

The Missing Link: Estimating the Impact of Incentives on Teacher Effort and Instructional Effectiveness Using Teacher Accountability Legislation Data

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Abstract

Teacher effort, a critical component of education production, has been understudied in the literature due to measurement difficulties. I use a principal-agent model, North Carolina data, and the state's accountability system that awards cash for school-level academic growth, to distill effort from teacher absence and capture its effect. I find low effort at low and high probabilities of bonus-receipt, high effort when the bonus outcome is in doubt, and free-ridership. Teachers respond to incentives, and effort impacts achievement. Policy simulations with individual-level incentives eliminate free-rider effects, but reduce effort by pushing teachers into the tails of the probability-of-bonus-receipt distribution.

Keywords: Accountability, principal-agent model, teachers

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1 Introduction

Extensive empirical evidence exists to suggest that teacher quality is one of the most important determinants of a student's success in his or her academic career.¹ Observable teacher characteristics such as experience, advanced degrees, and credentials have been identified as proxies for teacher quality. Previous studies have found these variables to have weak to

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¹See Rivkin, Hanushek, and Kain (2005), Darling-Hammond (2000), and Sanders and Rivers (1995), among others.

moderate positive correlation with higher student achievement. While these characteristics explain part of the impact of teachers, they fail to account for all of the observed variation in achievement. Researchers have often modeled this residual unaccounted-for-effect of teachers as individual-specific constants in fixed-effects models.²

In the mean time, policy makers have used accountability systems to introduce ‘market forces’ to improve standardized test scores. If sanctions or bonuses are expected to raise student achievement, these punishments or rewards must be designed to affect some portion of teacher quality that is not easily observable in an administrative data set. This paper uncovers this important and unexplored dimension of teacher quality as effort, the one endogenous characteristic common to all teachers that can be immediately influenced by incentives.

While there is considerable debate on whether accountability systems help students as intended, there is evidence that teachers and administrators respond to incentives. Studies have shown that performance incentives for teachers can lead to higher academic performance, states with stricter accountability standards have been associated with higher test scores, and high stakes testing improves student achievement, compared to low stakes testing. The accumulated evidence shows that incentives can raise test scores.³ This study adds to the literature documenting of the effectiveness of accountability system by estimating a structural model of education production with a pay-for-performance teacher compensation component.

Accountability systems may raise test scores, but test score increases by themselves do not always reflect actual education production. Test scores can be raised in one of three ways. The first method is to alter the way in which teachers are hired or fired. By selecting or attracting ‘better’ teachers to start and dismissing teachers who prove to be ineffective, students will receive better instruction, leading to higher test scores.⁴ While changing the teacher labor market in a fundamental way will most likely have the largest impact, whether accountability systems can effect this type of change remains an open question.⁵

The second way is to change teacher or school behavior unrelated to measurable effort exertion. Some of these changes lead to no actual education production. Examples of such behaviors include classifying marginal students as disabled, suspending them to prevent testing, or altering students’ answer sheets.⁶ Some behavior may or may not lead to education

²See Goldhaber and Anthony (2007), Rockoff (2004), Clotfelter, Ladd, and Vigdor (2007), among others.

³See Figlio and Kenny (2007), Clotfelter and Ladd (1996), Lavy (2009), Carnoy and Loeb (2002), Dee and Jacob (2011), Wong, Cook, and Steiner (2010), Vigdor (2008), and Jacob (2007). Some recent experimental studies have found no impact of incentives. See Section 2.1.

⁴See Rockoff et al. (2008) and Hanushek (2009) among many others.

⁵See discussion in section 6.2.

⁶See Cullen and Reback (2002), Figlio and Getzler (2002), Jacob (2005), Figlio (2006), and Jacob and Levitt (2003).

production. ‘Teaching to the test,’ for example, may take valuable instruction time away from education production if the standardized test is poorly designed, or it may help teachers to create a good curriculum and impart necessary knowledge to students if the test is well-designed.⁷ Some of these behaviors will systematically bias the relationship between the outcome and the inputs, making raw exam scores an imprecise measure of student achievement.⁸

Finally, the third way is to increase teacher effort. An increase in effort will lead to real education production. Many studies have discussed the importance of teacher effort, and some have made the connection between teacher absence and teacher effort or student achievement. Studies generally find negative correlation between an increase in ‘market forces’ (such as decreased job security) and absenteeism. Studies that explicitly look at absence and academic achievement find a small to moderate decrease in test scores with increasing teacher absence.⁹ While using absence as a direct proxy for effort yields some suggestive results, I propose a method of isolating the true effort response of teachers to incentives using a unique data set collected in North Carolina that tracks the academic history, demographic information, and teacher and peer exposure of students in the public school system. This data set, along with the unique teacher incentive system that pays out bonuses for school-level year-over-year improvement in standardized test scores, makes it possible to distill the teacher’s level of effort, identify her effort response to incentives, gauge the impact of effort on achievement, and evaluate the efficacy of the accountability policy.¹⁰

Recent research has focused on the distributional changes in achievement.¹¹ In addition to the change in distribution of resources or effort, which creates winners and losers among the student population, there may still be net positive or negative effects, depending on aggregate effort level change due to the incentive policy. This study will examine efficiency as well as distributional implications of accountability policies.

The key insight from the theoretical model and econometric analysis is that the conventional wisdom of eliminating free-ridership to increase the effectiveness of incentives may actually be counterproductive when the accountability system uses strict thresholds (based

⁷See Grissmer and Flanagan (1998), and Hanushek and Raymond (2005).

⁸It should be noted that this bias is in addition to the already noisy test score as a measure of student achievement. If combined with a risk-averse teacher, the inherent noise may dis-incentivize teachers from exerting effort, especially at smaller schools where this noise component would be larger. See Kane and Staiger (2002).

⁹See Bradley, Green, and Leever (2007), Clotfelter, Vigdor, and Ladd (2007), Duflo and Hanna (2006), Miller, Murnane, and Willett (2008), and Jacob (2010), for example.

¹⁰Another method is to induce more student effort. Evidence of efficacy remains mixed. See Angrist and Lavy (2002).

¹¹See Neal and Schanzenbach (2010), Bocher-Jennings (2005), and White and Rosenbaum (2007).

on year-over-year test score gains) to make binary decisions on whether teachers are rewarded as a group.¹²

Policy simulations showcase the tradeoff between the free-rider effect and the incentive effect generated by the accountability policy. Results shows that classroom-level incentives *lower* education achievement compared to school-level incentives. While negative free-rider effects are eliminated with individual level incentives, isolating teachers has the undesirable effect of pushing many teachers towards the tails of the probability distribution, where incentive effects of the bonus are minimized.

The next section details the North Carolina accountability system. Section 3 presents a simple theoretical model. Section 4 describes the data, and section 5 introduces the econometric model. I present the results in section 6. Section 7 discusses the implications of altering the bonus system. I conclude in section 8. Proofs for the theoretical model are presented in the appendix.¹³

2 The North Carolina Accountability System

The North Carolina accountability program (also known as ABC) began in the 1995/96 academic year.¹⁴ While the system has grown in complexity and has gone through minor alterations in the details of execution, the principal mechanism of offering cash incentives for student achievement gains has remained unchanged for more than a decade.¹⁵ In contrast to a level model such as No Child Left Behind (NCLB) that rewards absolute achievement, the focus of the North Carolina system has always been on growth of scores from the previous year.

