

제13강(10장)

지분계획과 분할구계획

지난 시간

- 10.1 교차실험과 지분실험
- 10.2 이단지분계획
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제13강(10장)

지분계획과 분할구계획

이번 시간

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10.4 분할구계획

10.4 분할구계획

<표 10-9> 전자부품의 수명 자료: 이원배치법의 자료구조

온도T (°F) 가열시간 B (분)				
	580	600	620	640
5	217	158	229	223
	188	126	160	201
	162	122	167	182
10	233	138	186	227
	201	130	170	181
	170	185	181	201
15	175	152	155	156
	195	147	161	172
	213	180	182	199

10.4 분할구계획

<표 10-10> 전자부품의 수명 자료: 분할구계획의 자료구조

반복 R	온도T (°F)	580	600	620	640
	가열시간 B (분)				
I	5	217	158	229	223
	10	233	138	186	227
	15	175	152	155	156
II	5	188	126	160	201
	10	201	130	170	181
	15	195	147	161	172
III	5	162	122	167	182
	10	170	185	181	201
	15	213	180	182	199

10.4 분할구계획

<표 10-11> 분할구계획에서 랜덤화 방법의 예

반복 1

a_1	a_0	a_2
b_0	b_1	b_0
b_1	b_0	b_1

반복 2

a_2	a_1	a_0
b_1	b_1	b_0
b_0	b_0	b_1

반복 3

a_1	a_2	a_0
b_0	b_1	b_1
b_1	b_0	b_0

10.4 분할구계획

◆ 분할구계획에 대한 통계 모형(모형 1)

$$x_{ijk} = \mu + \underbrace{\rho_k + \alpha_i + (\rho\alpha)_{ki}}_{\text{주구}} + \underbrace{\beta_j + (\rho\beta)_{kj} + (\alpha\beta)_{ij} + (\rho\alpha\beta)_{kij}}_{\text{세구}}$$

$(k = 1, \dots, r; i = 1, \dots, a; j = 1, \dots, b)$

- ρ 항: 반복 R=I, II, III ($r=3$)
- α 항: 온도 T=580, 600, 620, 640 ($a=4$)
- $\rho\alpha$ 항: 반복 R과 온도 T간 상호작용효과 {주구오차(whole-plot error)}
- β 항: 가열시간 B=5, 10, 15 ($b=3$)
- $\rho\beta, \alpha\beta, \rho\alpha\beta$: 여러 가지 상호작용효과 (이들 중 $\rho\alpha\beta$ 는 세구오차임)
 - = split-plot error
 - = subplot error

10.4 분할구계획

◆ 분할구계획에 대한 통계 모형(모형 2)

$$x_{ijk} = \mu + \underbrace{\rho_k + \alpha_i + (\rho\alpha)_{ki}}_{\text{주구}} + \underbrace{\beta_j + (\rho\beta)_{kj} + (\alpha\beta)_{ij} + (\rho\alpha\beta)_{kij}}_{\text{세구}} \quad (\text{모형1})$$

$$x_{ijk} = \mu + \underbrace{\rho_k + \alpha_i + e_{(1)ik}}_{\text{주구(1차 단위)}} + \underbrace{\beta_j + (\alpha\beta)_{ij} + e_{(2)ijk}}_{\text{세구(2차 단위)}} \quad (\text{모형2})$$

여기서 $e_{(1)ik} = (\rho\alpha)_{ki}$, $e_{(2)ijk} = (\rho\beta)_{kj} + (\rho\alpha\beta)_{kij}$,

$$e_{(1)ik} \sim N(0, \sigma_{E_1}^2), e_{(2)ijk} \sim N(0, \sigma_{E_2}^2).$$

10.4 분할구계획

<표 10-14> 분할법 데이터에 대한 분산분석표(모형2) (A, B 모수인자, R 변량인자)

