A Testbed for Quantum Error Correcting Codes

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The Ideal Project Outcome

- A library for Qiskit where users could investigate the performance of quantum codes on real quantum hardware in a 'plug and play' fashion without the need for full error syndrome measurements.

The source material

- 'Decoding Quantum Errors With Subspace Expansions'
- Jarrod R. McClean, Zhang Jiang, Nicholas C. Rubin, Ryan Babbush & Hartmut Neven.
- Nature Communications, (2020)11:636, https://doi.org/10.1038/s41467-020-14341-w

Quantum subspace expansion on 2 slides

- Using the stabiliser generators for some quantum code one can define a projector which will project a mixed state onto the code space. You can use this to correct an encoded quantum state.

$$\overline{P} = \overline{P}^{\dagger} = \prod_{i}^{m} P_i = \frac{1}{2^m} \sum_{M_i \in \mathcal{S}} M_i \qquad (1)$$

- Where $P_i = \frac{1}{2}(I + G_i)$ for G_i a generator of the code, M_i the elements in the full stabiliser group $\mathcal{S}$ (the same S as under the sum, sorry latex and powerpoint equations don't always mix!), m the size of the generating set of the stabiliser.

- This projector is equal to the sum of elements in the full stabiliser group with an equal weighting. The size of this group grows exponentially with the size of its generating set. So errors can't be comprehensively suppressed with this method. However, we can choose to use a subset of the elements of the full stabiliser group to form our projector and see if this still provides some resistance agaisnt errors.

$$\overline{P}_c = \sum_{i}^{L} c_i M_i$$

- Previously the equal weighting of $\frac{1}{2^m}$ in Eq. (1) was a good choice. The problem now becomes how to best choose the c_i for a given set of M_i.

Quantum subspace expansion on 2 slides

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$$\overline{P}_c = \sum_i^L c_i M_i$$

- It can be shown (and is in the paper) that finding the optimal weighting is equivalent to solving a generalised eigenvalue problem using a classical computer. This has the form on the left with C the vector of optimal weights, E the corresponding eigenvalue and H, S the transformed code space Hamiltonian and overlap matrices as defined on the left.

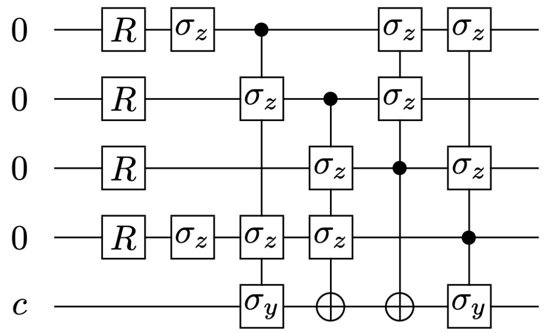
$$HC = SCE \qquad H_{ij} = \mathrm{Tr}\left[M_i^\dagger H_c M_j \rho\right] \qquad S_{ij} = \mathrm{Tr}\left[M_i^\dagger M_j \rho\right] \qquad \mathcal{H}_\mathrm{c} = -\sum_{M_i \in \mathcal{M}} M_i$$

- With the optimal weighting in hand one can correct a target observable as below, with measurements scaling as $3 * shots * L^2$ where $shots, L$ are the number of device shots per measurement and the number of check operators used.

$$< O > = \sum_{i,j}^L c_i^* c_j \frac{Tr\left[M_i^\dagger O \; M_j \; \rho\right]}{Tr[\overline{P}_c \rho]}$$

