OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
DEPARTMENT OF PHYSICS AND ENGINEERING PHYSICS

PHY101: GENERAL PHYSICS I

TUTORIAL SET ONE

DUE DATE IS TUESDAY, MAY 30, 2017.
SUBMIT T0 GROUP HEADS. GROUP HEADS (ONLY) CAN SUBMIT TO PY200B OR PY101B. CALL 08024500480, 08130816628 FOR ENQUIRIES.

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1. Earth is approximately a sphere of radius 6.37×10^6 m. What are (a) its circumference in kilometers, (b) its surface area in square kilometers, and (c) its volume in cubic kilometers?

2. A lecture period (50 min) is close to 1 microcentury. (a) How long is a microcentury in minutes? (b) Using percentage difference $= (\frac{\text{actual}-\text{approximation}}{\text{actual}})$, find the percentage difference from the approximation

3. Determine which of the following equations are dimensionally correct: (a) $v_f = v_i + ax$ (b) $y = (2\ m) \cos\,(kx)$, where $k = 2\ m^{-1}$.

4. A person takes a trip, driving with a constant speed of 89.5 km/h, except for a 22 minute rest stop. If the person's average speed is 77.8 km/h, (a) how much time is spent on the trip and (b) how far does the person travel?

5. The position of a particle moving along an x axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t = 3.0$ s. (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t = 0$)? (i) Determine the average velocity of the particle between $t = 0$ and $t = 3$ s.

6. An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s^2 in a straight line until it reaches a speed of 20 m/s. It then slows at a constant rate of 1.0 m/s^2 until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop?

7. A motorcycle is moving at 30 m/s when the rider applies the brakes, giving the motorcycle a constant deceleration. During the 3.0 s interval immediately after braking begins, the speed decreases to 15 m/s.What distance does the motorcycle travel from the instant braking begins until the motorcycle stops?

8. A vector $\vec{B}$, with a magnitude of 8.0 m, is added to a vector $\vec{A}$, which lies along an x axis. The sum of these two vectors is a third vector that lies along the y axis and has a magnitude that is twice the magnitude of $\vec{A}$. What is the magnitude of $\vec{A}$?

9. Two vectors are given by $\vec{a} = (4.0\ m)\hat{\imath} - (3.0\ m)\hat{\jmath} + (1.0\ m)\hat{k}$ and $\vec{b} = (-1.0\ m)\hat{\imath} + (1.0\ m)\hat{\jmath} + (4.0\ m)\hat{k}$. What is the (a) magnitude of $\vec{a}+\vec{b}$? In unit-vector notation, find (b) $\vec{a}+\vec{b}$, (c) $\vec{a}-\vec{b}$, and (d) a third vector $\vec{c}$ such that $\vec{a}-\vec{b}+\vec{c}=0$?

10. A ship sets out to sail to a point 120 km due north. An unexpected storm blows the ship to a point 100 km due east of its starting point. (a) How far and (b) in what direction must it now sail to reach its original destination?

11. Given $\vec{a}\cdot\vec{b} = ab\ cos\ \theta$ and $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$ calculate the angle between the two vectors given by $\vec{a} = 3.0\hat{\imath} + 3.0\hat{\jmath} + 3.0\hat{k}$ and $\vec{b} = 2.0\hat{\imath} + 1.0\hat{\jmath} + 3.0\hat{k}$.

12. A vector of magnitude 10 units and another vector of magnitude 6.0 units differ in directions by 60°. Find (a) the scalar product of the two vectors and (b) the magnitude of the vector product $\vec{a}\times\vec{b}$.

Professor M. A. Eleruja
May 25, 2017

PHY 101 — GENERAL PHYSICS I

SOLUTIONS TO TUTORIAL SET ONE

1. Radius of the spherical Earth, $R = 6.37 \times 10^6$ m $= 6.37 \times 10^3$ km

   (a) Circumference, $s = 2\pi R = 4.00 \times 10^4$ km

   (b) Surface Area, $A = 4\pi R^2 = 5.10 \times 10^8$ km^2

   (c) Volume, $V = \frac{4}{3}\pi R^3 = 1.08 \times 10^{12}$ km^3

2. (a) 1 microcentury $= (10^{-6}\ \cancel{\text{century}}) \left(\frac{100\ \cancel{yr}}{1\ \cancel{century}}\right) \left(\frac{365\ \cancel{day}}{1\ \cancel{yr}}\right) \left(\frac{24\ \cancel{hr}}{1\ \cancel{day}}\right) \left(\frac{60\ min}{1\ \cancel{hr}}\right)$
   $= 52.6\ min$