North Carolina public school students in grades 3 through 8 must take end-of-grade (EOG) exams in reading and mathematics. The test is on a developmental scale, allowing comparison of scores across grades. Using the formula defined below, North Carolina Department of Public Instruction (NCDPI) determines the required achievement gains for each school based on the school's students' performance last year on the EOG exams. That is, the difference in test

¹²This issue can be understood as a principal-agent problem, with the state government as the principal, teachers as agents, and the accountability incentives as the all-or-nothing bonus contract. In the context of this study, school achievement is the cooperative team project that the bonus contract is based on, student or classroom-level achievement is the noisy individual output, and teacher absence is the imperfect signal of worker effort.

¹³Robustness checks and minor regression results are presented in the online appendix available at <http://sites.google.com/site/tomsyahn/>.

¹⁴The acronym stands for strong Accountability, teaching the Basics, and emphasis on local Control.

¹⁵For instance, middle school and high schools achievement gains were measured starting in 1997/98. For a complete description of the incentive system as well as the high school criterion calculation, see Vigdor (2008).

scores is initially captured at the student level, allowing transfer students to be included in the school achievement calculations. The system is two-tiered, with teachers in schools making ‘expected growth’ receiving a \$750 bonus, and teachers in schools making ‘high growth’ earning \$1,500. ‘Expected’ and ‘high’ growth are defined below.

The achievement gain threshold used in the calculation of bonus eligibility is defined as:

$$\Delta y_{mgst} = \Delta y_{gs94} + b_1 ITP_{gst} + b_2 IRM_{mgst}$$

The Δy_{mgst} term is the required change in the test score in subject m for students in grade g in year t in school s compared to the score last year. Δy_{gs94} is average change in standardized test scores for all North Carolina students in 1993/94, compared to the scores from 1992/93.

The second and third terms on the right hand side are ‘correction factors.’ The ITP_{gst} term is the ‘Index of True Proficiency,’ and the IRM_{mgst} term is the ‘Index for the Regression to the Mean.’ The two terms are meant to adjust test score goals for shocks in performance of the school’s students last year.¹⁶

Using this formula, for a school with G tested grades, $2G$ thresholds are produced each year (math and reading), which are compared to the actual average test score improvement at the school. The school scores and threshold scores are differenced, standardized, weighted by the number of students in each grade, and summed across grades and subjects. That is, each school receives a z-score centered at zero and a standard deviation of one. If this z-score, termed ‘expected growth composite’ is greater than zero, teachers in the school receive a \$750 bonus. Teachers in schools that make expected growth yet fail to test more than 98% of eligible students are exempted from the bonus. The procedure is repeated after increasing the growth threshold by 10%, to generate the ‘high growth composite.’ Teachers in schools that make high growth receive an additional cash bonus of \$750. Therefore, teachers in a school with exceptional test score growth scores can earn as much as \$1,500.

¹⁶ ITP is obtained by subtracting the sum of the 1994/95 state average reading and math test scores from the sum of the average reading and math scores of students in grade $g - 1$, school s in year $t - 1$. Therefore, one ITP value is generated per school. The b_1 term varies by grade and subject but is positive in all cases. Thus, schools that performed better compared to the state standard last year must achieve higher growth this year to qualify for the cash bonus, all else equal. IRM is generated by taking the difference of the average test scores of students in grade $g - 1$, school s in year $t - 1$ and the 1994/95 state average test scores *for each subject*. Therefore, two IRM values are generated per school. The b_2 term varies by grade and subject but is negative in all cases. IRM is meant to account for the possibility that some students may have performed particularly well (or poorly) as a statistical anomaly on a particular test, and will return to expected performance the next year. Thus, if some students performed abnormally well on the math test in year $t - 1$, the state does not place overly harsh expectations on the school to replicate this feat in year t . I remain agnostic about the effectiveness of ITP and IRM in accounting for these macro-level shocks, but they may help to correct for some measurement error, especially in small schools.

2.1 Other Teacher Incentive Systems

There is no clear consensus on the effectiveness of pay-for-performance systems in the literature. Some studies find that bonus incentives can have large positive impacts, while others find little to no impact, even with substantial bonus payouts.

In the United States, teacher or school incentive programs of various types exist (or had existed before expiring) in 40 states.¹⁷ Large incentive programs or experiments that evaluated pay-for-performance were implemented in Colorado (Denver), Florida, Illinois (Chicago), Minnesota, New York City, North Carolina, Tennessee, and Texas (Dallas and Houston).¹⁸

The Denver ProComp system awarded up to 6.4% of the ‘index salary’ (in 2010, \$37,551) to teachers based on school-level proficiency or school-level growth in test scores. Teachers could earn bonuses in a variety of other ways as well, including attaining advanced degrees or certification and working in hard-to-staff schools. Goldhaber and Walch (2012) found that students taught by both ProComp participant and non-participant teachers made significant gains. In Florida, the most recently administered incentive program (MAP) awarded bonuses that ranged from 5% to 10% of a district teacher’s average annual salary. Each district had discretion over the proportion of teachers that received the bonus, and at least 60% of the criterion had to be based on student proficiency or gains. A Chicago experiment that tested for loss aversion by teachers (of having to ‘give back’ the bonus for poor student performance, compared to the traditional bonus) in a pay-for-percentile framework showed positive and significant impact of the loss-focused incentives (See Jacob (2005)). In Minnesota, the Q-Comp system allowed schools to choose either school-level or individual-level incentives. A teacher could be awarded a maximum of \$2,000, based on several factors, such as peer observations, professional development credits, as well as student achievement. Sojourner, West, and Mykerezi (2011) found small but positive and significant increase in student achievement. In New York City, a school-level randomized teacher incentive experiment that paid out approximately \$3,000 per union teacher was conducted. Fryer (2011) found the bonus to have little impact on test scores. A large-scale incentive experiment in Nashville, Tennessee that ran from 2006 to 2009 grouped middle school teachers into teams and evaluated them based on value-added measures of student performance. Each teacher could earn approximately \$6,000, but Springer et al. (2010) found no significant effect on test scores or changes to

¹⁷Some states have had several different incentive programs in place at different times. For example, Florida has had at least 3 different systems (E-Comp, STAR, and MAP). Recently, in 2011, a new pay-for-performance law was introduced in Florida.

¹⁸This is by no means an exhaustive list, but it does represent a cross-section of different types of incentive programs.

teaching practices.¹⁹ Ladd (1999) analyzed a Dallas merit pay program (1991-1995) based on school-wide performance that awarded \$1,000 annual bonuses and found that the bonus incentives resulted in test score gains. Imberman and Lovenheim (2012) investigated a Houston rank-order tournament incentive pay program (2006-2010) that paid out substantial bonuses (up to \$7,000) to teachers grouped at the grade-subject level. The authors found that achievement increased, with smaller groupings making larger gains, pointing to possible free-rider effects.

Internationally, well-known studies of incentive programs were conducted in India, Israel, and Kenya. Many of these programs were short-term (or one-shot) evaluations that showed teacher incentives can raise student achievement.