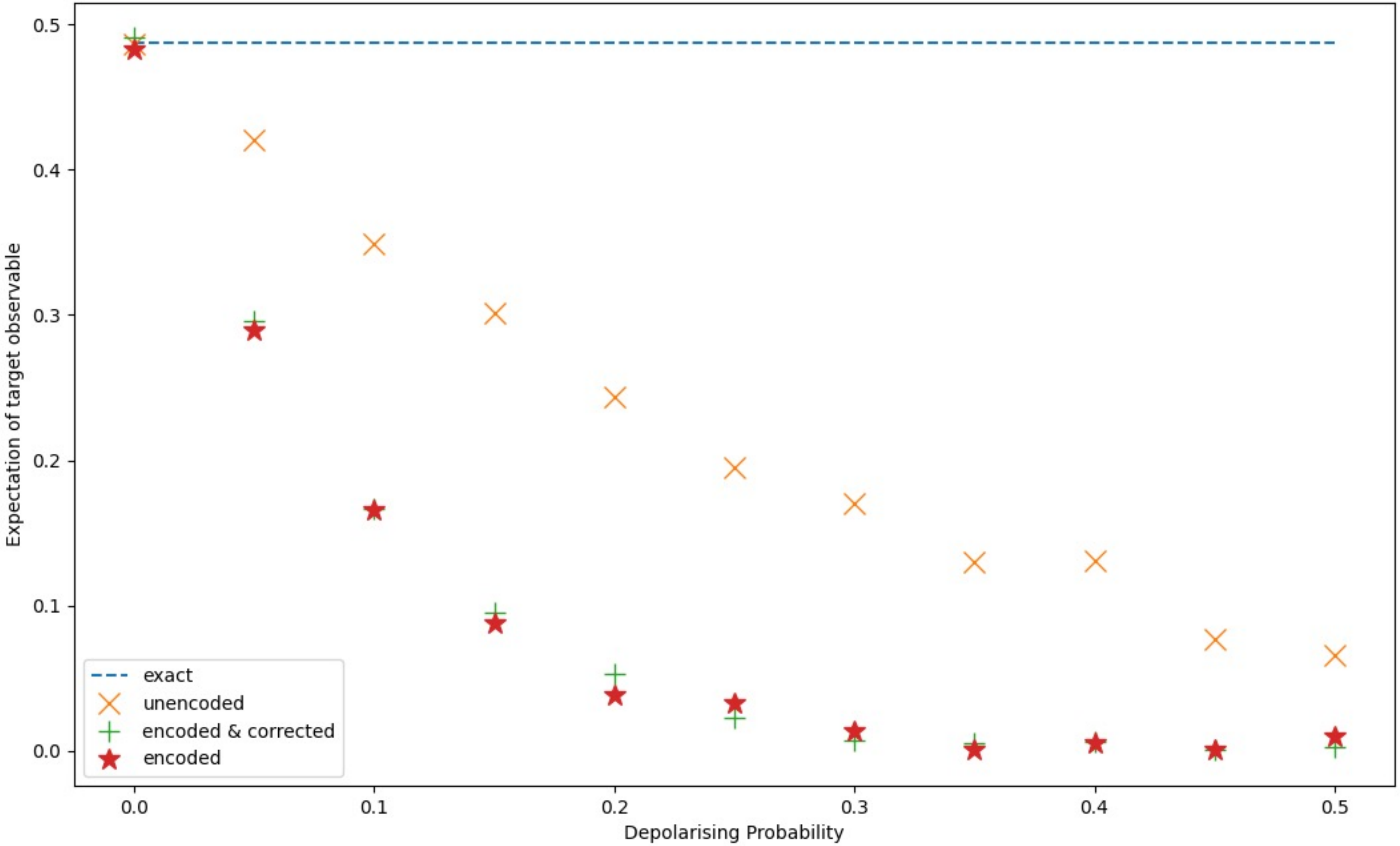
The Toy Problem

- Encode random state using the [[5,1,3]] code encoding network (order of qubits is reversed in Python code i.e. c is qubit 0).
- Target observable is the logical Z: $Z_0 \otimes Z_1 \otimes Z_2 \otimes Z_3 \otimes Z_4$
- Estimate necessary expectation values to calculate observable from quantum subspace expansion method.
- This is all done through a depolarising channel.
- See how accuracy of the expectation estimation goes with increasing depolarising probability.



