CRYPTANALYSIS of NUMBER THEORETIC CIPHERS

Samuel S. Wagstaff, Jr.



A CRC Press Company
Boca Raton London New York Washington, D.C.

Contents

1	M	athematical Foundations of Cryptanalysis	1
1	Ter	minology of Cryptography	3
	1.1	Notation	3
	1.2	Types of Attacks	4
	1.3	Public Key Ciphers	6
	1.4	Block and Stream Ciphers	7
	1.5	Protocols	10
	1.6	Exercises	10
2	Pro	bability Theory	13
	2.1	Definitions	13
	2.2	The Birthday Problem	15
	2.3	Random Variables	20
	2.4	Exercises	24
3	Div	isibility and Arithmetic	27
	3.1	Divisibility	27
	3.2	Arithmetic with Large Integers	28
	3.3	Greatest Common Divisors and the Euclidean Algorithm	36
	3.4	Exercises	42
4	Pri	mes	45
	4.1	The Fundamental Theorem of Arithmetic	45
	4.2	The Distribution of Prime Numbers	49
	4.3	Identifying and Finding Primes	51
	4.4	The Largest Prime Factor of a Number	54
	4.5	Exercises	59
5	Coı	ngruences	61
	5.1	Simple Properties of Congruences	61
	5.2	Linear Congruences	64
	5.3	The Chinese Remainder Theorem	69
	5.4	Exercises	72

xii Contents

6	Eule	er's Theorem and Its Consequences	75
	6.1	Fermat's Little Theorem	75
	6.2	Euler's Theorem	79
	6.3	Primitive Roots	86
	6.4	Discrete Logarithms	89
	6.5	Exercises	91
7	Seco	ond Degree Congruences	93
	7.1	The Legendre Symbol	94
	7.2	The Law of Quadratic Reciprocity	98
	7.3	The Jacobi Symbol	100
	7.4	Euler Pseudoprimes	103
	7.5	Solving Quadratic Congruences Modulo m	104
	7.6	Exercises	110
8	Info	rmation Theory	111
	8.1	Entropy	111
	8.2	Perfect Secrecy	114
	8.3	Unicity Distance	115
	8.4	Some Obsolete Ciphers	117
	8.5	The Entropy of Number Theoretic Ciphers	121
	8.6	Exercises	122
9	Cno	ups, Rings and Fields	125
ð	9.1		125
	$9.1 \\ 9.2$	Groups	$\frac{125}{127}$
		Simple Properties of Groups	
	9.3	The Baby-Step-Giant-Step Algorithm	130
	9.4	Rings and Fields	132
	9.5	Polynomials	133
	9.6	Algebraic Number Theory	137
	9.7	Exercises	140
10	_	onential Methods of Factoring Integers	143
		Fermat's Difference of Squares Method	143
		Pollard's Rho Method	146
		Pollard's $p-1$ Method	149
	10.4	Square Form Factorization	151
	10.5	Exercises	153
11	Fine	ling Large Primes	155
		Stronger Probable Prime Tests	156
		Lucas Probable Prime Tests	160
		Rigorous Proof of Primality	165
		Prime Proofs for Arbitrary Large Integers	169
		Exercises	169

