1109 
        1            0          0 
```

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