Noise analysis of fractional-order two-integrator CCII low-pass filter using Pspice

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- *Invited Paper* -

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***Abstract*—In this paper output noise is investigated for several fractional-order filters. Examples are given using the low-pass Butterworth current-mode active-RC filter approximations. It is demonstrated experimentally that as with integer-order filters, output noise decreases with decreasing order of fractional-order filters. The considered circuits use CCIIs and have two integrators in the feedback loop. The fractional-order filter has one fractional capacitance realized by a second Foster circuit. Other components are of integer order. The noise comparison is simulated with Pspice.**

Keywords*—*second-generation current conveyor; Biquad; Pspice; fractional calculus; constant phase element; noise

I. INTRODUCTION

Recently fractional-order calculus has gained significant attention by researchers [1]–[5]. The reasons are manifold. Fractional components are better in modelling biological tissues, describing processes in control theory, materials theory, diffusion theory, robotics, signal processing, and so on. Besides, as shown in recent publications, fractional-order circuits possess a capability for on-chip implementation and low-voltage operation [4], and applications in bioengineering [5]. The importance of noise reduction is also presented in [5].

In classical filter theory, we design filters starting from specifications, by first calculating the filter order, which is most often a non-integer value. We then round this number to the nearest higher, integer-order value which becomes the filter order. As we shall show in this paper by keeping the order of the filter to be non-integer, which means fractional-calculus, we can reduce output noise significantly. Furthermore, an additional benefit of the fractional-calculus design approach is the possibility of controlling the slope of the filter roll-off. The latter can be reduced to $(n-1+\alpha)\cdot 20$dB/dec from $n\cdot 20$dB/dec, providing us with an additional degree of freedom.

In this paper, using the example of a current-mode Biquad filter, we shall replace one of the two integer-order capacitors with one fractional-order capacitor. In this way, our circuit becomes fractional. We then compare the output noise of two filter realizations, one fractional and the other integer. All design equations are given, and a noise analysis using the Orcad Pspice circuit simulator is carried out.

Recently in [6] a second-order band-pass (BP) filter using the AD 844 input circuit for the positive second-generation current conveyors (CCII+) is presented. For our noise analysis in this paper we use the low-pass (LP) output of that circuit and the noise model of the CCII+ in [6].

In Section 2 a brief definition of fractional calculus is given. In Section 3, a current-mode two-integrator Biquad is presented with its design equations. Two versions are described: one is a common Biquad of the second order, the other is a Biquad of fractional order. In Section 4 with the example of a Butterworth filter, the transfer-function output noise analysis using Orcad Pspice is presented. The noise reduction, reducing to the lower, fractional-order filter, is demonstrated.

II. FRACTIONAL CALCULUS

In literature many definitions of fractional derivatives and integrals exist. One of the most common definitions is the Riemann-Liouville fractional differentiator-integrator given by

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \,, \tag{1}$$

where a is a lower border on the interval of integration (it defines initial conditions), α is an arbitrary complex number which represents a fractional order and is given by

$$ n-1 < \alpha < n \,, \tag{2}$$

and

$$ \Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt, \; z \in C \,, \tag{3}$$

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is the Euler Gamma function (also known as a factorial) defined for a complex argument z [7].

In circuit and filter theory our interest is concentrated with the case of $0<\alpha<1$ in (2) where α is a real number. Thus (1) takes on the simpler form

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_a^t \frac{f(\tau)}{(t-\tau)^\alpha}d\tau . \quad (4)$$

The Gamma function in (3), will also have a real and positive argument in (4). It can readily be shown that when α is an integer number, (4) reduces to a classical, integer-order derivative.

An important property of the fractional derivative of the order α defined by (4), which corresponds to analysis in the time domain, is that it possesses a Laplace transform defined by

$$L\left[{}_aD_t^\alpha f(t)\right] = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k\, {}_aD_t^{\alpha-k-1} f(0) . \quad (5)$$

When the initial conditions are zero (a=0), this becomes:

$$L\left[{}_0D_t^\alpha f(t)\right] = s^\alpha F(s) . \quad (6)$$

This *simplifies the analysis in the frequency domain* significantly; in that s is a Laplace variable and α is an arbitrary order. For integer-order and positive α, we have a classical, integer-order derivative, while for integer-order and negative α we have a classical, n-times repeated, or n-fold, integral [7].