Contents	xiii
Contents	XIII

12	Elliptic Curves	171
	12.1 Definitions and Examples	171
	12.2 Factoring with Elliptic Curves	176
	12.3 Primality Proving with Elliptic Curves	181
	12.4 Exercises	182
13	Subexponential Factoring Algorithms	185
	13.1 Factoring with Continued Fractions	185
	13.2 The Quadratic Sieve	190
	13.3 Variations of the Quadratic Sieve	193
	13.3.1 Large Primes	193
	13.3.2 Multiple Polynomials	194
	13.3.3 The Self-Initializing Quadratic Sieve	195
	13.4 The Number Field Sieve	196
	13.5 Exercises	201
		000
14	Computing Discrete Logarithms	203
	14.1 Shanks' Baby-Step-Giant-Step Method	204
	14.2 Pollard's Methods	204
	14.2.1 The Rho Method for Discrete Logarithms	204
	14.2.2 The Lambda Method for Discrete Logarithms	$\frac{205}{206}$
	14.3 Discrete Logarithms via Index Calculus	200
	14.4 Other Fast Methods for the Group R_m	210
	14.5 Exercises	210
15	Random Number Generation	211
	15.1 Linear Feedback Shift Registers	212
	15.2 A Quadratic Residue Random Number Generator	215
	15.3 Hash Functions	216
	15.4 Generating Truly Random Numbers	217
	15.5 Exercises	218
II	The Cryptographic Algorithms	219
		007
16	Private Key Ciphers	221 221
	16.1 Rijndael, the Advanced Encryption Standard	
	16.1.1 Byte Arithmetic in Rijndael	222
	16.1.2 Word Arithmetic in Rijndael	$\frac{224}{225}$
	16.1.4 The Key Schedule of Rijndael	$\frac{225}{227}$
	16.1.5 Summary of Rijndael	$\frac{227}{227}$
	16.1.5 Summary of Hijhdaer	228
	16.3 Elliptic Curve Pohlig-Hellman	228
	16.4 Exercises	230
		_00

•	~
XIV	Contents
Alv	Concinos

17	Pub	lic Key Ciphers	231
	17.1	Rivest-Shamir-Adleman	231
	17.2	Massey-Omura	232
	17.3	Elliptic Curve Massey-Omura	233
	17.4	ElGamal	233
	17.5	Elliptic Curve ElGamal	234
	17.6	Rabin-Williams	235
	17.7	Exercises	237
18	Sign	ature Algorithms	239
	18.1	Rivest-Shamir-Adleman Signatures	239
	18.2	ElGamal Signatures	240
	18.3	Rabin-Williams Signatures	241
	18.4	The Digital Signature Algorithm	242
	18.5	Exercises	244
19	Key	Exchange Algorithms	245
	19.1	Key Exchange Using a Trusted Server	245
	19.2	The Diffie-Hellman Key Exchange	248
	19.3	The X.509 Key Exchange	249
	19.4	Exercises	251
20	Sim	ple Protocols	253
	20.1	Bit Commitment	253
		Mental Poker	253
	20.3	Oblivious Transfer	255
	20.4	Zero-knowledge Proofs	256
	20.5	Methods of Sharing Secrets	258
		20.5.1 Secret Splitting	258
		20.5.2 The Lagrange Interpolating Polynomial Scheme	258
		20.5.3 The Asmuth and Bloom Threshold Scheme	260
	20.6	Blind Signatures	261
	20.7	Exercises	261
21	Con	aplicated Protocols	263
		Contract Signing	263
	21.2	Secure Elections	265
	21.3	Electronic Cash	268
		21.3.1 Electronic Cash According to Chaum	268
		21.3.2 Electronic Cash According to Brands	271
	21.4	Exercises	274

į

ŧ

Contents xv

22 Complete Systems	275
22.1 Kerberos	275
22.2 Pretty Good Privacy	277
22.3 Exercises	278
III Methods of Attack	279
23 Direct Attacks	281
23.1 Try All Keys	281
23.2 Factor a Large Integer	283
23.3 Solve a Discrete Logarithm Problem	284
23.4 Timing Attacks	286
23.5 Exercises	287
24 Exploiting an Error	289
24.1 Key Management	289
24.2 Reuse of a Key	290
24.3 Bad Parameter Choice	291
24.4 Partial Key Exposure	293
24.5 Computer Failure	293
24.6 Exercises	294
25 Active Attacks	297
25.1 Force a User to Make a Mistake	297
25.2 Man-in-the-Middle Attacks	298
25.3 Birthday Attacks	300
25.4 Subliminal Channels	300
25.5 Exercises	301
References	303
Index	311