	요인	SS	ν	V	$E(V)$	F_0
1차단위	R	SS_R	$r-1$	V_R	$\sigma_{E_2}^2 + b\sigma_{E_1}^2 + ab\sigma_R^2$	V_R / V_{E_1}
	A	SS_A	$a-1$	V_A	$\sigma_{E_2}^2 + b\sigma_{E_1}^2 + br\sigma_A^2$	V_A / V_{E_1}
	E_1	SS_{E_1}	$(r-1)(a-1)$	V_{E_1}	$\sigma_{E_2}^2 + b\sigma_{E_1}^2$	V_{E_1} / V_{E_2}
2차단위	B	SS_B	$b-1$	V_B	$\sigma_{E_2}^2 + ar\sigma_B^2$	V_B / V_{E_2}
	$A \times B$	$SS_{A \times B}$	$(a-1)(b-1)$	$V_{A \times B}$	$\sigma_{E_2}^2 + r\sigma_{A \times B}^2$	$V_{A \times B} / V_{E_2}$
	E_2	SS_{E_2}	$a(b-1)(r-1)$	V_{E_2}	$\sigma_{E_2}^2$	
T		SS_T	$abr-1$			

$$SS_{E_1} = SS_{A \times R} = SS_{AR} - SS_A - SS_R,$$

$$SS_{AR} = \sum_{i=1}^a \sum_{k=1}^r \frac{T_{i.k}^2}{b} - CT$$

$$SS_{E_2} = SS_T - (SS_R + SS_A + SS_{E_1} + SS_B + SS_{A \times B}) \text{ 또는 } SS_{E_2} = SS_{B \times R} + SS_{A \times B \times R}$$

10.4 분할구계획

<표 10-13> 분산분석표(모형2)

요인	제공합	자유도	평균제공	F_0
R (반복)	1962.7	2	981.4	3.32
A (온도)	12494.3	3	4164.8	14.08
E_1	1773.9	6	295.7	0.48
B (시간)	566.2	2	283.1	0.46
$A \times B$	2600.4	6	433.4	0.70
E_2	9933.3	16	620.8	
T	29331.0	35		

