

HOMEWORK # 3 (Due to December 11, 2012, Tuesday)

(15 points each, 20 points is bonus)

1. A couple wishes to have exactly two female children in their family. They will have children until this condition is fulfilled.
 - a. What is the probability that the family has four children.
 - b. What is the probability that the family has at most four children.
 - c. What is the probability that the family has x male children.
 - d. How many children would you expect this family to have?
 - e. How many male children would you expect this family to have?
2. An instructor who taught two sections of probability last term, the first with 20 students and the second with 30, decided to assign a term project. After all projects had been turned in, the instructor randomly ordered them before grading. Consider the first 15 graded projects:
 - a. What is the probability that exactly 10 of these are from the second section?
 - b. What is the probability that at least 10 of these are from the second section?
 - c. What is the probability that at least 10 of these are from the same section?
 - d. What are the mean value and standard deviation of the number among these 15 that are from the second section?
 - e. What are the mean value and standard deviation of the number of projects not among these first 15 that are from the second section?
3. Customers at a gas station pay with a credit card (A), debit card (B), or cash (C). Assume that successive customers make independent choices, with $P(A) = 0.5$, $P(B) = 0.2$, and $P(C) = 0.3$
 - a. Among the next 100 customers, what are the mean and variance of the number customers who pay with a debit card?
 - b. Answer part-a for the number of customers who don't pay with cash.
4. Two different suppliers, A and B, provide the same part to a certain manufacturer. All supplies of this part are kept in a large bin. In the past, 5 percent of the parts supplied by A and 9 percent of the parts supplied by B have been defective. A supplies four times as many parts as B. Suppose you reach into the bin and select a part and observe that it is non-defective. What is the probability that it was supplied by A?
5. Jane fails quizzes with probability $1/4$, independently of other quizzes.
 - a. What is the probability that Jane fails exactly two of the next six quizzes?
 - b. What is the probability that the second and third time Jane fails a quiz will occur when he takes his eighth and ninth quizzes, respectively?
 - c. What is the probability that Jane will pass at least four quizzes before the first fail?
 - d. If it is known that Jane has passed first two quizzes, what is the probability that she will pass at least three more quizzes before the first fail?
 - e. What is the expected number of quizzes that Jane will take until the third fail?
 - f. What is the probability that Jane fails two consecutive quizzes before he passes two consecutive quizzes?

6. A smog alert has been called in a certain area of Izmir in which there are several industrial firms. An inspector will visit 10 randomly selected firms to check for violations of regulations. Let X be the number of firms visited by the inspector that are in violation.
- If there are 50 industrial firms in the area of which 15 are in violation, what is the probability mass function of X ?
 - If there are 500 firms in the area of which 150 are in violation, write an exact expression for $P(X \leq 2)$ and also approximate it using another probability mass function.
 - Compute $E(X)$ and $\text{Var}(X)$ both for the exact probability mass function and the approximating probability mass function you used in part-b.

7. Let X be a random variable denoting the proportion of students in a class who get a grade lower than C. Suppose X has the following probability density function:

$$f_X(x) = \begin{cases} k^2 x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{o.w} \end{cases}$$

- Find k .
 - Find $E(X)$ and $\text{Var}(X)$
 - Find the cumulative distribution function of X .
 - A class is assumed to be unsuccessful if the proportion of students with a grade lower than C is greater than 0.5. What is the probability that a class is unsuccessful?
 - Assume 5 classes take the course and their successes are independent of each other. What is the probability that at least two of the classes will be unsuccessful?
8. The waiting time for a bus in a bus stop, X , has the following density function:

$$f_X(x) = \begin{cases} x, & 0 < x \leq 1 \\ 1/2, & 1 < x \leq 2 \\ 0, & \text{o.w} \end{cases}$$

- Compute $P(0.5 < X < 2)$, $P(0.5 < X \leq 2)$, and $P(0.5 < X \leq 3)$
- Compute the cumulative distribution function of X .
- Compute $E(X)$ and $\text{Var}(X)$.