Assignment 1

1- a) $\sum_{i=1}^{n^2-1} 6i$=6$\sum_{i=1}^{n^2-1} 1$=$\frac{6(n^2-1)n^2}{2}$=$3(n^4-n^2)$

b) $[\ln(n^4)]'$=4 $*\ln(n) = \frac{4}{n}$

2- t_1=$10^{-4}n^2$ $t_a = 10^{-4} * 20^2 = 10^{-4} * 400 = 4 * 10^{-2}$

$t_b = 10^{-4} * n^2 * 10^{-3} = 10^{-7} * n^2 = n^2 * 10^{-7}$

$n^2 * 10^{-7} = 4 * 10^{-2}$

$n \cong 632$

3- $\theta(n\sqrt{n})$ 2ms n=100

2=100$\sqrt{100} * c$ c=0,002

n=2500 t=2500$\sqrt{2500}$*0,002=250 ms

$\frac{250}{2} = 125$

Running time will increase 125 times for size increasing 25 times.

$$\frac{25k\sqrt{25k}}{k\sqrt{k}} = \frac{125k\sqrt{k}}{k\sqrt{k}} = 125$$

4-

1) $\lim_{n\to\infty} \frac{5(3^n)10^5}{(2,5)^{n+2}} = \frac{\infty}{\infty}$ L'Hospital rule $\lim_{n\to\infty} \frac{[5(3^n)10^5]'}{[(2,5)^{n+2}]'} = \lim_{n\to\infty} \frac{5.10^5 \ln 3.3^n}{\ln(2.5)(2.5)^{n+2}}$

$= \lim_{n\to\infty} \frac{3^n}{(2,5)^{n+2}} = \infty$ since $3>2,5$

$A=\omega(B) \Rightarrow A=\Omega(B)$

2) $\lim_{n\to\infty} \frac{3^n}{n+2^n} = \infty$ since $3>2$

$A=\omega(B)$
$\Downarrow$
$A=\Omega(B)$

5-e) def foo2(n):

return (n*(n+1)/2)+(n*(n+1)*(2*n+1)/6)-n*n-n

```
>>> import calc
>>> calc.foo(25)
5200
>>> calc.foo2(25)
5200.0
>>> calc.foo(100)
333300
>>> calc.foo2(100)
333300.0
```

5a)

```
def foo(n):
    q=0
    for j in range n:
        q=q+(j*j)+j
    return q
```

a) n multiplications, 2n additions

b) $O(3n) = O(n)$ c) $\sum_{j=1}^{n-1} j^2+j = \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} - n^2 - n$

d)

```
def foo2(n):
    return (n*(n+1)/2)+(n*(n+1)*(2n+1)/6)-n*n-n
```

6- a) $\lim_{n\to\infty} \frac{\sqrt[3]{n}}{\log_2(n^3)} = \frac{\infty}{\infty} \Rightarrow \lim_{n\to\infty} \frac{[\sqrt[3]{n}]'}{[\log_2(n^3)]'} = \frac{3\cdot\ln 2\cdot n}{3n^{2/3}\cdot} = \frac{n}{n^{2/3}} = \infty$

$\sqrt[3]{n} = \omega(\log_2(n^3))$

b) $t_n = 0.0555\sqrt[3]{n} + 5005\log_2(n^3) \Rightarrow O(\sqrt[3]{n})$ since growth rate of $\sqrt[3]{n}$ is bigger than $\log_2(n^3)$ as we found out in section(a) with applying limit rule.

c) $\lim_{n\to\infty} \frac{\sqrt[3]{n}}{(\log_2 n)^3} = \frac{\infty}{\infty} \Rightarrow \lim_{n\to\infty} \frac{[\sqrt[3]{n}]'}{[(\log_2 n)^3]'} =$

$= \frac{3\cdot 3(\log(n))^2}{3n^{2/3}\cdot(\log 2)^3\cdot n} = \frac{\log^2(n)}{n^{5/3}} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{[\log^2(n)]'}{[n^{5/3}]'} = \frac{2\cdot\log n\cdot 3}{n\cdot 5n^{2/3}} = \frac{\log n}{n^{5/3}} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{3}{n\cdot 5n^{2/3}} = \frac{3}{5n^{5/3}} = 0$

7 - a) $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{i} 1 = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} i = \sum_{i=1}^{n-1} i(n-i) = \sum_{i=1}^{n-1} ni - i^2 = \frac{n(n-1)n}{2} - \frac{(n-1)(n)(2(n-1)+1)}{6}$

$= \frac{n^2(n-1)}{2} - \frac{(n-1)n(2n-1)}{6}$

b) $O(n^3)$

c) Both functions have n^3 order growth while a) gives an exact number of additions needed.

8- Since it is guaranteed that first element of the list repeated 1 time on the rest of the list first loop will only iterate once. Second loop runs from 2nd to nth element of the list chance of finding our element is same on every slot this is binomial distribution.

E=$\frac{1}{n-1}+\frac{2}{n-1}+\frac{3}{n-1}+\cdots+\frac{n-2}{n-1}+\frac{n-1}{n-1}=\frac{n(n-1)}{2(n-1)}=\frac{n}{2}$

Therefore average number of comparisons we have to do is $\frac{n}{2}$. Which is O(n) time complexity