ME474
Homework 6

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In this report, the solution to the homework 6 is given with detailed steps, equations, and the MATLAB code.

Definition of Parameters and Working Condition

Firstly, the given parameters are defined as follows:

Polytropic efficiency: $\eta_{\text{poly}} = 0.9$

Bypass ratio: $\text{bpr} = 4$

Fan pressure ratio: $\text{fpr} = 2.2$

Overall pressure ratio: $\text{opr} = 18$

Combustor pressure loss: $\Delta P_{\text{loss}} = 6\%$

Combustion efficiency: $\eta_{\text{comb}} = 1$

Turbine inlet temperature: $T_{\text{inlet}} = 1200\,K$

Mechanical efficiency: $\eta_{\text{mech}} = 0.98$

Specific heat at constant pressure for air: $c_{p,air} = 1000\,J/kgK$

Specific heat at constant pressure for gas: $c_{p,gas} = 1148\,J/kgK$

Specific heat ratio for air: $k_{air} = 1.4$

Specific heat ratio for gas: $k_{gas} = 1.333$

Using the k and c_p values. The R values calculated as follows:

$$R_{air} = \frac{c_{p,air}}{k_{air} - 1} = 285.71\,J/kgK \tag{1}$$

$$R_{gas} = \frac{c_{p,gas}}{k_{gas} - 1} = 286.78\,J/kgK \tag{2}$$

Additionally, it is assumed that the intake efficiency is 1:

Intake efficiency: $\eta_{in} = 1.00$

Then, the working condition is defined as follows:

Mach number: $M_{cruise} = 0.7$

Ambient pressure: $P_a = 0.2\,bar = 20\,kPa$

Ambient temperature: $T_a = 220\,K$

Solution of the Problem

With the parameters and the working condition defined, the solution can be explained. The configuration of the turbofan engine and the corresponding states can be seen in Figure 1.

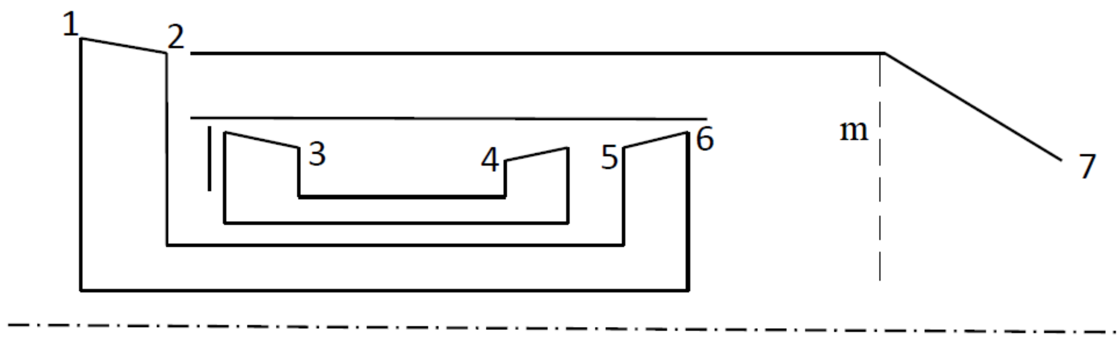

Figure 1: Configuration of the turbofan engine.

Firstly, the stagnation conditions after the intake are calculated. The stagnation temperature is calculated as follows:

$$T_{01} = T_a + \frac{C_a^2}{2c_p} \tag{3}$$

where C_a is the velocity of the intake air. It is calculated as follows:

$$C_a = M_{\text{cruise}}\sqrt{R_{\text{air}}k_{\text{air}}T_a} \tag{4}$$

The stagnation pressure is calculated as follows:

$$p_{01} = p_a(1 + \eta_{in}\frac{C_a^2}{2c_pT_a})^{(k_{air}/(k_{air}-1))} \tag{5}$$