$> F(3, 6; 0.01)$

10.4 분할구계획

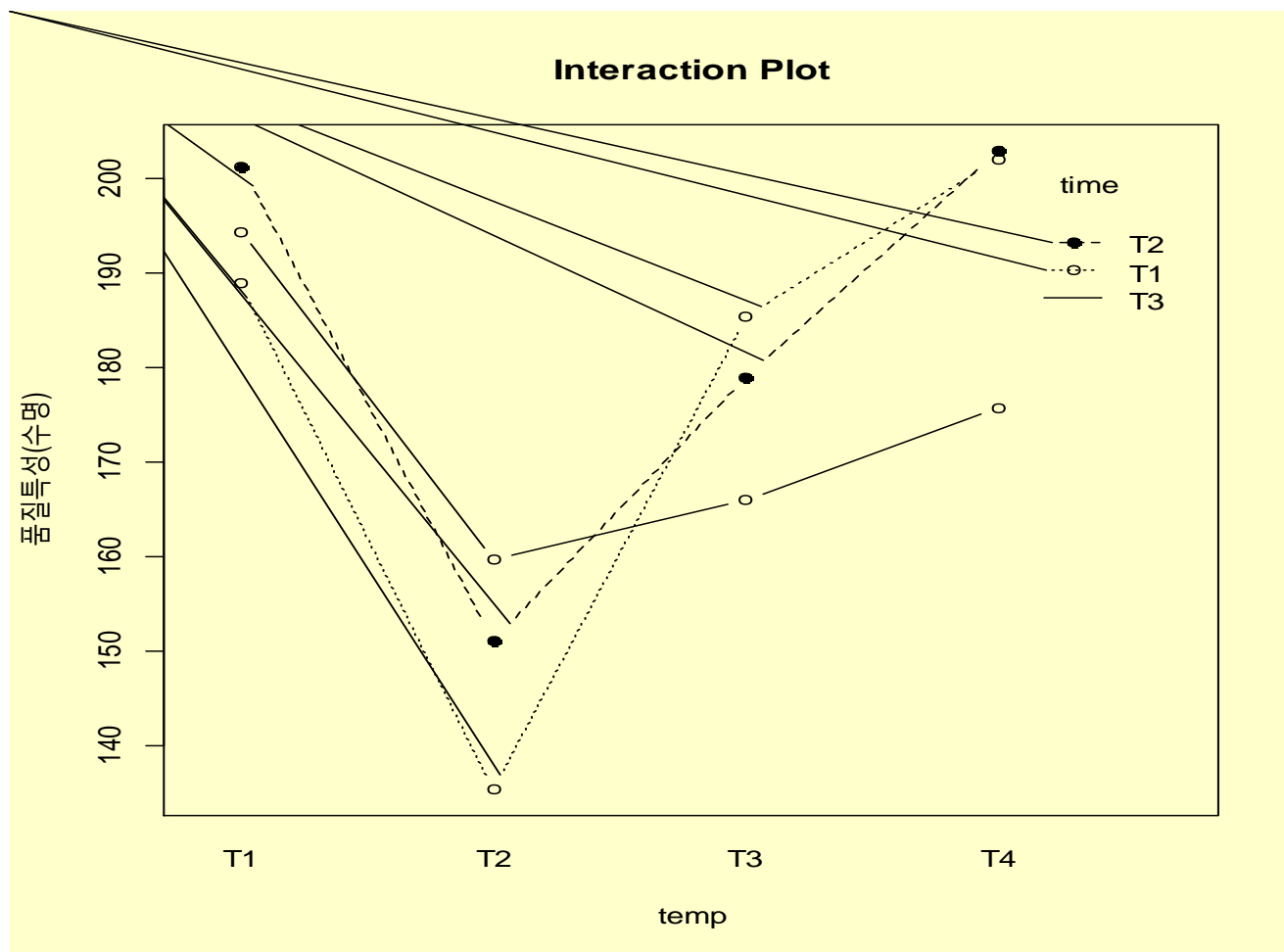
R 실습

```
life <- c(217, 158, 229, 223, 233, 138, 186, 227, 175, 152, 155, 156,  
         188, 126, 160, 201, 201, 130, 170, 181, 195, 147, 161, 172,  
         162, 122, 167, 182, 170, 185, 181, 201, 213, 180, 182, 199)  
  
rho <- c(rep(1, 12), rep(2, 12), rep(3, 12))  
  
time <- rep(c(rep(1, 4), rep(2, 4), rep(3, 4)), 3)  
  
temp <- rep(c(1, 2, 3, 4), 9)  
  
split.data <- data.frame(life, rho, time, temp)  
  
split.data$rho <- factor(split.data$rho, levels=c(1, 2, 3), labels=c("R1", "R2", "R3"))  
  
split.data$time <- factor(split.data$time, levels=c(1, 2, 3), labels=c("Time1", "Time2", "Time3"))  
  
split.data$temp <- factor(split.data$temp, levels=c(1, 2, 3, 4), labels=c("Temp1", "Temp2", "Temp3", "Temp4"))  
  
model3 <- aov(life~temp*time+ Error(rho/temp), data=split.data)  
  
summary(model3) # 표 <10-13>
```