A. The fractional-order capacitance

In this section, we shall simulate a fractional-order capacitance of the order α=0.2, 0.5, and 0.8, and investigate its influence on the Biquad transfer function and output noise. The fractional-order capacitor is also referred to as a constant-phase element (CPE), or *fractance* component. The component symbol and its realization is shown in Fig. 1. Other realizations e.g. Foster I, Cauer I and II of CPEs are shown in [8] and [9].

The impedance of the fractional-order capacitor (CPE) is defined by

$$Z(s) = \frac{1}{C_\alpha s^\alpha} , \quad (7)$$

where α ($0<\alpha<1$) is the order of CPE, and C_α is a normalized pseudo capacitance in [F/sec$^{(1-\alpha)}$]. We refer to C_α as a pseudo capacitor, because it is a capacitor that is dependent on frequency, and on the fractional order α, and does not possess a common dimension in Farads. To simplify the circuit design, the value of capacitor C in [F] which approximates a CPE, can readily be calculated from the value of the pseudo capacitor C_α at a frequency ω_0, from:

$$C = \frac{C_\alpha}{\omega_0^{1-\alpha}} . \quad (8)$$

The phase of the impedance of the fractional capacitance is fixed and equal to $\alpha\cdot\pi/2$. The approximation of CPE by a network containing integer-order components, as in Fig. 1, is currently the only way to simulate a fractional-order capacitor in PSpice.

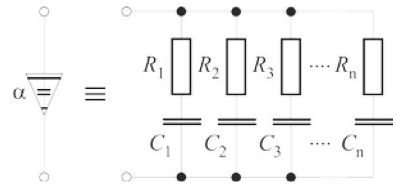

Fig. 1. Fractional capacitor schematic symbol, and Foster II implementation.

In what follows we present the realization of two two-integrator Biquads (one of which is a fractional-order integrator) in current mode using the Pspice-versions CCII+ as the input circuit of the AD 844. We have decided to present current-mode example because it operates at high frequencies, and is fully integrable, because of grounded components.

III. CURRENT-MODE TWO-INTEGRATOR BIQUAD

A. Integer-order filter circuit

For our analysis, we consider the "two-integrator" current-mode (CM) Biquad shown in Fig. 2, which realizes a LP characteristic. The Biquad consists of only positive current conveyors (CCII+) and can therefore be realized by the CCII in the AD 844 device. We can replace one or both capacitors of the Biquad by fractional capacitors (see Fig. 1) and can generalize the transfer function by introducing one or two additional degrees of freedom.

Note that for the case α_1=1, and α_2=1, the circuit in Fig. 2 is a classical-integer-order Biquad as in [6], with integer-order capacitors. In this case the LP current transfer function $T(s)$ in terms of the pole frequency Ω_p, the pole Q, Q_p, and the dimensionless gain factor k_{LP} is given by

$$T_{LPF}(s) = \frac{I_{out}(s)}{I_{in}(s)} = \frac{k_{LP}\cdot\Omega_p^2}{s^2 + (\Omega_p/Q_p)s + \Omega_p^2} ; \quad (9)$$

$$\Omega_p = \sqrt{\frac{R_5}{R_6}\frac{1}{R_1R_2C_1C_2}} ,\; Q_p = \frac{R_8R_1C_1\Omega_p}{R_5} ,\; k_{LP} = \frac{R_7}{R_{16}} . \quad (10)$$

B. Fractional-order filter circuit of the order (1+α)

For the case when $0<\alpha_1<1$ and $0<\alpha_2<1$, the filter circuit becomes a fractional-order Biquad of the order $n=\alpha_1+\alpha_2$. The transfer function is given by [10]

$$T_{FLPF}(s) = \frac{I_{out}(s)}{I_{in}(s)} = \frac{a}{s^{\alpha_1+\alpha_2} + bs^{\alpha_2} + c} , \quad (11)$$

where coefficients a, b and c readily follow from (10), and are given by:

$$a = \frac{R_7}{R_{16}}\frac{R_5}{R_6}\frac{1}{R_1R_2C_1C_2} ;\; b = \frac{R_5}{R_8R_1C_1} ;\; c = \frac{R_5}{R_6}\frac{1}{R_1R_2C_1C_2} . \quad (12)$$

Note that we use the same notation as in [10]. From (11) the amplitude- and phase-frequency characteristics are given by:

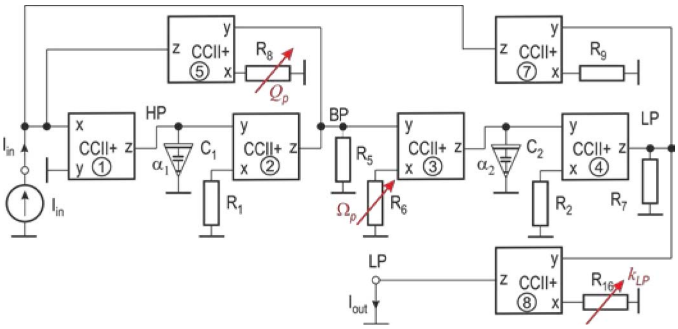

Fig. 2. Second-order, two-integrator, LP current-mode filter using CCII+, with two fractional-order capacitors of the order α_1 and α_2. Note that parameters Ω_p, Q_p, and k_{LP} can be tuned independently of each other.

$$\left|T_{FLPF}(j\omega)\right| = \\ = \frac{a}{\sqrt{\begin{aligned}&2c\cos\left(\frac{(\alpha_1+\alpha_2)\pi}{2}\right)\omega^{\alpha_1+\alpha_2}+2bc\cos\left(\frac{\alpha_2\pi}{2}\right)\omega^{\alpha_2}+\\&+2bc\cos\left(\frac{\alpha_1\pi}{2}\right)\omega^{\alpha_1+2\alpha_2}+b^2\omega^{2\alpha_2}+c^2+\omega^{2(\alpha_1+\alpha_2)}\end{aligned}}}, \tag{13}$$

and

$$\angle T_{FLPF}(j\omega) = \\ = \tan^{-1}\left[-\frac{\omega^{\alpha_1+\alpha_2}\sin\left(\frac{(\alpha_1+\alpha_2)\pi}{2}\right)+b\omega^{\alpha_2}\sin\left(\frac{\alpha_2\pi}{2}\right)}{\omega^{\alpha_1+\alpha_2}\cos\left(\frac{(\alpha_1+\alpha_2)\pi}{2}\right)+b\omega^{\alpha_2}\cos\left(\frac{\alpha_2\pi}{2}\right)+c}\right]. \tag{14}$$

Equations (13) and (14) are suitable for our analysis because they separate two fractional constants α_1 and α_2, representing two fractional capacitances with admittances $s^{\alpha 1}C_{\alpha 1}$ and $s^{\alpha 2}C_{\alpha 2}$. In earlier works (see also [11]) fractional filters were analyzed in a simpler way, namely by choosing $\alpha_1=\alpha_2=\alpha$. It should be noted that the –3dB cut-off frequencies of the obtained filters, are shifted; they readily follow from (13), and are determined by a real root of [10]

$$\begin{aligned}&\omega^{2(\alpha_1+\alpha_2)}+2\omega^{\alpha_1+\alpha_2}c\cos\left(\frac{(\alpha_1+\alpha_2)\pi}{2}\right)+\omega^{2\alpha_2}b^2+\\&+2\omega^{\alpha_1+2\alpha_2}b\cos\left(\frac{\alpha_1\pi}{2}\right)+2\omega^{\alpha_2}bc\cos\left(\frac{\alpha_2\pi}{2}\right)-c^2=0.\end{aligned} \tag{15}$$

To develop a filter example of the order $n=1+\alpha$ from the above expressions, we substitute $0<\alpha=\alpha_1<1$ and $\alpha_2=1$. The first visible effect of the fractional order of the circuit is the change of the slope of the LP characteristics from 20dB/dec to $\alpha\cdot$20dB/dec, where $0<\alpha<1$. It can also be seen from (15) that the −3dB cut-off frequency differs with different α [10].

All the characteristics of the filter (e.g. cut-off frequency, slope) become functions of α. This adds an extra degree of freedom in the filter design. Thus, if a cut-off frequency is defined by $\omega_0=(1/RC)^{1/\alpha}$, besides the capacitor and resistor values, an additional design parameter is the order α of the capacitor. For example, we can design high-frequency filters using a smaller α, while keeping the same time constants (i.e. the same RC product) [8].