With Equation 3, Equation 4, and Equation 5; the following values are obtained:

$$C_a = 207.65\, m/s \qquad T_{01} = 241.56\, K \qquad p_{01} = 27.74\, kPa$$

Then, the stagnation pressures and temperatures of two separate streams, namely cold stream (through the bypass duct) and hot stream (through the high pressure compressor) are found using the following equations:

$$T_{02} = T_{01}(fpr)^{poly,comp} \tag{6}$$

$$p_{02} = p_{01}(fpr) \tag{7}$$

$$T_{03} = T_{02}(\frac{opr}{fpr})^{poly,comp} \tag{8}$$

$$p_{03} = p_{01}(opr) \tag{9}$$

In these equations, poly,comp denotes the value of $(n-1)/n$ for the polytropic compression. It is calculated as follows:

$$poly, comp = \frac{k_{air} - 1}{k_{air} n_{poly,c}} \tag{10}$$

Its value is found as 0.3175.

Using Equation 6, Equation 7, Equation 8, and Equation 9; the following values are obtained:

$$T_{02} = 310.26\,K \qquad p_{02} = 61.03\,kPa \qquad T_{03} = 604.68\,K \qquad p_{03} = 499.36\,kPa$$

Then, the stagnation pressure before the turbine is calculated using the given pressure drop as follows.

$$p_{04} = p_{03}(1 - \Delta P_{\text{loss}}) \tag{11}$$

where p_{04} is found as $469.40\,kPa$.

The turbine inlet temperature, T_{04} is given as 1200 K.

Since the turbines do just enough work to run the compressor, by the work requirement of the high pressure and the low pressure rotors, the following equations can be written for the stagnation temperatures after the turbines:

$$T_{05} = T_{04} - \frac{c_{p,air}}{\eta_m c_{p,gas}}(T_{03} - T_{02}) \tag{12}$$

$$T_{06} = T_{05} - (bpr + 1)\frac{c_{p,air}}{\eta_m c_{p,gas}}(T_{02} - T_{01}) \tag{13}$$

Using Equation 12 and Equation 13; the following values are obtained:

$$T_{05} = 938.31\,K \qquad T_{06} = 632.97\,K$$

To calculate the stagnation pressures after the turbines the value of $(n-1)/n$ for the polytropic expansion is required. It is calculated as follows:

$$poly, exp = n_{poly,t}\frac{k_{gas} - 1}{k_{gas}} \tag{14}$$

Then the stagnation pressures can be calculated using the following relations:

$$p_{05} = p_{04}(\frac{T_{05}}{T_{04}})^{(1/poly,exp)} \tag{15}$$

$$p_{06} = p_{05}(\frac{T_{06}}{T_{05}})^{(1/poly,exp)} \tag{16}$$

Using Equation 15 and Equation 16; the following values are obtained:

$$p_{05} = 157.16\,kPa \qquad p_{06} = 27.29\,kPa$$

To continue with the solution, the momentum should be found. Thus, the values of p_2, A_2, C_2, p_6, A_6, and C_6 are required. Firstly, it should be noted that in the question, specific thrust is asked. Thus, the mass flow rate through the turbofan can be taken as $1\,kg/s$.

Then, using the bypass ratio, the mass flow rate of cold stream and hot stream can be found as follows:

$$m_c = \frac{bpr}{bpr + 1} \tag{17}$$

$$m_h = 1 - m_c \tag{18}$$

Their values are found as $0.8kg/s$ and $0.2kg/s$, respectively.

To find the pressure and temperature at state 6, the following relations can be written:

$$p_6 = p_{06}\left(1 + \frac{k_{\text{gas}} - 1}{2} M_6^2\right)^{-\frac{k_{\text{gas}}}{k_{\text{gas}}-1}} \tag{19}$$

$$T_6 = \frac{T_{06}}{1 + \frac{(k_{\text{gas}}-1)\ M_6^2}{2}} \tag{20}$$

In the problem, it is stated that the Mach number at state 6, M_6 should be assumed to be equal to 0.5.Then, using Equation 19 and Equation 20; the following values are obtained:

$$p_6 = 23.18\,kPa \qquad T_6 = 607.68\,K$$

To find the velocity at state 6, the following equation can be written:

$$C_6 = M_6\sqrt{k_{gas}R_{gas}T_6} \tag{21}$$

Its value is found as $240.99\,m/s$.

