Finding Values for the PI Controllers based on Input Poles

```
syms s a b l g Kp Ki Jp Ji Ci %define symbolic variables

Hvtheta = -s/l/(s^2 -g/l); % TF from velocity to angle of pendulum
K = Kp + Ki/s; % TF of the angle controller
J = Jp + Ji/s + Ci/s^2; % TF of the controller around the motor
M = a*b/(s+a); % TF of motor
Md = M/(1 + M*J); % TF of motor + feedbac controller around it

pretty(collect(Md));
```

```
                      2
               (a b) s
----------------------------------------
 3               2
s  + (a + Jp a b) s  + Ji a b s + Ci a b
```

```
Htot = 1/(1 - Hvtheta*Md*K); % the total system function
pretty(simplify(Htot)) % display total system function
```

```
                                1
- ------------------------------------------------------------
                          2
                   a b s  (Ki + Kp s)
  ------------------------------------------------------- - 1
        2          2    3                    2
  (- l s  + g) (a s  + s  + Ci a b + Jp a b s  + Ji a b s)
```

```
% system parameters
g = 9.81;
l = 0.3879;
a = 17.84;
b =  0.009341;

Htot_subbed = subs(Htot) %substitue parameters found above
```

Htot_subbed =

$$\frac{1}{\dfrac{30019813371509505425\, s \left(\mathrm{Kp}+\dfrac{\mathrm{Ki}}{s}\right)}{69877851818280615936 \left(s^2-\dfrac{10900}{431}\right) \left(\dfrac{1200792534860380217 \left(\mathrm{Jp}+\dfrac{\mathrm{Ci}}{s^2}+\dfrac{\mathrm{Ji}}{s}\right)}{7205759403792793600 \left(s+\dfrac{446}{25}\right)}+1\right) \left(s+\dfrac{446}{25}\right)}+1}$$

```
i = sqrt(-1);
% Choosing Poles
p1 = -1.5;
p2 = -1.5 + 28*i;
p3 = -2.5;
p4 = -1.5 - 28*i;
p5 = -.5 ;
```

```
%target polynomial
target_polynomial = (s-p1)*(s-p2)*(s-p3)*(s-p4)*(s-p5)
```

target_polynomial =

$$\left(s+\frac{1}{2}\right)\ \left(s+\frac{3}{2}\right)\ \left(s+\frac{5}{2}\right)\ \left(s+\frac{3}{2}-28\,\mathrm{i}\right)\ \left(s+\frac{3}{2}+28\,\mathrm{i}\right)$$

```
%find denominator
[n d] = numden(Htot_subbed);
% find the coefficients of the denominator polynomia
coeffs_denom = coeffs(d,s);
% divide out the coefficient of the highest power term
coeffs_denom = coeffs(d,s)/(coeffs_denom(end));
% find coefficients of target polynomia
coeffs_tgt = coeffs(target_polynomial,s);
% solve the system of equations setting the coefficients of the
% polynomial in the target to the actual polynomials
solutions = solve(coeffs_denom == coeffs_tgt, Jp, Ji, Kp, Ki, Ci);

% display the solutions as double precision numbers
Jp = double(solutions.Jp)
```

```
Jp = -62.0486
```

```
Ji = double(solutions.Ji)
```

```
Ji = -1.0741e+03
```

```
Kp = double(solutions.Kp)
```

```
Kp = 2.3505e+03
```

```
Ki = double(solutions.Ki)
```

```
Ki = 8.8575e+03
```

```
Ci = double(solutions.Ci)
```

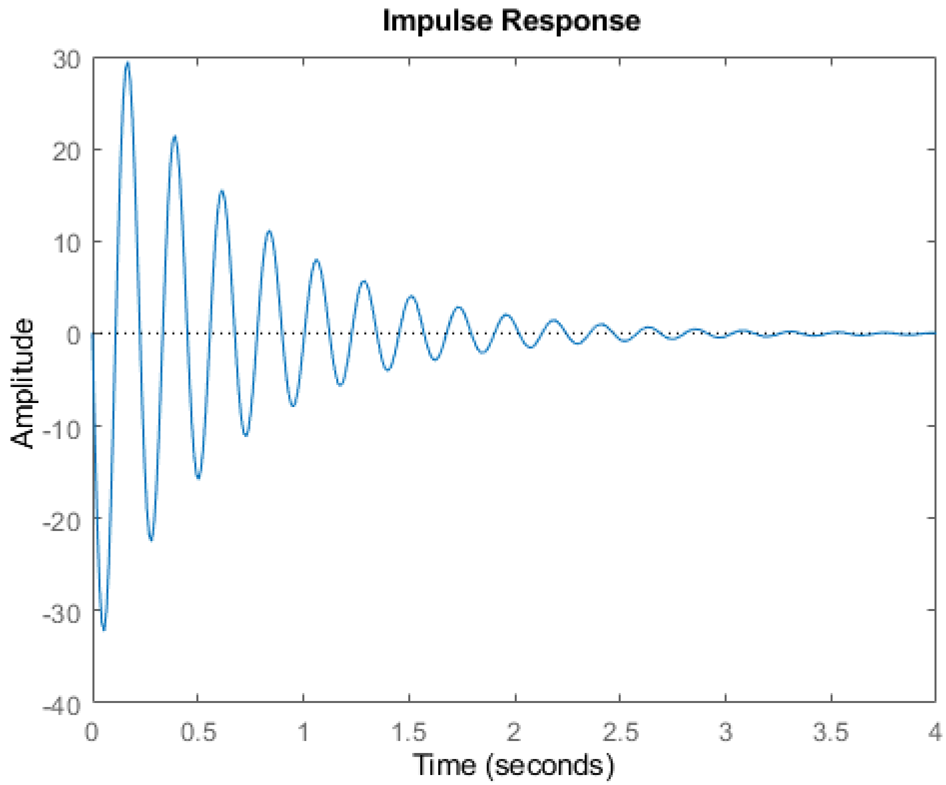
```
Ci = -349.8038
```

```
impulse_response_from_sym_expression(subs(Htot))
```

```
TFH =

  5.354e75 s^8 + 1.357e77 s^7 - 3.774e77 s^6 - 2.084e79 s^5 + 5.505e77 s^4 + 4.403e80 s^3 + 1.408e80 s^2
  ------------------------------------------------------------------------------------------------------
  5.354e75 s^8 + 1.357e77 s^7 + 5.029e78 s^6 + 9.599e79 s^5 + 3.64e80 s^4 + 4.403e80 s^3 + 1.408e80 s^2

Continuous-time transfer function.
```




```
function h = impulse_response_from_sym_expression(H)

    TFstr = char(H);

    % Define 's' as transfer function variable
    s = tf('s');
    % Evaluate the expression: "TF = (s+2)/(s^2+5*s+9)"
    eval(['TFH = ',TFstr]);
    impulse(TFH);

end
```