10.4 분할구계획

R 실습

```
with(split.data, interaction.plot(x.factor=temp, trace.factor=time, response=life, fun=mean, type="b",  
legend=T, ylab="품질특성(수명)", main="Interaction Plot", pch=c(1,19)))
```



line type

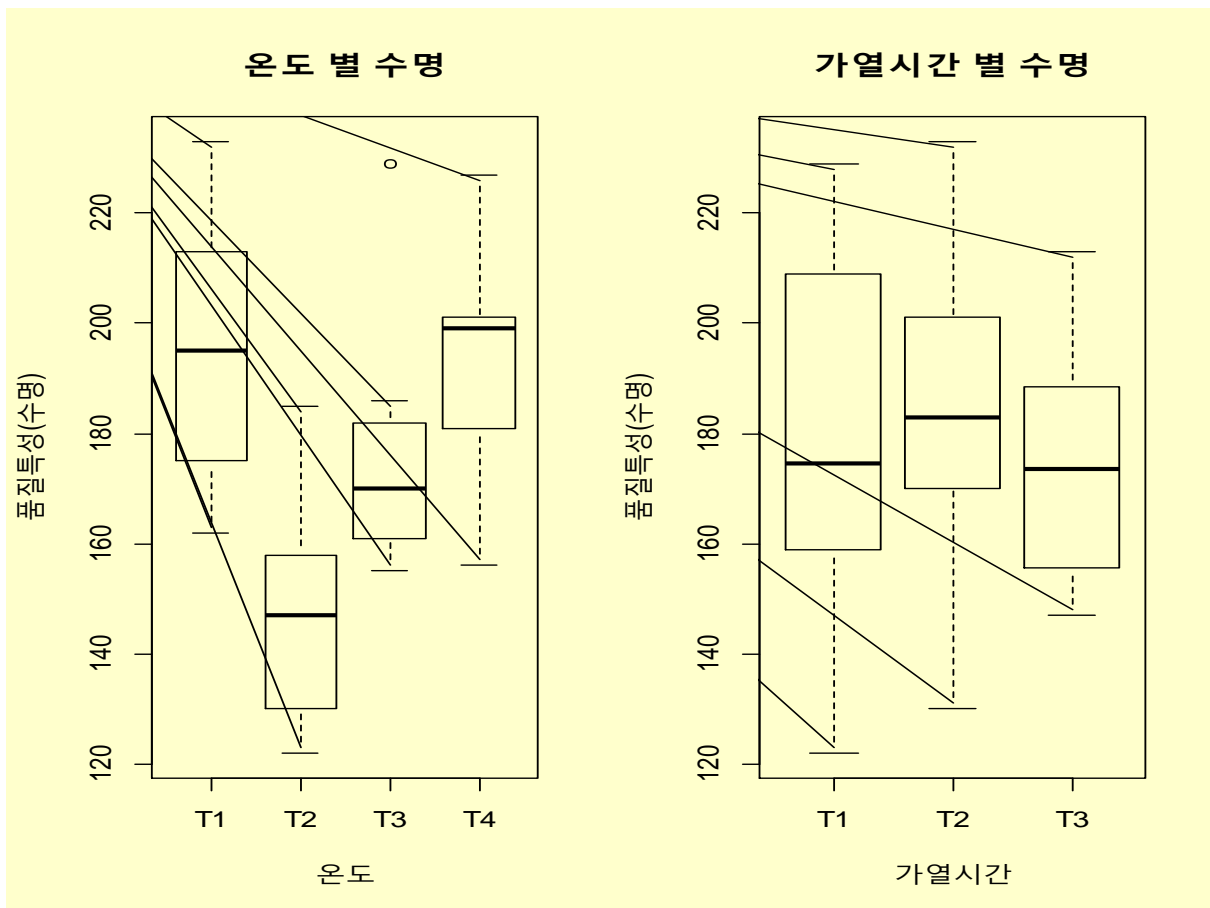
10.4 분할구계획

R 실습

```
par(bg=rgb(1,1,0.8),mfrow=c(1,2))
```

```
boxplot(life~temp, data=split.data, main="온도 별 수명",  
xlabs="온도",ylabs="품질특성(수명)")
```

```
boxplot(life~time, data=split.data, main="가열시간 별 수명",  
xlabs="가열시간",ylabs="품질특성(수명)")
```