There have been many attempts at constructing physical fractional-order capacitors and inductors (see also [12]), as well as designing circuits that approximate a CPE (see also [13]). However, because we do not have commercial fractional-order components to replace our capacitors, we use the passive-RC one-port shown in Fig. 1 *for Pspice simulations.*

IV. EXAMPLE

As an example for a second-order LP filters with cut-off frequency f_p=10kHz ($\Omega_p=2\pi\cdot f_p$), we selected a second-order Butterworth approximation with pole Q-value, Q_p=0.707, and unity pass-band gain k_{LP}=1. To perform the noise analysis, we need to design a physical circuit.

The design process for a fractional-order filter, starting from specifications, can be summarized by the following steps:

i) design a filter as an integer-order filter, rounding the calculated order to the next higher integer,

ii) replace one or more capacitors by appropriate fractional capacitors (CPEs),

iii) adjust the unmatched cut-off frequency, which is changed by the CPE, to the originally desired one.

In step *i*) we let the $C_1=C_2=C_0$, $R_1=R_2=R_0$, $R_5=R_6=R_7=R_9=R_0$, in (10) and obtain

$$\Omega_p=1/(R_0C_0),\ Q_p=R_8/R_5 \text{ and } k_{LP}=R_7/R_{16}. \tag{16}$$

Selecting C_0=10nF with the resulting resistor level $R_0=1/(\Omega_p\cdot C_0)$= 1.59155kΩ, we obtain $R_8=R_5\cdot Q_p$= =1.59155kΩ·0.707 = 1.125 kΩ, and $R_{16}=R_7/k_{LP}$= 1.59155kΩ. This is a second-order Butterworth low-pass circuit.

In step *ii*) we now examine the case that one of the capacitors in the second-order filter is replaced by a fractional capacitor. The more complicated case when two capacitors have different fractional order are described in [10] and [14]. With PSpice, we simulate the three cases α=0.2, α=0.5 and 0.8 and compare them to the original with α=1. Because we select one conventional and one fractional capacitor of the order $\alpha<1$ the resulting filter order is $n=1+\alpha<2$. In [11] it is shown that the fractional-order filter having $\alpha<1$ is always stable.

In step *iii*) above the cut-off frequencies in our examples are adjusted according to (15) (e.g. by multiplication of R_1 and R_2 or R_6, see Fig. 2).

A circuit which approximates a capacitor, which is the only component we replaced in the design, by a constant-phase element (CPE) is shown in Fig. 1. It is used to simulate a capacitor C=10nF at 10kHz, i.e. $|Z|$=1.59155kΩ, which approximates a CPE of the order α=0.2, 0.5, and 0.8 corresponding to a constant phase angle at high frequencies of −18°, −45°, and −72°, respectively (see also [8]). For this, a fifth-order continued fraction expansion (CFE) is used. The passive element values are from [8] and are given in Table I; they are adjusted for our frequency range around cut-off

10kHz. More details on the calculation of the values in Table I, and of the amplitude and phase characteristics of the obtained approximation of a fractance component, are given in [8].

TABLE I. PASSIVE ELEMENT VALUES FOR APROXIMATING A CPE WITH C=10NF, AT 10KHZ (α=0.2, 0.5, AND 0.8) USING FOSTER II TOPOLOGY.

Element	α=0.2	α=0.5	α=0.8
R_0	3.9928k	17.5k	110.96k
R_1	1.4538k	0.1773k	0.0217k
R_2	4.6372k	1.5106k	1.1844k
R_3	7.2915k	3.7539k	4.9682k
R_4	8.0789k	6.1949k	12.2k
R_5	6.1943k	8.0587k	25.3k
C_1	0.4033n	1.8558n	5.3629n
C_2	0.8964n	2.1974n	2.1723n
C_3	1.9477n	3.1833n	2.0141n
C_4	5.7566n	6.2204n	2.6335n
C_5	40.8n	22.9n	5.5627n