   (b) The % difference $= \dfrac{52.6\ min - 50\ min}{52.6\ min} = 4.9\%$

3. (a) $v_f = v_i + ax$; $[v_f] = [v_i] = LT^{-1}$, $[a] = LT^{-2}$, $[x] = L$
   so we have, $LT^{-1} = \underline{LT^{-1} + L^2T^{-2}}$; recall that quantities can be added only if they have the same dimensions (terms to be added on RHS do not have the same dimensions), hence, the equation is not dimensionally correct.

   (b) $y = (2\ m) \cos\ (kx)$, where $k = 2\ m^{-1}$; $[y] = L$, $[2\ m] = L$, $[kx] = [(2\ m^{-1})x] = L^{-1}L$. Therefore we can think of the quantity kx as an angle in radians, and we can take its cosine. The cosine itself will be a pure number with no dimensions. We now have, $L = L$. This equation is dimensionally correct.

4. (a) Total time for the trip, $t_{total} = t_1 + 22\ min = t_1 + 0.37\ h$, t_1 is the time spent travelling at $v_1 = 89.5\ km/h$. The distance travelled, $\Delta x = v_1 t_1 = v_{avg} t_{total}$ gives, $(89.5\ km/h)t_1 = (77.8\ km/h)(t_1{+}0.367\ h), t_1 = 2.44\ h$; $t_{total} = t_1{+}0.37\ h = \underline{2.81\ h}$

   (b) Distance travelled during the trip, $\Delta x = v_1 t_1 = v_{avg} t_{total}$,
   $\Delta x = v_{avg} t_{total} = (77.8\ km/h)(2.81\ h) = \underline{219\ km}$

5. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$ we obtain the velocity and the acceleration functions: $v(t) = 24t - 6t^2$ and $a(t) = 24 - 12t$ with length in meters and time in seconds. Substituting 3 for t yields $x(3) = \underline{54m}$.

   (b) Similarly, For $t = 3, v(3) = 18\ ms^{-1}$.

   (c) For $t = 3, a(3) = -12\ ms^{-2}$.

   (d) At the maximum x, we must have $v = 0$; eliminating the t = 0 root, the velocity equation reveals $t = 24/6 = 4\ s$ for the time of maximum x. Inserting $t = 4$ into the equation for x leads to $x = 64\ m$ for the largest x value reached by the particle.

   (e) From (d), we see that the x reaches its maximum at $t = 4.0\ s$.

   (f) A maximum v requires a = 0, which occurs when t = 24/12 = 2.0 s. This, inserted into the velocity equation, gives $v_{max} = 24\ m/s$.

   (g) From (f), we see that the maximum of v occurs at $t = 24/12 = 2.0\ s$.

   (h) In part (e), the particle was (momentarily) motionless at t = 4 s. The acceleration at that time is readily found to be $24 - 12(4) = -24\ ms^{-2}$.

   (i) At $t = 0, x = 0$ and at $t = 3\ s, x = 54\ m, v_{avg} = (54 - 0)/(3 - 0)\ ms^{-1}$.

6. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0; v = 20\ ms^{-1}$ and $a = 2\ ms^{-2}$. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20\ ms^{-1}; v = 0$ and $a = -1\ ms^{-2}$ (negative because the acceleration vector points opposite to the direction of motion).

   (a) For part 1, from $v = v_0 + at, 20 = 0 + 2t_1,\ t_1 = 10\ s$. Also, for part 2, $0 = 20 + (-1.0)t_2$ yields $t_2 = 20\ s$, and the total is $t = t_1 + t_2 = \underline{30\ s}$.
   (b) For part 1, taking $x_0 = 0$, we use $v^2 = v_0^2 + 2a(x - x_0)$, $x = \dfrac{v^2 - v_0^2}{2a} = \dfrac{(20\ ms^{-1})^2 - (0)^2}{2(2)\ ms^{-2}} = 100\ m$. This position is then the initial position for part 2, and with $x - x_0 = \dfrac{v^2 - v_0^2}{2a}$, we obtain, $x - 100\ m = \dfrac{(0)^2 - (20\ ms^{-1})^2}{2(-1)\ ms^{-2}} = \underline{300\ m}$.