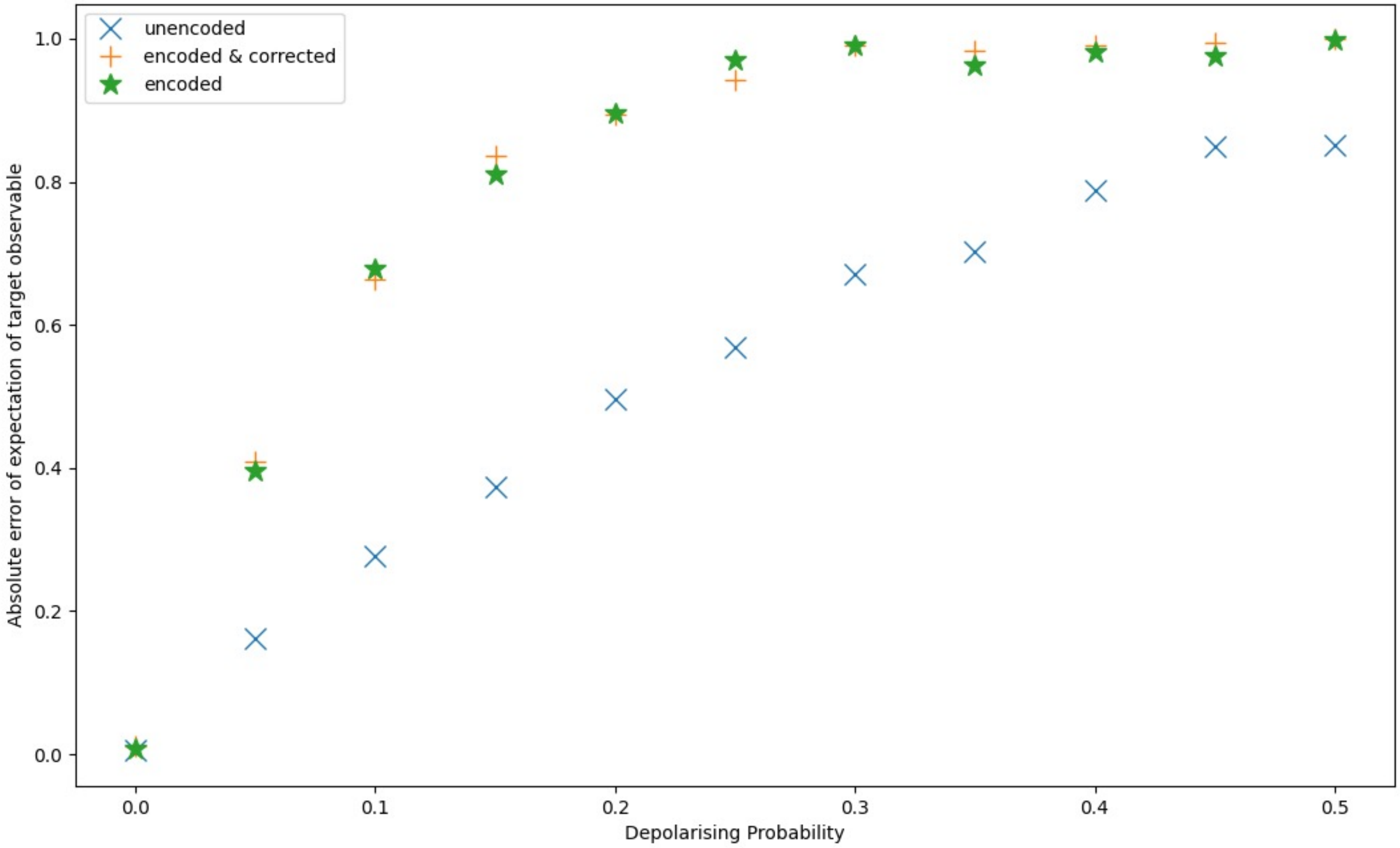
Gottesman - 1997 - Stabilizer Codes and Quantum Error Correction

Results



Estimate of expectation of logical Z for a randomly chosen single qubit state. 'Exact' is expectation of single qubit Pauli Z with no noise. 'Unencoded' is expectation for single qubit Pauli Z with noise. 'Encoded & corrected' is expectation of logical Z for 5 qubit code with encoded single qubit state and with QSE correction applied. 'Encoded' is with no QSE correction applied.

Results



Absolute error with exact expectation of target observable. Labels correspond to same cases as on previous slide.

If I had two more weeks…

- At 0 depolarisation the subspace expansion method is quite accurate. So understand if presence of noise in encoding network contributed to lack of efficacy of the encoding and correction at stronger depolarisation.
- Use as a pedagogical tool to observe the relationship between the optimal weighting of check operators and the choice of noise model.
- How does the choice of check operators affect the numerical stability of the generalised eigenvalue problem?

If I had two more weeks (or if my laptop charger didn't break!) [cont].

- Observe the effects for more target observables which can't be efficiently classically simulated.

- Observe the effects for different noise models & real device noise. Control noise on real device by adding layers of redundant operations of increasing depth.

Contributions

- Open source implementation using Qiskit. (While for the purposes of this hackathon an artificial parameterised noise model was chosen, this could be removed in favour of a backend of choice)
- Observing the accuracy of estimation of a Pauli observable.
- Generic code structure intended to be easily adapted to other choices of quantum codes and elements of stabiliser group for the expansion.