Muralidharan and Sundararaman (2009) evaluated the impact of incentives on teachers in a large-scale randomized evaluation in a rural province in India. Bonus was determined by percent improvement in math and language test scores from the previous year, and average bonus awards amounted to about 3% of annual salary. The study found that individual-level incentives were more effective in raising test scores compared to school-level incentives. In Israel, a school-wide incentive program instituted in 1995 was studied by Lavy (2002). Bonus was paid out based on dropout rates, the average number of credits taken per student, and the fraction of students receiving matriculation certificates. The top third performing schools were awarded the incentive, and the school divided this between salary bonuses for teachers and school improvement uses. Teacher bonus amounts ranged from \$250 and \$1,000. The study found that these school-based incentives raised test scores, decreased dropout, and increased the fraction of students receiving matriculation certificates. Lavy (2009) studied another Israeli program in 2001 that paid bonuses to teachers of grades 7 through 12 who taught English, Hebrew and Arabic, math, and ‘other subjects’ in a rank-order tournament. Bonus awards were paid out at the individual-level and were large, ranging from \$1,750 to \$7,500. Teachers could win multiple awards per year. The incentives were shown to be effective in increasing test scores. Glewwe, Ilias, and Kremer (2010) studied school-level incentives in Kenya that paid awards amounting to approximately 20 to 40% of one month salary to teachers based on test score improvements in all subjects in the school. The authors found that test scores did increase, but they speculate that this was not due to increase in teacher effort (as teacher attendance did not change), but to increasing the number of test preparation sessions.

As we can see from the above examples, pay-for-performance can mean many different

¹⁹In the New York City and the Nashville systems, it appears that a majority of teachers did not understand how the bonus outcome was determined, which could have contributed to the lack of observed impact.

things. Measures of student outcomes can include test score growth (value-added), proficiency level, school attendance, test participation, and graduation rate. Certain subjects, grades, students, and schools may be exempt from testing. Performance may be measured at the class, grade, subject, or school level. There may or may not be competition against other teachers or schools. Performance may be assessed alongside traditional ‘input’ measures such as advanced degrees and certification. Bonus can be all-or-nothing or pay-for-percentile. And of course, the amount of pay outs can vary substantially among programs. With such differences in implementation, it is not surprising that different studies have reached different conclusions.

The North Carolina program can be characterized as narrowly focused on year-over-year test score improvements with modest bonus pay outs. Teachers and schools were not in competition against each other, and the structure of the incentives was relatively simple and easy to understand. Elementary schools were assessed on the growth of reading and mathematics test scores only, with schools rewarded if they managed to cross a strict threshold that was based on average growth rate in the base year across the entire state. The \$1,500 maximum bonus was lower than average pay outs in many other states. The program was relatively long-lived (10+ years) and was implemented state-wide.

The simplicity of the program and the stability offered by its longevity and geographic breadth allow tractable theoretical and structural econometric models to be constructed that capture most of the salient features of the incentive program. The modest pay outs allay some concerns about other possible influences that may drive results, such as large-scale coordination among teachers and schools across years or direct intervention by districts. The elements that make the North Carolina system easier to analyze are also what distinguishes the ABC system from other state systems. Implementation details matter, and caution should be exercised when comparing relative effectiveness among incentive programs.

3 Theoretical Model

Teacher j ’s utility function is defined as the difference between expected bonus and effort cost. I assume teachers are risk neutral in bonus receipt. Teachers are differentiated by ability, $x_j \in [\underline{x}, \bar{x}]$ and effort, $e_j \in [\underline{e}, \bar{e}]$.²⁰ Effort e_j in the context of the model is not the absolute amount effort that can be exerted by a teacher. Instead, \underline{e} is the amount of effort a teacher will exert in the complete absence of incentive pressure, and \bar{e} is the maximum effort a teacher will exert at ‘optimal’ incentive pressure. B is the bonus that is paid to all teachers

²⁰I abstract away from student ability here, but the econometric model controls for observable student and peer characteristics.

at the school upon qualifying under the state criterion, Cr . Because the bonus is determined at the school level, all teachers' efforts and abilities contribute to the probability of bonus receipt, and the incentive system may suffer from a free-rider problem.²¹

$$U_j = B \cdot Pr(e_1, e_2, \dots, e_J, x_1, x_2, \dots, x_J, Cr) - C(e_j) \quad (1)$$

Define school average achievement in year t as Y_t :

$$Y_t = \frac{\sum_1^J y(x_{jt}, e_{jt}) \cdot w_{jt}}{J_t} + \epsilon_t \quad (2)$$

where $y(x_{jt}, e_{jt})$ is the average classroom-level achievement in class/teacher j in year t , w_{jt} is class size weight (the ratio of the size of teacher j 's class over the average class size in the school), J_t represents the number of teachers at the school, and ϵ_t is a stochastic error term which captures school-level shocks such as inclement weather or flu outbreaks. If class sizes across the school are identical, $w_{jt} = 1$ for all j , and the ratio drops out. With \bar{Y}_{t-1} defined as last year school-wide achievement, the probability of qualifying for the bonus is defined by:

$$Pr_t = F((Y_t - \bar{Y}_{t-1}) - Cr) \quad (3)$$

I make the following assumptions:

1. $F(\cdot) \in [0, 1]$ is twice differentiable and $F'(\cdot) \geq 0$.
2. The effort cost function, $C(e)$ is twice differentiable, with $C''(\cdot) > 0$ and $C'''(\cdot) \geq 0$.
3. $\frac{Bw_{jt}}{J_t}(F''(\cdot)a'(\cdot) + F'(\cdot)a''(\cdot)) \leq C''(\cdot)$.
4. Defining Y_{-j} as the school average achievement without teacher j 's class, for some $J > J^*$, there exists from high value of Y , Y^H and some low value, Y^L , such that $Pr_t|Y_{-j} > Y^H \rightarrow 1$ and $Pr_t|Y_{-j} < Y^L \rightarrow 0$ for the entire range of e_j and x_j .

Proposition 1 *Given Equations (1) - (3), Assumptions (1) - (4), and $\{x_1, \dots, x_J, w_1, \dots, w_J, B, Cr, J\}$ there exists an interior pure strategy Nash equilibrium in effort, $\{e_1, e_2, \dots, e_J\}$.*

Assumption 3 ensures global concavity of expected utility. Assumption 4 is a statement about the limitation of effort within a large school. One teacher cannot unilaterally determine the

²¹Teachers are assumed to be risk-neutral with respect to the bonus receipt. If teachers are risk-averse, the increase in uncertainty of outcome may dilute the effectiveness of the bonus scheme. On the other hand, risk-aversion may be one argument put forth in favor of school-level incentives.

bonus receipt of the entire school. If all other teachers shirk (and/or are low-ability), the best response of teacher j is also to shirk. On the other hand, if all teachers are giving maximal effort (and/or are high-ability) such that the bonus is assured, the best response of teacher j is again, to shirk. It is exactly when all other teachers are putting forth effort and the bonus outcome is in doubt, that teacher j is also induced to give some positive amount of effort.

