Practice Questions (For October 30, 2012, Recitation)

1. Three players A, B and C take turns tossing a fair coin. Suppose that A tosses the coin first, B tosses the second and C tosses third and cycle is repeated indefinitely until someone wins by being the first player to obtain a head. Determine the probability that each of the three players will win.

2. A total of 48 percent of the women and 37 percent of the men that took a certain quit smoking class remained nonsmokers for at least one year after completing the class. These people then attend a success party at the end of the year. If 62 percent of the original class were male, what percentage of those attending the party were women?

3. Urn 1 contains 2 white and 4 red balls, whereas Urn 2 contains 1 white and 1 red ball. A ball is randomly chosen from Urn 1 and put into Urn 2, and a ball is then randomly selected from Urn 2.
   a. What is the probability that the ball selected from Urn 2 was white;
   b. What is the conditional probability that the transferred ball was white, given that a white ball is selected from Urn 2?

4. Mary and Tom park their cars in an empty parking lot that consists of N parking spaces in a row. Assume that each possible pair of parking locations is equally likely. Calculate the probability that the parking spaces they select are adjacent. Assume that two cars cannot share a parking space.

5. Two fair, three-sided dice[1] are rolled simultaneously.
   a. Let X be the sum of the two rolls. Calculate the PMF, the expected value, and the variance of X.
   b. As a gambling game, you pay a dollars in advance and get paid $5X$, with X defined as in part (a). What value of a makes it a fair game, i.e., one in which you break even on average?
   c. Repeat parts (a) and (b) for the case where X is the square of the sum of the two rolls.

6. Consider another game played with dice. Each of two players rolls a fair, four-sided die. Player A scores the maximum of the two dice minus 1, which is denoted X. Player B scores the minimum of the two dice, which is denoted Y.
   a. Find the expectations of X, Y, and $X-Y$.
   b. Find the variances of X, Y, and $X-Y$.

7. Let X be a binomial random variable with parameters (n,p). What value of p maximizes $P(X = k)$, $k = 0, 1, \dots, n$?

8. A particular class has had a history of low attendance. Fed up with the situation, the professor decides that she will not lecture unless at least k of the n students enrolled in the class are present. Each student will independently show up with probability p_g if the weather is good, and with probability p_b if the weather is bad. If the chance of bad weather tomorrow is $P(B)$ what is the probability that the professor teaches her class?

[1] With coins and dice, "fair" means that all outcomes are equally likely. Unless otherwise indicated, an n-sided die has faces labeled 1, 2, ..., n. One can't really build a three-sided die, but it is nevertheless a well-defined probabilistic model

9. The president of a company discovers that one of her two vice presidents, A and B is stealing money from the company. In order to determine who is guilty, she decides to hire a private detective to investigate. If she chooses to investigate VP A she will have to pay D_A to the detective, and if A turns out to be guilty, the president will have to pay R_A to replace A Similarly, investigating B has costs D_B and R_B. Furthermore, if the detective decides that one of the VP's is innocent, the president will have to pay the detective to investigate the other VP. If the a priori probability that A is guilty is p, and that B is guilty is $1-p$, find the conditions on p, D_A, D_B, R_A, R_B for which investigating A first would minimize the expected cost of the procedure.