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10.5 이단분할구계획

10.5 이단분할구계획

<표 10-14> 고무의 경화율 자료

					온도(℃)					
		145			155			165		
		혼합물원료			혼합물원료			혼합물원료		
반복	실험실	A	B	C	A	B	C	A	B	C
I	1	18.6	14.5	21.1	9.5	7.8	11.2	5.4	5.2	6.3
	2	20.0	18.4	22.5	11.4	10.8	13.3	6.8	6.0	7.7
	3	19.7	16.3	22.7	9.3	9.1	11.3	6.7	5.7	6.6
II	1	17.0	15.8	20.8	9.4	8.3	10.0	5.3	4.9	6.4
	2	20.1	18.1	22.7	11.5	11.1	14.0	6.9	6.1	8.0
	3	18.3	16.7	21.9	10.2	9.2	11.0	6.0	5.5	6.5
III	1	18.7	16.5	21.8	9.5	8.9	11.5	5.7	4.3	5.8
	2	19.4	16.5	21.5	11.4	9.5	12.0	6.0	5.0	6.6
	3	16.8	14.4	19.3	9.8	8.0	10.9	5.0	4.6	5.9
IV	1	18.7	17.6	21.0	10.0	9.1	11.1	5.3	5.2	5.6
	2	20.0	16.7	21.3	11.5	9.7	11.5	5.7	5.2	6.3
	3	17.1	15.2	19.3	9.5	9.0	11.4	4.8	5.4	5.8

10.5 이단분할구계획

◆ 모형 1

$$\begin{aligned}x_{ijkl} = & \mu + \rho_l + \alpha_i + (\rho\alpha)_{li} \\& + \beta_j + (\rho\beta)_{lj} + (\alpha\beta)_{ij} + (\rho\alpha\beta)_{lij} \quad \dots\dots (10.56) \\& + \gamma_k + (\rho\gamma)_{lk} + (\alpha\gamma)_{ik} + (\rho\alpha\gamma)_{lik} + (\beta\gamma)_{jk} + (\rho\beta\gamma)_{ljk} + (\alpha\beta\gamma)_{ijk} + (\rho\alpha\beta\gamma)_{lijk} \\& l = 1, \dots, r; i = 1, \dots, a; j = 1, \dots, b; k = 1, \dots, c\end{aligned}$$

10.5 이단분할구계획

◆ 모형 2

$$x_{ijkl} = \mu + \rho_l + \alpha_i + e_{(1)li}$$

주구(whole plot, 1차 단위)

$$+ \beta_j + (\alpha\beta)_{ij} + e_{(2)lij}$$

세구(split plot, 2차 단위)

$$+ \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + e_{(3)lijk}$$

세세구(split-split plot, 3차 단위)

$$l = 1, \dots, r; i = 1, \dots, a; j = 1, \dots, b; k = 1, \dots, c$$

$$e_{(1)li} \sim N(0, \sigma_{E_1}^2), \quad e_{(2)lij} \sim N(0, \sigma_{E_2}^2), \quad e_{(3)lijk} \sim N(0, \sigma_{E_3}^2)$$

10.5 이단분할구계획

<표 10-18> 이단분할법의 분산분석표 (A, B, C 모수인자, R변량인자) (모형2)