I show that there can exist a free-rider problem in the incentive system:

Proposition 2 *Assuming identical teachers and $Y^L < Y < Y^H$, a free rider problem may exist.*

As the number of teachers increases from J to $J + 1$, a teacher's effort is distributed over a larger population. Since her pay-off remains constant, she will be induced to lower her level of effort. In this sense, free-rider problems always exists, unless incentives are reduced to individual-level bonuses.

Imposing identical teachers in Proposition 2 is a strong assumption, because it ignores the second force in the model that impacts teacher effort: the distribution of teacher ability across the school. An increase in the number of teachers in the model necessarily implies an additional class.²² If the new teacher's ability, x_{J+1} , is significantly different from the school average teacher ability (and if the school is small enough), this can change the probability of bonus receipt for the school, which can have a large impact on effort exertion across the school.

For the empirical model, I assume that the classroom average achievement is generated as:

$$y(x_{jt}, e_{jt}) = \exp(x_{jt})e_{jt}$$

and that the effort cost function is exponential.²³ Writing out the first order condition from the teacher utility function:

$$J^{-1} \cdot B \cdot F'(Y) \cdot \exp(x_j) \cdot w_j = C'(e_j)$$

I assume that the probability distribution is standard normal. Taking natural logs, the FOC

²²Assuming that J or w_j is under the control of the administrator *significantly* complicates the model. Implications of selecting an 'optimal' J or w_j are more fully explored in section 3.1.

²³Alternative forms of the effort cost function yielded no qualitative differences. The functional form of the classroom average achievement is admittedly ad hoc, to allow for a linear regression (after taking natural logs) in the empirical section.

becomes:

$$e_j^* = \gamma + x_j + \ln w_j - \ln J + \ln \phi(Y)$$

where γ is a normalizing term. In this way, the free-rider effect arising from an increase in J is separated from $\phi(\cdot)$, the pdf of the bonus receipt probability. This will greatly simplify the econometric specification.²⁴

Having found that school incentive policy may suffer from a free-rider problem, a simple solution would seem to be to go from a school-wide incentive to a classroom-level (individual) incentive where teachers are judged only on her students' performance. The first order condition shows that the $-\ln J$ term would equal zero when $J = 1$, thus eliminating the free-rider effect altogether. However, I show below that moving to this non-cooperative criterion will not necessarily increase effort exertion of teachers. In order to simplify the discussion, I add one more assumption:

5. When $J = 1$, There exists some high value of x , x^H and some low value, x^L , such that:

$$\frac{\partial F(\cdot | x_j \geq x^H)}{\partial e} \cong 0, \quad \frac{\partial F(\cdot | x_j \leq x^L)}{\partial e} \cong 0$$

This assumption is a statement about the limitations of the incentives. For the case of a single-classroom school, a teacher's location on the bonus distribution is more strongly determined by ability than effort *induced by incentives*. That is, it is not possible to change a 'low ability' teacher into a substantially more effective teacher simply by offering more money. Similarly, a 'high-ability' teacher will not turn into a significantly less able teacher because of a smaller bonus (in the form of reduced probability of bonus receipt). In the model, an isolated teacher with very low ability ($x_j < x^L$) will have close to zero probability of qualifying for the bonus. Maximum effort induced by incentives cannot improve student achievement enough for this teacher to get beyond the flat part of the distribution on the left tail. Similarly, an isolated teacher with very high ability ($x_j > x^H$) will have bonus probability approaching one. A reduction in teacher effort due to decreased incentives will not lower students' test scores enough to move her off of the right tail of the bonus distribution.

²⁴It should be noted that J still exists within $\phi(Y)$. Therefore, the $\ln J$ term *does not* isolate the full impact of J in the teacher's behavior, only the direct effect of diluting the effect of one teacher across a larger population. The J term embedded within y serves as a countervailing force, by averaging across class scores to pull school average scores near the middle of the probability pdf (and thus inspiring higher effort exertion).

Proposition 3 *It is possible for effort to decline when the test score aggregation for incentives changes from school-average to classroom-average.*

While decreasing J to 1 will increase effort by eliminating the free-rider problem, moving from school-wide average to classroom average will move each teacher to a different point on $F(\cdot)$. Whereas a teacher may have been at a point on the distribution where her marginal effort can make a difference in the school-average criterion, the classroom-average criterion may place her at the tails of the distribution where additional effort exertion makes little difference in changing the probability of bonus receipt. This change in optimal effort exertion based on a teacher's placement on the distribution of the bonus receipt is critical in the analysis of any policy that attempts to increase student achievement by altering the incentive system.

This property is best demonstrated by looking at a school with two teachers. Assume teacher 1 has $x_1 > x^H$ and teacher 2 has $x_2 < x^L$. These teachers' optimal solutions are demonstrated in Figure 1 and Figure 2. The thick lines represent indifference curves of teachers, and utility increases up and to the left. When the bonus is determined at the class-level, the optimum solutions for both teachers are at the corner ($e_1 = e_2 = \underline{e}$) because neither teacher can significantly increase the probability of bonus receipt by exerting extra effort. If bonus is awarded for joint performance, the optimum solutions move to interior points, and $e_i > \underline{e}$ for $i = 1, 2$, as marginal effort exertion for both teachers will now increase the probability of bonus receipt. I assume that the higher-ability teacher's marginal effort application is more effective compared to the lower-ability teacher, but the solution holds if I assume the opposite.

The results indicate that a school-average incentives with free-rider problems may be preferable to a classroom-average bonus system depending on the distribution of teacher ability within the school. If schools are composed mostly of homogeneous teachers, the free-rider effect may dominate the incentive effect. If the bonus depends on classroom achievement, and if the state criterion is set correctly such that teachers are moved to the 'middle' of the probability distribution, they may be induced to exert more effort, and student achievement may increase. On the other hand, if the variance in teacher ability is high within the school, a school-average incentive policy may actually induce greater effort and increased achievement because the free-rider effect is offset by the increase in incentive effects. This property does not rely on the traditional arguments against a competitive criterion: that it may deter teachers from sharing of knowledge or working together. The only motivating factor is that bonus receipt is judged on joint achievement.²⁵

²⁵If cooperation or a collaborative working environment is a positive non-monetary job characteristic that

The school-wide criterion is a blunt instrument to gather teachers into a narrower band on the bonus receipt probability. When success depends on joint performance, teachers at the left and right extremes are grouped together and ‘gathered’ to the middle of the distribution. Tightly grouping teachers will only increase the power of the incentives if the spot to which they are gathered allows them to positively affect the probability of bonus receipt through the application of effort. This potential positive effect of joint performance criterion must be weighed against the free-rider effects inherent to the system. The policy simulation section explores these implications by changing test score aggregation rule, from school-level to grade-level to classroom-level.

It is possible that the accountability system affects teacher behavior in other ways. For instance, teachers may attempt to coordinate effort such that they do not over-improve student achievement in year t , to the detriment of gains in year $t + 1$. If year-over-year gains matter, and teachers have fine control over student achievement, teachers should invest just enough effort to receive the bonus each year and make ‘steady’ progress by solving an inter-temporal problem of effort exertion.²⁶ The likelihood of teachers playing a multi-year coordinated game may be of concern in smaller schools with low teacher mobility. In general, yearly turnover of teachers in North Carolina is almost 10%. While teachers may be engaging in the dynamic game, the bonus outcome across the five-year data set was relatively volatile. If a school was categorized as making less-than-expected, expected, or high-growth in the 1999/00 academic year, the probability that the school would maintain its status across the five year period was approximately 8.5%. Across any four-year span, the probability of a school maintaining its status was about 13%.