To find the area at state 6, the ideal gas relation and mass flow rate relation can be written as follows:

$$\rho_6 = \frac{p_6}{(R_{gas}T6)} \tag{22}$$

$$A_6 = \frac{m_h}{(R_{gas}T6)} \tag{23}$$

The values of the density and the area are found as $0.1330\,kg/m^3$ and $0.0062\,m^2$, respectively.

Assuming that there is no swirl in the jet pipe downstream, the static pressure will be uniform across the duct, meaning $p2 = p6$.

With the ratio of p_2/p_{02} known, the Mach number at state 2 can be calculated using the following equation:

$$\left(1 + 0.5(k_{\text{air}} - 1)M_2^2\right)^{-\frac{k_{\text{air}}}{k_{\text{air}}-1}} - \frac{p_2}{p_{02}} = 0 \tag{24}$$

This equation is solved using the built-in "fzero" function of MATLAB and M_2 is found as 1.2623.

Then the values of the T_2, C_2, ρ_2, and A_2 are found with the following equations:

$$T_2 = \frac{T_{02}}{1 + \frac{(k_{\text{air}}-1)\ M_2^2}{2}} \tag{25}$$

$$C_2 = M_2 \sqrt{k_{air} R_{air} T_2} \tag{26}$$

$$\rho_2 = \frac{p_2}{(R_{air} T2)} \tag{27}$$

$$A_2 = \frac{m_c}{(R_{air} T2)} \tag{28}$$

Their values are found as follows:

$$T_2 = 235.28\,K \qquad C_2 = 387.25\,m/s \qquad \rho_2 = 0.3448\,kg/m^3 \qquad A_2 = 0.0060\,m^2$$

To find the properties for the mixture, first the constant properties of the mixture should be calculated as follows:

$$c_{p,m} = \frac{m_c c_{p,air} + m_h c_{p,gas}}{m_c + m_h} \tag{29}$$

$$R_m = \frac{m_c R_{air} + m_h R_{gas}}{m_c + m_h} \tag{30}$$

$$k_m = \frac{1}{1 - \frac{R_m}{c_{p,m}}} \tag{31}$$

Their values are found as $1029.6\,J/kgK$, $285.93\,J/kgK$, and 1.3845, respectively.

Then, the stagnation temperature of the mixture is found using the following equation:

$$T_{0m} = \frac{m_c\ c_{p,\text{air}}\ T_{02} + m_h\ c_{p,\text{gas}}\ T_{06}}{m\ c_{p,m}} \tag{32}$$

Its value is found as $382.23\,K$.

Then, the momentum value is found as follows:

$$P_{m1} = m_c C_2 + p_2 A_2 + m_h C_6 + p_6 A_6 \tag{33}$$

This value should be equal to the value of the following equation due to momentum balance:

$$P_{m2} = m C_m + p_m A_m \tag{34}$$

The value of A_m is the sum of the A_2 and A_6. However, the value of p_m and C_m are unknown. They should be found by iterating different values of Mach numbers for the mixture until the momentum balance satisfies.

Firstly, an initial guess is made for M_m. Then, static temperature T_m is found using the following relation:

$$T_m = \frac{T_{0m}}{1 + \frac{(k_{\mathrm{m}}-1)\ M_m^2}{2}} \tag{35}$$

Then, the velocity is found using the following relation:

$$C_m = M_m \sqrt{k_m R_m T_m} \tag{36}$$

Then, the pressure is calculated by combining the ideal gas relation and the mass flow rate relation as follows:

$$p_m = \frac{m R_m T_m}{C_m A_m} \tag{37}$$

After p_m is found, the momentum values are compared and M_m is iterated accordingly until the tolerance is reached. p_m is found as $30.88\,kPa$.