7. $v_0 = +30\ m/s$; $v_1 = +30\ m/s$; $t_1 = 3\ s$; $a = (v_1 - V_0)/t_1 = -5\ m/s^2$. The displacement to the point it stops ($v_2 = 0$) using $v_2^2 = v_0^2 + 2a\Delta x$ is $\Delta x = -\dfrac{30\ (m/s)^2}{2(-5\ m/s^2)} = 90\ m$

8. Let A denote the magnitude of $\vec{A}$; similarly for the other vectors. The vector equation is $\vec{A} + \vec{B} = \vec{C}$ where B = 8.0 m and C = 2A. The angle between $\vec{A}$ and $\vec{C}$ is 90°, which makes this a right triangle where B is the size of the hypotenuse. Using the Pythagorean theorem, $B = \sqrt{A^2 + C^2}$, we then have, $8.0 = \sqrt{A^2 + 4A^2}$ and $A = 8/\sqrt{5} = 3.6\ m$

9. (a) $\vec{a} + \vec{b} = [4.0 + (-1.0)]\hat{\imath} + [(-3.0) + (1.0)]\hat{\jmath} + [1.0 + 4.0]\hat{k} = (3.0\hat{\imath} - 2.0\hat{\jmath} + 5.0\hat{k})$ m
   $|\vec{a} + \vec{b}| = (\sqrt{3.0^2 - 2.0^2 + 5.0^2})\ \text{m}^2 = \sqrt{38} = 6.16\ m^2$
   (b) $\vec{a} + \vec{b} = [4.0 + (-1.0)]\hat{\imath} + [(-3.0) + (1.0)]\hat{\jmath} + [1.0 + 4.0]\hat{k} = (3.0\hat{\imath} - 2.0\hat{\jmath} + 5.0\hat{k})$ m
   (c) $\vec{a} - \vec{b} = [4.0 - (-1.0)]\hat{\imath} + [(-3.0) - (1.0)]\hat{\jmath} + [1.0 - 4.0]\hat{k} = (5.0\hat{\imath} - 4.0\hat{\jmath} - 3.0\hat{k})$ m
   (d) $\vec{a} - \vec{b} + \vec{c} = 0$ leads to $\vec{c} = \vec{b} - \vec{a}$ which is the opposite of $\vec{a} - \vec{b}$ in item (c) above. Thus, $\vec{c} = (-5.0\hat{\imath} + 4.0\hat{\jmath} + 3.0\hat{k})$ m

10. The vector sum of the displacements $\vec{d}_{storm}$ and $\vec{d}_{new}$ must give the same result as its originally intended displacement $\vec{d}_0 = (120\ km)\hat{\jmath}$ where east is $\hat{\imath}$ and north is $\hat{\jmath}$. Thus, we write: $d_{storm} = (100\ km)\hat{\imath}, d_{new} = A\hat{\imath} + B\hat{\jmath}$

    (a) $\vec{d}_{storm} + \vec{d}_{new} = \vec{d}_0$ readily yields $A = -100\ km$ and $B = 120\ km$.
    The magnitude of $\vec{d}_{new} = |\vec{d}_{new}| = \sqrt{A^2 + B^2} = 156\ km$
    (b) The direction is $tan^{-1}(B/A) = -50.2°$ or $180° + (-50.2°) = 129.8°$

11. Since $\vec{a}\cdot\vec{b} = ab\ cos\ \theta$ and $\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$, then, $\theta = cos^{-1}\left(\dfrac{a_xb_x + a_yb_y + a_zb_z}{ab}\right)$
    $a = |\vec{a}| = \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.20$; $b = |\vec{b}| = \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.74$
    The angle between them, $\theta = cos^{-1}\left(\dfrac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.20)(3.74)}\right) = 22°$

12. Given: $a = |a| = 10, b = |b| = 6.0$ and $\theta = 60°$, **(a)** $\vec{a}\cdot\vec{b} = ab\ cos\ \theta = (10)(6.0)\ cos\ 60° = 30$; **(b)** $|\vec{a} \times \vec{b}| = ab\ sin\ \theta = (10)(6.0)\ sin\ 60° = 52$.