3.1 Other Policy Instruments

In addition to the formal assumptions made above, I also do not allow the principal or the superintendent to alter class size by 1) adding more teachers (increasing J) holding student population constant or 2) having different class-sizes for different teachers. While decreasing class size across the school or optimally distributing students to teachers will unequivocally increase academic achievement in a school with no accountability system in place, the incentive pressure introduced by the pay-for-performance system significantly complicates the impact of the above policy instruments.

exists (or is stronger) because of the school-wide bonus, moving to a classroom standard would have an even greater (negative) impact. If cooperation is captured by absence (that is, one of the ways in which teachers exert effort is to share didactic techniques), the interpretation from the model would remain the same.

²⁶See Maccartney (2012).

The negative impact of increasing number of teachers on effort suggests that the often observed negative correlation between school size and academic achievement in the literature may be due in part to this free-rider impact. The existence of free-rider effect implies that a given level of incentive pressure should be more keenly felt in smaller schools. This result means that accountability pressure should be geared toward impacting larger schools. Since free-rider effect is stronger in larger schools, stronger pressure (or greater rewards/punishments) will be required to extract the same level of effort as smaller schools. This result complicates attempts to increase academic achievement through class-size reduction. Decreasing class-size by adding more teachers may prove to be less effective than expected, because the increase in the number of teachers exacerbates free-rider effects, pulling teacher effort and achievement down. This force will push against the increase in test scores from class-size reduction.

Incentive pressure also plays a role in determining the ideal proportion of students to be distributed among teachers with varying ability. In general, the administrator will lean toward assigning more students to higher ability teachers until average academic achievement across the school is maximized. At one extreme, when there is no pay-for-performance system in place, the principal may be induced to evenly distribute students across all teachers. At the other extreme, when incentive pressure is strong, the optimal solution may be to assign zero students to low ability teachers. See the appendix for a discussion for both cases.

4 Data

I use an administrative data set for the North Carolina public school system from the academic years 1999/00 to 2003/04.²⁷ The data set contains information on all public schools, students, teachers, and administrators in North Carolina. Since the data is collected annually and individuals can be matched across years, a relatively complete longitudinal picture of the entire public school system in North Carolina emerges, detailing students' academic trajectories, peer interactions, and exposure to teachers.²⁸

²⁷The data, which is collected by the North Carolina Department of Public Instruction (NCDPI), was made available by North Carolina Education Research Data Center (www.pubpol.duke.edu/centers/child/nceddatacenter.html) at the Center for Child and Family Policy. While student and teacher level data are confidential, aggregate data and summary statistics are publicly available at the NCDPI web site (www.ncpublicschools.org/reportstats.html). Post-2004 data has unreliable absence data, and pre-1999 data yield poorer teacher-student matches.

²⁸It should be noted that on average, about 80% of all records of students are successfully matched with teachers. Charter schools were dropped due to inconsistent student-teacher match rates across the sample. In general charter school students comprised less than 3% of the state's student population during the sample years.

It should be noted that in the 2003/04 academic year, the North Carolina public school system was also subject to NCLB. I abstract away from impacts from NCLB sanctions. The criteria for making expected growth and adequate yearly progress (AYP) are largely uncorrelated. In fact, approximately 40% of schools in North Carolina pass under one regime yet fail under the other. In addition, two consecutive years of failures to make AYP are required before schools are subject to sanctions, and the sanctions that these schools would have been under - being forced to offer students the option to transfer to a different school - had very low uptake and did not impact teachers individually.²⁹

There are two unique features of the data that I take advantage of to identify the effect of teacher effort. The first is that each student record is linked to a teacher. This permits the identification of a complete classroom, with information on the student, teacher, and peers, provided that student instruction is confined primarily to the self-contained classroom. While students in middle and high schools change teachers and peers each period, elementary school students are tied to a single classroom, where they are exposed to the same peers and teacher throughout the school day. Therefore, any effect of effort from the elementary school teacher should be isolated to her classroom. I focus on teachers of students in grades 3 through 5 in schools that have grade 5 as the highest grade. Each student in the data set has a pre- and post-teacher exposure test score. Grades 4 and 5 students use the previous year test score as the pre-exposure score, and grade 3 students take a separate exam at the start of the academic year. While teachers in charge of kindergarten through grade 2 contribute to students' academic growth, their efforts are not explicitly evaluated.

The data set tracks a student's performance year-to-year as long as he or she remains in the North Carolina public school system. Because of the need for at least two years of EOG exam results for each student to judge whether he or she has improved, students with only a single year record are dropped.³⁰ Demographic characteristics information such as sex, race, age, and parental education level are collected. I divide race into minority (black, Hispanic, American Indian) and white (and Asians). I divide parental education level into those who have high school education or less, and those who have above high school education. Students in charter schools and alternative schools are dropped.

The other unique feature of the data set is that it contains teacher absence data. Teacher absences in North Carolina are broadly categorized into: sick leave, personal leave, and annual

²⁹While teachers may not be incentivized by NCLB pressure, it is possible that the principal may be impacted. This may lead the school to change behavior, such as targeting certain teachers to certain classes. For more details, see Ahn and Vigdor (2013).

³⁰These students may be transfer students from out of state or simply missing data from the previous year. Simple analysis of students with only a single year of test data revealed no obvious systematic missing data issues.

(vacation) leave. I use the sum of sick leave and personal leave as the measure of teacher absence in an academic year. Most of the annual leave days coincide with school vacation days. About 60% of annual leaves are concentrated in December, June, July, and August. Another 20% occurs in November, which indicates Thanksgiving break. Most of the remainder of the annual and personal leaves are for months at a time, indicating longer absences due to severe health problems or maternity leave.

Because sick and personal leaves are unplanned-for and take place during the school year, they can have the largest impact on student learning. A substitute teacher may have less experience and lack certification. More importantly, he or she may have a weaker grasp of the material being taught, will not know the aptitude of individual students, and may not be induced to exert more effort from bonus incentives. For the study period, teachers were absent an average of 9.6 days in an academic year of 180 days. There is a significant number of teachers who are absent more than 30 days (about 5% of the original sample), and I exclude these teachers from analysis. The resulting teacher absence rate of 5.3% is in line with other studies of teacher absences.³¹ Including teachers with high level of absence does not qualitatively change results. Table 1 summarizes student, teacher, class, and school characteristics.

5 Econometric Model

The econometric model estimates three equations, following the theoretical model. School-level expected bonus is generated using student achievement and the accountability rules defined in section 2. Teacher effort is captured using teacher absence. I show that teacher absence can be separated into effort and other unrelated shocks and use the incentive legislation to isolate the effort component. This reduces noise arising from uncorrelated shocks, making teacher absence a good signal for effort. Student achievement is estimated using distilled effort with other traditional student, peer, and teacher demographic characteristics in a standard education production function. The system is solved iteratively until convergence.³²

³¹See Ehrenberg et al. (1991).