Then, the stagnation pressure of the mixture is found as follows:

$$p_{0m} = \frac{p_m}{\left(1 + \frac{k_m-1}{2} M_m^2\right)^{\frac{k_m}{k_m-1}}} \tag{38}$$

Its value is found as $43.09\,kPa$.

Then the nozzle pressure ratio p_{0m}/p_a is compared with the critical pressure ratio to check if the flow is choked or unchoked.

The critical pressure ratio is calculated as follows:

$$cpr = \frac{1}{\left(1 - \frac{k_m - 1}{(k_m+1)\ \eta_{in}}\right)^{\frac{k_m}{k_m-1}}} \tag{39}$$

Since the nozzle pressure ratio is 2.1547 and the cpr is 1.8836, the flow is choked.

Then, the values of T_7, p_7, $\rho 7$, $C7$, and A_7 are calculated using the following formulas:

$$T_7 = T_{0m} \frac{2}{k_m + 1} \tag{40}$$

$$p_7 = p_{0m} \frac{1}{cpr} \tag{41}$$

$$\rho_7 = \frac{p_7}{R_m T_7} \tag{42}$$

$$C_7 = \sqrt{k_m R_m T_7} \tag{43}$$

$$A_7 = \frac{m}{\rho_7 C_7} \tag{44}$$

Using Equation 40, Equation 41, Equation 42, Equation 43 and Equation 44; the following values are obtained:

$$T_7 = 320.60\,K \qquad p_7 = 22.88\,kPa \qquad \rho_7 = 0.2496\,kg/m^3$$
$$C_7 = 356.25\,m/s \qquad A_7 = 0.0112\,m^2$$

Finally, the specific thrust is calculated using the following formula:

$$\frac{F}{m} = C_7 - C_a + A_7 \frac{(p_7 - p_a)}{m} \tag{45}$$

Its value is found as $180.97\,Ns/kg$.

Additionally, the relevant area ratio $A2/A6$ is found as 0.9602.