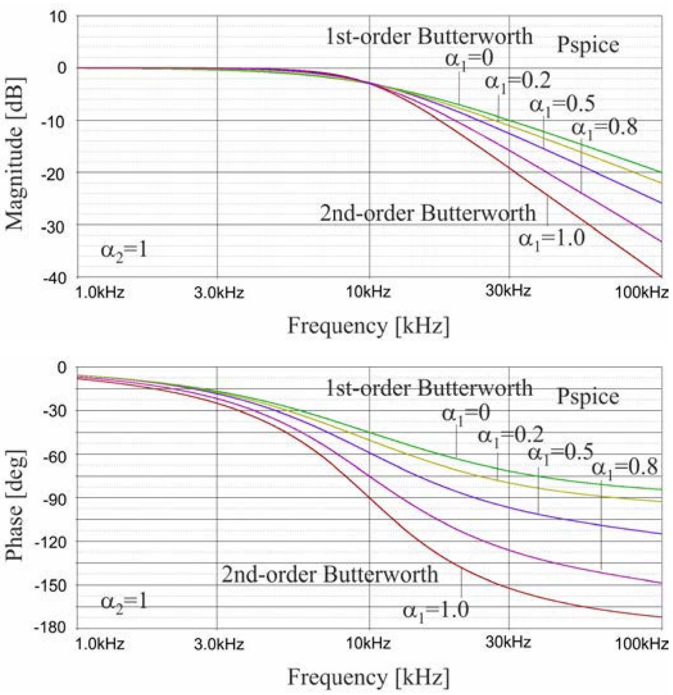

Fig. 3. (a) Amplitude-frequency and (b) phase-frequency characteristics of LP filter examples. Butterworth filters of integer-orders n=1 and 2, and of fractional-orders $n=1+\alpha_1$; with α_1=0.2; 0.5; 0.8, are presented.

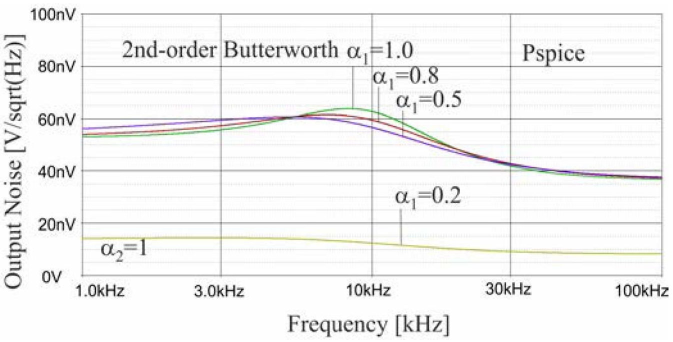

Fig. 4. Total output noise-voltage spectral density of the second-order LP filter examples as in Fig. 3 generated by Orcad Pspice.

We do not care about the total capacitance in Table I, at this point, we use the two-port in Fig. 1, for the simulation purpose of the CPE, only (in the future the two-port will be replaced by a single physical component). All other components remain the same as calculated by (16) for the integer-order circuit. Note that the first-order Butterworth filter example is designed by a simple RC two-port. In Fig. 3 the amplitude- and phase-frequency characteristics obtained by Pspice are shown. In Fig. 3(a) the slope of the filter curves, which is equal to $(1+\alpha)\cdot 20$dB/dec, and in Fig. 3(b) the constant phase of $(1+\alpha)\cdot(\pi/2)$ at high frequencies for $\alpha=\alpha_1$=0, 0.2, 0.5, 0.8 and 1 are shown. The output voltage-noise spectral density was also simulated by Pspice for both the integer-order filter circuit and for the fractional-order filter circuit and is shown in Fig. 4.

The simulation was carried out with the input circuit of the commercially available current-feedback operational amplifier (CFOA) AD 844 for the CCII+, the noise model of which is defined in [6]. Typical values for this device as given in the data-sheet (Analog Devices) are $e_{ny}(f)$= 2nV/√Hz, $i_{nx}(f)$= 10pA/√Hz and $i_{ny}(f)$= 12pA/√Hz.

Fig. 4 demonstrates that by lowering the order $n=1+\alpha$, the noise at the filter output is increasingly reduced from α=0.8 to α=0.2, by a much larger amount. Thus, the reduction in noise is caused by the reduction of the fractional order of the filter.

V. CONCLUSIONS

In this paper, it is shown empirically that reducing the order of a low-pass filter from integer to lower non-integer order reduces the output noise of the filter. This is demonstrated with the example of a Butterworth low-pass filter using Orcad Pspice. To analyze the noise of the fractional-order filter, we have used a CCII noise model in [6]. The two-integrator filter section uses one fractional capacitor, integer resistors and an integer capacitor for its realization. For the active amplifiers it uses the CCII part of the AD 844 current-feedback operational amplifier (CFOA). Theoretical calculations will be published shortly.

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