³²This is equivalent to solving for a teacher reaction function. An alternate specification of estimating student achievement non-parametrically using student, peer, teacher, and school data yielded no qualitative differences.

5.1 School-level Bonus

While the expected bonus would usually be estimated as a probit or logit regression from data on current year bonus receipt, last year test scores, and this year predicted scores times the bonus amount, I am able to use the state defined criterion as specified in the accountability system description. Because the differences between school scores and state criterion are standardized to $N(0,1)$, let \widehat{Y}_{st} represent the predicted *school* performance this year, and let $Y_{s(t-1)}$ be school performance last year.³³ Expected bonus is:

$$E(B) = \{\Phi(\widehat{Y}_{st} - Y_{s(t-1)} - Cr_1) + \Phi(\widehat{Y}_{st} - Y_{s(t-1)} - Cr_2)\} \cdot B \quad (4)$$

where Cr_1 and Cr_2 are ‘expected’ and ‘high’ growth composite threshold values, respectively, B is the bonus amount, and $\Phi(\cdot)$ is the standard normal CDF.³⁴

5.2 Teacher Absence Decision

If effort were observable, the effect of incentives could be captured by running the following regression:

$$e_{jst} = \alpha_0 + X_{jst}\alpha_1 + \alpha_2 I_{jst} + \varepsilon_{jst}$$

where e_{jst} is teacher j ’s effort (located in school s in year t), X_{jst} are observable characteristics of teacher and year, school-district, and grade dummy variables, I_{jst} are measures of incentive strength (bonus receipt probability and incentive dilution), and ε_{jst} is the idiosyncratic error.

While effort is not directly measurable, teacher absence can serve as a noisy signal. Teacher absence is noisy because it contains effort as well as unrelated shocks and measurement error. I hypothesize that teacher absence, A_{jst} is determined by three components as defined below:

$$A_{jst} = g(X_{jst}, e_{jst}, \eta_{jst})$$

η_{jst} represents factors that affect absence that are unrelated to e_{jst} , such as unforeseen bad/good health outcomes and weather, and other orthogonal shocks that affect the number of days a teacher is absent.

³³The Y values are the sum of all students’ standardized reading and math scores. $Y_{s(t-1)}$ captures last year test scores of all current students in school s , even if they were not attending school s last year.

³⁴In the NC case, $B = 750$ as the ‘expected’ growth yields \$750 and ‘high’ growth yields \$1,500. Note from the description of the accountability system that the argument inside the $\Phi(\cdot)$ is a z-score distributed with zero mean and standard deviation of one. More generally, the equation would be: $E(B) = \Phi(\widehat{Y}_{st} - Y_{s(t-1)} - Cr_1)B_1 + \Phi(\widehat{Y}_{st} - Y_{s(t-1)} - Cr_2)B_2$.

I assume that for two teachers i and j , $E(A_i) \geq E(A_j)$ if and only if $e_i \leq e_j$ conditional on teacher, class, school, and district characteristics.³⁵ Because the absence variable is a count of days of absence in the school year, it is appropriate to think of effort as the aggregate effort provided by the teacher throughout the academic year. This assumes away situations where a teacher may redistribute effort for different time periods. For instance, a teacher cannot ‘save up’ her effort prior to the exam, and saturate her students at exam time.³⁶

Since absence is correlated with effort, and incentives are correlated with effort, the projection of incentives on absence results in an unscaled measure of effort. Using the data available and the first order condition directly from the theory, I model the absence decision as follows:

$$A_{jst} = X_{jst}\alpha_1 + \alpha_2 \ln J_{st} + \alpha_3 \ln(\phi(\hat{Y}_{st}, Cr_1) + \phi(\hat{Y}_{st}, Cr_2)) + \eta_{jst} \quad (5)$$

The measure of incentive strength (I_{jst}) to be used are J_{st} , the number of teachers at the school, and $\phi(\hat{Y}_{st}, Cr_1) + \phi(\hat{Y}_{st}, Cr_2)$, the position of the school with respect to the probability of bonus receipt.³⁷ The J term estimates free-ridership, as discussed in the theory. I anticipate $\alpha_2 > 0$. The effect of the ϕ term is best explained by Figure 3. The ϕ term measures the incentive effects of being at different points in the distribution of bonus receipt probability. The shape of the log of the pdf is strictly concave, with low effort tied to very low and very high probabilities of bonus receipt. The peak of effort exertion is tied to mid-level academic growth, when the bonus receipt is in doubt and very dependent on the effort exertion of teachers. Since absence is negatively associated with effort, I expect $\alpha_3 < 0$. I assume that the teacher does not differentiate effort exertion between subjects or among students.³⁸

The shape of the ϕ variable is important, as it reflects the non-linear effect of the probability of receiving the bonus. As the bonus becomes very easy or very difficult to attain, the utility maximizing response of teachers is to decrease effort.³⁹ X_{jst} is teacher, class, and

³⁵For instance, it is possible that experienced teachers may need to exert less effort compared to a new teacher for the same level of education production.

³⁶See the Proof Appendix for a formalization of this condition. Also, teacher absence pattern does not fluctuate just prior to the end of the school year.

³⁷I use the total number of teachers at the school (which includes teachers in K, 1st, and 2nd grades) to account for the fact that teachers who do not contribute to the school average score are in line to receive the cash bonus. Using the count of 3rd, 4th, and 5th grade teachers only does not change the qualitative results.

³⁸Implicit in the equation, by the exclusion of teacher fixed effects, is the strong assumption that there is no unobservable heterogeneity of teachers that would result in different patterns in teacher absence. This was primarily due to data problems in being able to consistently link teachers across the entire sample.

³⁹The non-linear, non-monotonic effect will be important for policy analysis. To check if the shape of the pdf is consistent with the theory, I run an alternate specification where I replace ϕ with $E(B)$ and $E(B)^2$. The squared term of expected bonus is included to capture non-linear effects of the probability of receiving the bonus. Results support the functional form assumption.

school characteristics.⁴⁰ η_{jst} is the idiosyncratic error term that represents the uncorrelated health and weather shocks.

By estimating absence decision in this way, I decrease the noise in teacher absence and distill it into a useful signal for effort. The change in ‘predicted absence,’ driven purely by the incentives in the bonus program, is an indicator of change in effort, because absence reduction *per se* is not rewarded as part of the incentive system.⁴¹ Predicted absence, \widehat{A} , is now a latent variable for effort.

Distilling effort in this way differentiates and isolates its impact on student achievement from other possible ways of increasing student achievement. Many of the methods mentioned in the introduction (such as excluding marginal students from taking the exam or changing answer sheets) serve as *substitutes* for education production through effort.⁴² If test score gains are observed for schools under a high degree of accountability pressure, yet distilled effort response is absent, this may indicate that schools are employing other methods.