Appendix

The code for the question is given in this section.

```
clc
clear
close all

% Define the constants

cp_air = 1000; %J/kg*K
cp_gas = 1148; %J/kg*K
k_air = 1.4;
k_gas = 1.333;
R_air = cp_air*(1-1/k_air); % J/kg*K
R_gas = cp_gas*(1-1/k_gas); % J/kg*K
% R_air = 287; % J/kg*K
% R_gas = 287; % J/kg*K

% Define the given variables in the question

n_poly = 0.9;
bpr = 4;
fpr = 2.2;
opr = 18;
dp_comp = 0.06;
eff_comb = 1;
T_turb = 1200; %K
eff_mech = 0.98;
m = 1;

% n_poly = 0.9;
% bpr = 5;
% fpr = 1.65;
% opr = 25;
% dp_comp = 150; %kPa
% eff_comb = 1;
% T_turb = 1550; %K
% eff_mech = 0.99;
% m = 1;

% Define the cruise condition and the ambient condition

M_cruise = 0.7;
pa = 20; %kPa
Ta = 220; %K

% M_cruise = 0;
% pa = 100; %kPa
% Ta = 288; %K

% Calculate the (n-1)/n values for polytropic compression and expansion

poly_comp = (k_air-1)/(k_air*n_poly);
poly_exp = (k_gas-1)*n_poly/(k_gas);

% Calculate the mass ratios
mc = bpr/(bpr+1);
mh = 1-mc;
```

```
% Calculate the speed of sound

a = sqrt(R_air*k_air*Ta);
Ca = M_cruise*a;


% Assume intake efficiency of 0.95

eff_in = 1;

% Calculate the state 1 temperature and pressure
T01 = Ta+(a*M_cruise)^2/(2*cp_air);
p01 = pa*(1+eff_in*(a*M_cruise)^2/(2*cp_air*Ta))^(k_air/(k_air-1));

% Calculate the state 2 temperature and pressure
T02 = T01*(fpr)^poly_comp;
p02 = p01*fpr;

% Calculate the state 3 temperature and pressure
T03 = T02*(opr/fpr)^poly_comp;
p03 = opr*p01;

% Calculate the state 4 temperature and pressure
T04 = T_turb;
p04 = p03*(1-dp_comp);

% Calculate the state 5 temperature and pressure
T05 = T04 - (cp_air*(T03-T02))/(eff_mech*cp_gas);
p05 = p04*(T05/T04)^(1/poly_exp);

% Calculate the state 6 temperature and pressure
T06 = T05 - (bpr+1)*cp_air*(T02-T01)/(eff_mech*cp_gas);
p06 = p05*(T06/T05)^(1/poly_exp);

% Asuume that the mach number at state 6 is 0.5
M6 = 0.5;

% Calculate the pressure and temperature at state 6 using the standard relations
p6 = p06*(1/(1+(k_gas-1)/2*M6^2))^(k_gas/(k_gas-1));
T6 = T06/(1+(((k_gas-1)*M6^2)/2));
rho6 = p6/(R_gas*T6)*10^3;
C6 = M6*sqrt(k_gas*R_gas*T6);
A6 = mh/(C6*rho6);


% Calculate the pressure and temperature at state 2 using the standard relations
p2=p6;

p2_p02 = p2/p02;

% Define the function for which we want to find the root
f = @(M2) (1 + 0.5*(k_air-1)*M2^2)^(-k_air/(k_air-1)) - p2_p02;

% Initial guess for Mach number
```

```matlab
M0 = 0.5;

% Use fzero to find the root of the function
M2 = fzero(f, M0);
T2 = T02/(1+(((k_air-1)*M2^2)/2));
rho2 = p2/(R_air*T2)*10^3;
C2 = M2*sqrt(k_air*R_air*T2);
A2 = mc/(C2*rho2);


% Calculate the mixture properties
cp_m = (mc*cp_air+mh*cp_gas)/(mc+mh);
R_m = (mc*R_air+mh*R_gas)/(mc+mh);
k_m = 1/(1-R_m/cp_m);

% Calculate T07
T0m = (mc*cp_air*T02+mh*cp_gas*T06)/(m*cp_m);

momentum_m = (mc*C2+p2*1000*A2)+(mh*C6+p6*1000*A6);
Am = A6+A2;

% Make an initial guess for M7
Mm_a = 0.1;
Mm_b = 1;

for i=1:100
    Mm = (Mm_a+Mm_b)/2;
    Tm = T0m/(1+(((k_m-1)*Mm^2)/2));
    Cm = Mm*sqrt(k_m*R_m*Tm);
    pm = m*R_m*Tm/(Cm*Am*1000);
    m_m_calc = m*Cm+Am*pm*1000;
    error = m_m_calc-momentum_m;
    tolerance = 1e-6;

    if m_m_calc < momentum_m  % Stop if the function value is close to zero
        Mm_b = Mm;
    elseif m_m_calc > momentum_m
        Mm_a = Mm;
    end

    if abs(error) < tolerance
        break
    end
end

p0m = pm/((1/(1+(k_m-1)/2*Mm^2))^(k_m/(k_m-1)));

% Calculate the nozzle pressure ratio
p0m_pa = p0m/pa;

% Calculate the critical pressure ratio
p0m_pc = 1/(1-(k_m-1)/((k_m+1)*eff_in))^(k_m/(k_m-1));

% Check if the nozzle is choking
```

```
if p0m_pa > p0m_pc
    fprintf('The nozzle is choking.\n')
else
    fprintf('The nozzle is not choking.\n')
end

% Since the nozzle is choking, T7=Tc and p7=pc
T7 = (2/(k_m+1))*T0m;
p7 = p0m*(1/p0m_pc);
rho7 = p7/(R_m*T7)*10^3;
C7 = sqrt(k_m*R_m*T7);
A7 = m/(rho7*C7);

specific_thrust = C7-Ca+A7*(p7-pa)*1000/m
A2_A6 = A2/A6
```