요인	SS	ϕ	V	$E(V)$
R	SS_R	$r - 1$	V_R	$\sigma_{E_3}^2 + c\sigma_{E_2}^2 + bc\sigma_{E_1}^2 + abcr\sigma_R^2$
A	SS_A	$a - 1$	V_A	$\sigma_{E_3}^2 + c\sigma_{E_2}^2 + bc\sigma_{E_1}^2 + bcr\sigma_A^2$
E_1	SS_{E_1}	$(a - 1)(r - 1)$	V_{E_1}	$\sigma_{E_3}^2 + c\sigma_{E_2}^2 + bc\sigma_{E_1}^2$
B	SS_B	$(b - 1)$	V_C	$\sigma_{E_3}^2 + c\sigma_{E_2}^2 + acr\sigma_B^2$
$A \times B$	$SS_{A \times B}$	$(a - 1)(b - 1)$	$V_{A \times B}$	$\sigma_{E_3}^2 + c\sigma_{E_2}^2 + cr\sigma_{A \times B}^2$
E_2	SS_{E_2}	$a(b - 1)(r - 1)$	V_{E_2}	$\sigma_{E_3}^2 + c\sigma_{E_2}^2$
C	SS_C	$c - 1$	V_C	$\sigma_{E_3}^2 + abr\sigma_C^2$
$A \times C$	$SS_{A \times C}$	$(a - 1)(c - 1)$	$V_{A \times C}$	$\sigma_{E_3}^2 + br\sigma_{A \times C}^2$
$B \times C$	$SS_{B \times C}$	$(b - 1)(c - 1)$	$V_{B \times C}$	$\sigma_{E_3}^2 + ar\sigma_{B \times C}^2$
$A \times B \times C$	$SS_{A \times B \times C}$	$(a - 1)(b - 1)(c - 1)$	$V_{A \times B \times C}$	$\sigma_{E_3}^2 + r\sigma_{A \times B \times C}^2$
E_3	SS_{E_3}	$ab(c - 1)(r - 1)$	V_{E_3}	$\sigma_{E_3}^2$
T	SS_T	$abcr - 1$		

10.5 이단분할구계획

<표 10-16> 분산분석표(모형2)

요인	제곱합	자유도	평균제곱	F_0
R(반복)	9.414	3	3.138	
A(실험실)	40.664	2	20.332	$7.57 < F(2, 6; 0.01) = 10.9$
E_1	16.110	6	2.685	
B(온도)	3119.51	2	1559.76	$2835.93 > F(2, 18; 0.01) = 6.01$
$A \times B$	4.936	4	1.234	2.24
E_2	9.883	18	0.55	
C(혼합물원료)	145.718	2	72.859	$293.4 > F(2, 54; 0.01) = 5.02$
$A \times C$	0.339	4	0.085	0.342
$B \times C$	43.687	4	10.922	$43.987 > F(4, 54; 0.01) = 3.69$
$A \times B \times C$	1.077	8	0.135	0.544
E_3	13.406	54	0.2483	
T	3404.74	107		

10.5 이단분할구계획

R 실습

```
hard <- c(18.6, 14.5, 21.1, 9.5, 7.8, 11.2, 5.4, 5.2, 6.3,  
20, 18.4, 22.5, 11.4, 10.8, 13.3, 6.8, 6, 7.7,
```

[[중략]]

```
17.1, 15.2, 19.3, 9.5, 9, 11.4, 4.8, 5.4, 5.8)
```

```
R <- c(rep(1, 27), rep(2, 27), rep(3, 27), rep(4, 27)) # R=반복
```

```
A <- rep(c(rep(1, 9), rep(2, 9), rep(3, 9)), 4) # A=실험실
```

```
B <- rep(c(1, 1, 1, 2, 2, 2, 3, 3, 3), 12) # B=온도
```

```
C <- rep(c(1, 2, 3), 36) # C=혼합물원료
```

```
hard.data <- data.frame(hard, R, A, B, C)
```

```
hard.data$R <- factor(hard.data$R, levels=c(1, 2, 3, 4), labels=c("R1", "R2", "R3", "R4"))
```

```
hard.data$A <- factor(hard.data$A, levels=c(1, 2, 3), labels=c("Room1", "Room2", "Room3"))
```

```
hard.data$B <- factor(hard.data$B, levels=c(1, 2, 3), labels=c("B1", "B2", "B3"))
```

```
hard.data$C <- factor(hard.data$C, levels=c(1, 2, 3), labels=c("C1", "C2", "C3"))
```

```
modell <- aov(hard~A*B*C+ Error(R/A/B/C),data=hard.data)
```

```
summary(modell) # <표 10-16>
```