5.3 Student Achievement

The student achievement equation does not follow from any specific utility maximization solution for students. I assume that there exists some student-level production function for education where the inputs are student, peer, and teacher characteristics. The achievement function for student i in subject m is specified as follows:

$$y_{mist} = Z_{ist}\beta_1 + \beta_2 y_{mis(t-1)} + \beta_3 \widehat{A}_{jst} + \beta_4 \widehat{A}_{jst} \cdot I[\text{minority}] + \nu_{mist} \quad (6)$$

Z_{ist} is a vector of student, peer, and teacher demographic characteristics plus year, district, and grade dummy variables.⁴³ \widehat{A}_{jst} is the projected measure of (negative) effort from equation (5). $I[\text{minority}]$ is an indicator variable which equals one if the student is black or Hispanic.

⁴⁰These include teacher gender, minority, experience, and certification status, class and school size, percent minority students and percent low parent education, rural school indicator, and dummy variables for grade, district, and year.

⁴¹That absence is not part of the incentive system is important for its role of serving as a signal for effort. If the number of absences was incorporated into the incentive system, depending on the state formula, teachers could choose to simultaneously reduce effort and absence, while ensuring the same or higher expected pay out. This would render absence less useful as a signal. For instance, in the system analyzed by Imberman and Lovenheim (2012), teacher absence does impact the bonus.

⁴²If teachers decided to exert more effort and increase the use of these other techniques at the same time, the impact of effort would be overestimated in the model.

⁴³Student characteristics are gender, minority, parental education status, and the previous year’s test score. Peer characteristics are percent minority, percent low parental education (at class and school levels), and class and school size. Included teacher characteristics are teacher gender, minority, experience, and certification status.

Interaction between effort and minority status was included to check if ‘per-unit’ effort efficacy differed by demographic characteristics.

5.4 Estimation

Equations (4), (5), and (6) are solved iteratively until convergence. In the initial step, I plug in the actual test scores from the data, Y_{st} , in equation (4) to generate the probability of qualifying for the bonus, $\Phi(\cdot)$, and expected bonus.⁴⁴ Equation (4) is a rule set, not a regression.⁴⁵

The CDF is converted to the pdf $\phi(\cdot)$ for use in the absence equation (equation (5)). The $\phi(\cdot)$ value, observable teacher and school characteristic, as well as the measure of free-ridership (J) are used to generate projected effort (\hat{A}). Projected effort is then plugged in the student achievement equation (equation (6)) with student, teacher, and school characteristics to predict student test scores. Predicted student test scores are aggregated to school averages, \hat{Y}_{st} , and plugged into equation (4) again to generate new expected bonus and $\Phi(\cdot)$, and the process continues until the parameters from equations (5) and (6) converge, essentially achieving a fixed point in the parameter and the distribution of test scores.⁴⁶

6 Results

Having specified the estimation strategy, I present the results. Before the parameter estimates are considered, it is natural to wonder if the procedure to distill effort is required at all. A single equation OLS or fixed-effects model that uses raw absence data may suitably summarize the relationship between teacher effort and achievement. Alternatively, it is possible that effort is not important. Pure exposure to teachers may drive achievement.

To motivate the need for signal distillation, I start by showing OLS estimates without isolating the effort component in the first step by estimating equation (6) using raw absence

⁴⁴Expected bonus is used for simulation and cost-benefit analysis of potential policy changes. An assumption inherent at the start of the estimation is that the actual test scores serve as a good initial proxy for the average test score before the shock in equation (2) is realized. This is more likely to be an acceptable assumption if school-wide shocks tend to be small in magnitude in either direction.

⁴⁵The actual bonus would equal \$0 if $Cr_1 > Y_{st} - Y_{s(t-1)}$, \$750 if $Cr_1 \leq Y_{st} - Y_{s(t-1)} \leq Cr_2$ and \$1,500 if $Y_{st} - Y_{s(t-1)} > Cr_2$. The math and reading test scores of students in grades 3, 4, and 5 are condensed into a z-score that measures school-wide achievement, which directly impacts the teachers’ incentives, which in turn impacts student scores.

⁴⁶An alternative interpretation of the setup is a simultaneous equations system with the probability of bonus receipt, number of teachers, and student individual characteristics serving as the exclusion restrictions, with effort/absence and education achievement as the endogenous variables.

data. The results are shown in Table 2. The negative impact of teacher absence on academic achievement is similar to results from Clotfelter, Ladd, and Vigdor (2007).⁴⁷ If the regression results are interpreted as causative, students do not suffer much from teacher absence. Enacting some policy that would cut the average number of absent days in half, from approximately 10 days to 5 days, would increase average reading achievement by roughly 0.5% of a standard deviation and math achievement by about 1% of a standard deviation. In contrast, paying for teacher certification would be approximately three-times more effective. The reason for the small OLS estimate may be because an absence does not result in zero education production during that day. A substitute teacher will be assigned, and students will still receive instruction. Imagine two identical teachers teaching identical classes. If one of the teachers is absent one additional day compared to the other teacher (with a substitute teacher filling in), student performances should not be significantly different across the two classes.

The estimate of the impact of projected effort due to incentive effects in Table 3 is more than 25 times greater than the OLS parameter estimate on absence for reading performance, and more than 15 times larger for math performance. However, as I show later, this magnified impact still translates to modest gains in test scores. This difference can be explained by extending the example above. Assume that one teacher is more motivated. The less motivated teacher does not prepare as much and generally cares less about the educational outcome of students. Further assume that the two teachers are in charge of identical classes. All else equal, student achievement in the less motivated teacher's class should be lower, and this teacher is likely to be absent more often throughout the school year. Now, assume that the enthusiastic teacher has a bad health shock and is forced to be absent the same number of days as the less motivated teacher. While students in both class are exposed to their teacher the same number of days, the enthusiastic teacher will give superior instruction to her students throughout the entire academic year. The OLS procedure in this case treats the teachers as identical (due to the same number of absences), and predicts the same achievement (the average effect across the two teachers). The structural model predicts higher effort and achievement in the motivated teacher's class, due to the incentives from the accountability system.

In addition, teacher certification status and teacher experience become stronger predictors of academic achievement when absence is distilled into effort. Teachers with more experience also have higher rates of absence (perhaps due to more health issues or smaller valuation of the bonus) (See Table 4.). The full impact of experience on academic achievement is

⁴⁷Clotfelter, Ladd, and Vigdor (2007) shows that fixed-effects estimates yield smaller but still significant estimates on absence. The reading and math scores are adjusted to represent percent change of one standard deviation of scores.

shared between the teacher experience and the raw absence variables. This pushes the OLS parameter estimate on experience down and pulls the parameter on absence up. Since the parameter estimate on absence is a negative number, the true impact of teacher absences is further underestimated in this framework.⁴⁸ Using the absence *response* of experienced teachers to changes in expected bonus separates out this confounding influence.

The teacher-effort estimates in Table 4 presents the impact of incentive forces on teacher absence. The probability of qualifying for the bonus, represented by ϕ , and the number of teachers in the school, J change the level of effort teachers exert in response to the accountability system.

