1- Let $a, b, d \in \mathbb{Z}$ $d=0$ $a|b \overset{?}{\iff} da|db$

Assume $a|b \Rightarrow$ $b = a.k$ $k \in \mathbb{Z}$

$d.b = d.a.k$ lets multiply bothsides with $d \in \mathbb{Z}$

$(db) = (da)k$ this gives us $da|db$

Assume $da|db \Rightarrow$ $0.a|0.b$

$0 \backslash 0$ is indeterminate

a, b is irrelevant to $da|db$ since $d=0$

we can not prove that $a|b$

2- n is a composite integer therefore $n = p_1.p_2 ...$ $p_1, p_2, ... \in P$

prime numbers

$\exists p \in P$ $p|n$ that $p \overset{?}{\leq} \sqrt{n}$

$n = p_1.p_2 .....$ and p is smallest prime number

Lets assume $p > \sqrt{n} \Rightarrow p^2 > n$

If $p^2 > n$ in order to reach n we should have another prime number q thats higher than p. So we can say $p.q = n$ but $p.p$ was already higher than n therefore $p.q = n$ wont satisfy in any case we can say from proof with contradiction that $p > \sqrt{n}$

3- $m \in \mathbb{Z}^+$ $\forall x \in \mathbb{R}$ $x \geq 1$

$1 \leq mk < x$ Number of k's that satisfies the condition

minimum of these numbers is m $k=1$ or 0 when $x < m$

maximum of these numbers is $x - x \bmod m$

n_k: number of k's we can find

$$n_k = \frac{x - x \bmod m}{m} = \frac{x}{m} - \underbrace{\frac{\overset{<m}{x \bmod m}}{m}}_{<1} = \left\lfloor \frac{x}{m} \right\rfloor$$

4- $x \in \mathbb{R}$ $\quad 2\lfloor x \rfloor \overset{?}{\leq} \lfloor 2x \rfloor \overset{?}{\leq} 2\lfloor x \rfloor + 1$

We know that $\lfloor x \rfloor \leq x$

$$2\lfloor x \rfloor \leq 2x$$

$$\lfloor \underbrace{2\underbrace{\lfloor x \rfloor}_{\in \mathbb{Z}}}_{\in \mathbb{Z}} \rfloor \leq \lfloor 2x \rfloor$$

Because $\lfloor x \rfloor$ is an integer another floor function will be equal to input $2\lfloor x \rfloor$

$$2\lfloor x \rfloor \leq \lfloor 2x \rfloor \quad (1)$$

$0 < e < 1$ $e \in \mathbb{R}$

$x = \lfloor x \rfloor + e$ $\lfloor 2x \rfloor = \lfloor 2(\lfloor x \rfloor + e) \rfloor = \lfloor 2\lfloor x \rfloor + 2e \rfloor = 2\lfloor x \rfloor + \lfloor 2e \rfloor \leq 2\lfloor x \rfloor + 1$ (2)

From 1 and 2 $\quad 2\lfloor x \rfloor \leq \lfloor 2x \rfloor \leq 2\lfloor x \rfloor + 1$

5- Let $x \in \mathbb{R}$ and $n \in \mathbb{Z}$ with $n > 0$

$$\lfloor \lfloor x \rfloor / n \rfloor \overset{?}{=} \lfloor x/n \rfloor$$

6- Let $a, b \in \mathbb{Z}$ $\quad b < 0$ $\quad (a \bmod b) \overset{?}{\in} (b, 0]$

$a \bmod b = r$ operation defined for a, b being positive integers

If $b < 0$ then mod operator can't operate.