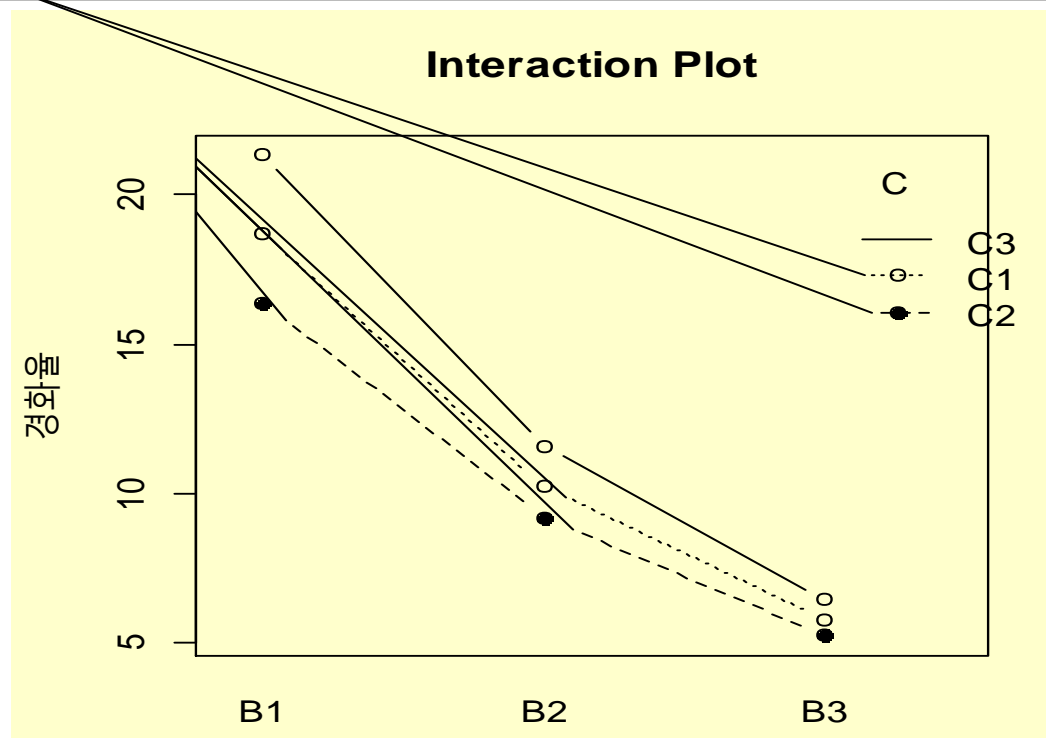
10.5 이단분할구계획

R 실습

```
with(hard.data, interaction.plot(x.factor=A, trace.factor=B, response=hard, fun=mean, type="b",  
legend=T, ylab="경화율", main="Interaction Plot", pch=c(1,19)))
```

```
with(hard.data, interaction.plot(x.factor=A, trace.factor=C, response=hard, fun=mean, type="b",  
legend=T, ylab="경화율", main="Interaction Plot", pch=c(1,19)))
```

```
with(hard.data, interaction.plot(x.factor=B, trace.factor=C, response=hard, fun=mean, type="b",  
legend=T, ylab="경화율", main="Interaction Plot", pch=c(1,19)))
```



10.5 이단분할구계획

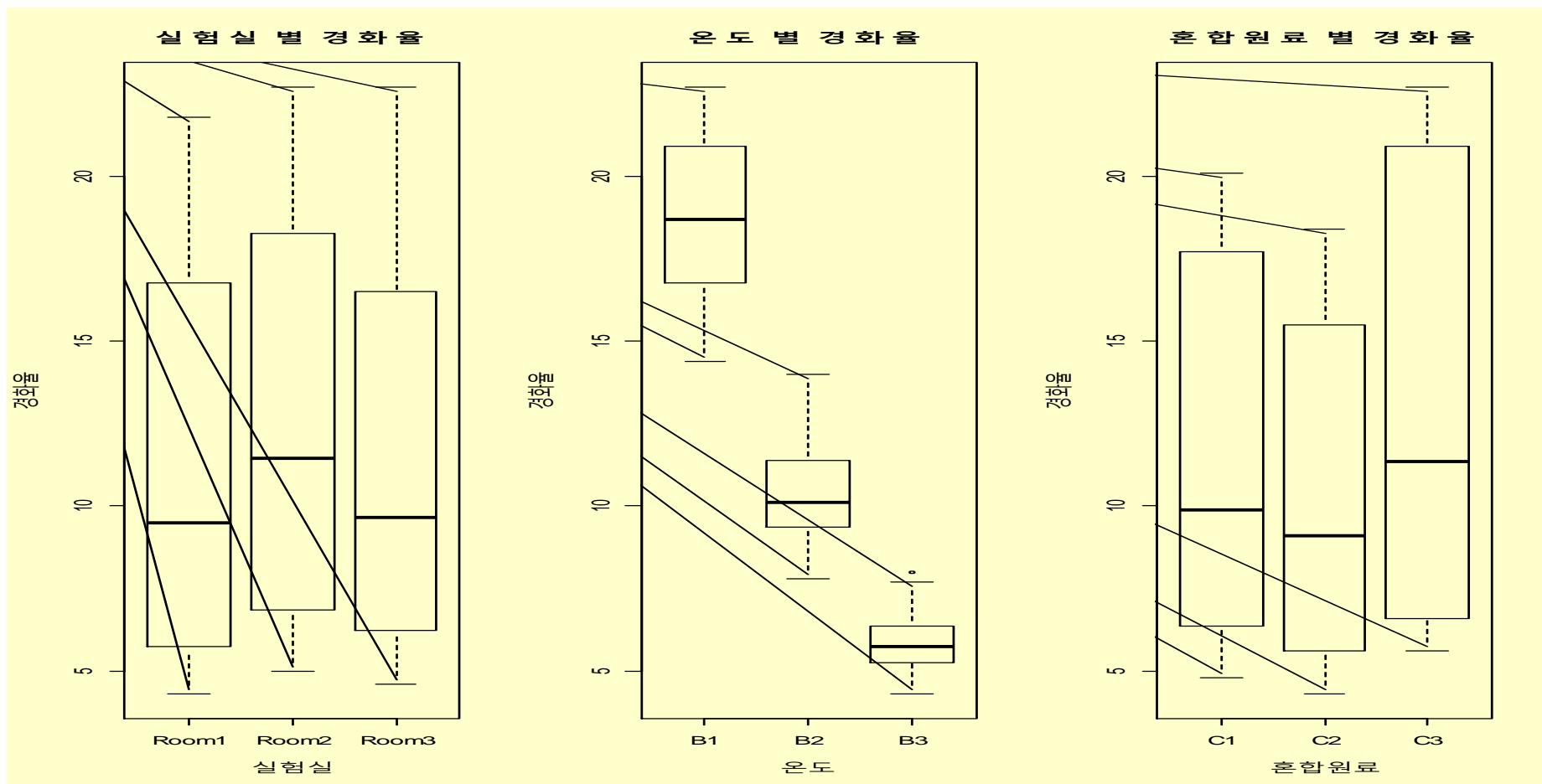
R 실습

```
par(bg=rgb(1,1,0.8),mfrow=c(1,3))
```

```
boxplot(hard~A, data=hard.data, main="실험실 별 경화율", xlab="실험실",ylab="경화율")
```

```
boxplot(hard~B, data=hard.data, main="온도 별 경화율", xlab="온도",ylab="경화율")
```

```
boxplot(hard~C, data=hard.data, main="혼합원료 별 경화율", xlab="혼합원료",ylab="경화율")
```



다음 시간 안내

제14강(11장)

반응표면분석