The sign of the parameter on the ϕ term shows that the school's *location* on the pdf of the bonus receipt probability is critical in determining a teacher's effort exertion. Very low and very high probabilities of bonus receipt are associated with lower effort exertion. Translating the ϕ term into expected bonus, at low levels of expected bonus, teachers exert low effort, with effort increasing as expected bonus (probability of bonus receipt) increases. The peak level of effort exertion is achieved when expected bonus is at approximately \$870. Beyond this point, the bonus becomes a 'sure-thing,' and effort declines again toward lower levels as expected bonus increases. Therefore, if the goal of the incentive policy is to energize teachers to exert the maximum amount of effort possible, the threshold value for attaining the bonus must be set such that qualifying for the cash bonus is neither too easy nor too difficult. While the estimate for effort signal shows how teachers respond to incentives, the estimate for the effect of effort on achievement shows how students respond to motivated teachers. Increasing expected bonus amount from \$0 to \$850 will increase an average teacher's effort by about 13%, which translates to about a 2.8% of a standard deviation increase on the average student's EOG reading exam and about a 3.8% of a standard deviation increase on the mathematics exam. It is interesting to place these results in the context of the effectiveness of class size reduction.⁴⁹ While estimated magnitudes differ by studies, it is generally agreed that cutting a 20-student class in half can yield somewhere between 5 to 15% of a standard deviation increase in student achievement. Comparing the effectiveness of the two policies, bonus incentives may be an efficient method of raising students achievement.

The parameter on the J term shows that a teacher exerts a lower level of effort when the number of colleagues in the school increases. A higher number of teachers at a school implies

⁴⁸When the OLS regression is run excluding the teacher experience and certification variable, the parameter estimate on absence increases from -0.0010 to -0.00045 while other parameter estimates are stable. This further confirms that the raw absence variable is soaking up some of the explanatory power of experience.

⁴⁹See Hanushek, Rivkin, and Kain (2005).

lower effort for teachers, pointing to incentive dilution in school-level incentives.⁵⁰

6.1 Identification

Identification in this model is *not* based on pre- and post-treatment outcomes. The data set does not go back in time far enough to capture pre-treatment outcomes. Only about 25% of the student data can be matched across any two pre-incentive years.⁵¹ Instead, identification comes from the variation in the number of teachers and the distance from the cut-off for bonus in both directions. There are some schools that are so far below the standard across the sample years such that their predicted probability of qualifying for the bonus is near zero, and there are some schools that far enough over the bar on all years such that the bonus is almost a certainty. The relatively high number of days of absence observed for teachers in these schools is what drives identification in the model. A necessary assumption here is that teacher effort is fully captured by observed teacher absence. If teachers exert effort on unobservable dimensions, and if this effort exertion is uncorrelated with absence (and/or incentive pressure), the parameter estimate on projected effort will be biased. If, on the other hand, these dimensions of effort move together with absence, projected effort can be interpreted as the combination of many types of teacher effort.

Incentive related factors that move teacher effort in the model are captured by the number of teachers at the school (J_{st}) and the position of the school with respect to the probability of bonus receipt ($\phi(\hat{y}_{st}, Cr_1) + \phi(\hat{y}_{st}, Cr_2)$). In essence, J and ϕ instrument for teacher effort. The econometric model is identified if the instruments J and ϕ can be excluded from the student achievement equation (equation (6)); however, there are credible arguments for including these exclusion restrictions directly into the student achievement equation.

The number of teachers, J , serves as a measure of incentive dilution from free-rider effects that could impact the amount of effort teachers put forth. If the free-rider effect leads to more absences, this could negatively impact student achievement. However, it is also possible to argue that more teachers at a school could lead to more individualized attention or better

⁵⁰A possible concern here is that the parameter on the number of teachers is not measuring free-ridership arising from the bonus regime, but a pre-existing condition. I ran a regression that measures the sensitivity of absence on the number of teachers in the school for pre- and post-incentive years. The parameter estimate on the number of teachers pre-incentive is statistically insignificant, while it is positive and significant for parameters from post-incentive installation.

⁵¹Given this limitation, I run a placebo trial that estimates the teacher absence decision equation and the effect of incentives on achievement. The results show that teachers do not respond and student achievement is not driven by incentives (as they do not exist yet). There is some evidence that once the bonus was discontinued, the ‘bonus effect’ disappeared. The bonus system was discontinued in 2010 due to budgetary cuts. See Ahn and Vigdor (2012) for details.

discipline of students. Further, J as a measure of free-ridership, and J as a measure of individualized attention could pull achievement in opposite directions. To test whether J has a direct *positive* effect on student achievement, I run an alternate specification using $J \cdot I[\text{no bonus effect}]$ and find that there is no statistically significant direct effect of J on y . Here, $I[\text{no bonus effect}]$ is an indicator variable which equals one when the probability of obtaining the bonus is near 100% or 0% and 0 otherwise. That is, the bonus is almost assured or virtually out of reach, meaning that the effect of J should only come through the direct channel. See Table 5.

A school's proximity to the bonus threshold may also impact student achievement through other non-effort channels. For example, if the school is close to the threshold, the principal or superintendent may decide to reallocate the school budget toward expenditures that could bring about an immediate short-term increase in student achievement (perhaps at the cost of longer-term expenditure goals). Alternatively, money could be reallocated in schools that are farther away from the threshold, in an attempt to bring them closer to the middle of the distribution, where the incentive effect will be strongly felt by teachers. It may also be possible to argue that the bonus itself directly affects teacher behavior unrelated to the time commitment component of effort as revealed by absence. For example, a teacher may upgrade (or choose to skimp on) teaching aids based on anticipated payouts. Table 6 tests for these alternative channels by including dummy variables for schools that are almost out of the bonus and completely assured of the bonus. The insignificant parameter estimate on $I[\text{no bonus effect}]$ show that the bonus amount has no direct effect when predicted effort is included in the regression.⁵²

The two regressions show that the two measures serve as valid exclusion restrictions. The number of teachers in a school is a good measure of free-rider effects, and the location of the school along the probability distribution serves as a good proxy for incentive effects.

6.2 Impact of Sorting

Student achievement gains in response to accountability pressure is assumed to be captured by the change in absence response of teachers in the model. However, there may be other school responses that occur concurrently with teacher effort increase. If these responses also help

⁵²While the NC data does not contain school-level budget information, it does contain district (LEA) level expenditure data. A regression with various non-salary current expenditures (at the district level) with the fraction of schools near the threshold as the relevant control failed to yield statistically significant estimates. While this regression may fail to isolate in-year changes or details that would be visible with school-level data, it does support the assumption that no large scale budgetary reallocation is taking place in response to the bonus incentives.

to increase student achievement, the effectiveness of effort may be overestimated. A central assumption is that possible non-random sorting and matching of teachers across and within schools in response to accountability pressure will not significantly bias the estimation in the student achievement function. If schools and teachers respond to accountability pressure by sorting, the gains to effort may be distorted. Teacher sorting may occur in the short, medium, and long-term.

Over the short-term (within the current academic year), a principal may seek to match students and classes to particular teachers to maximize the probability of bonus receipt. There is little doubt that non-random sorting of students and teachers into classroom exists.⁵³ This also holds true in the North Carolina data; however, this is problematic only if sorting occurs more frequently in schools that are under accountability pressure, compared to schools not under pressure.

If there is substantial sorting of teachers on unobservable characteristics in response to accountability pressure within the school, the impact of effort may be overestimated. Since the observed increase in test scores in pressured schools resulted from the additional effort induced by the incentives plus the sorting, and since sorting and effort increase are correlated (via accountability pressure), the impact of effort will be biased upwards. This bias due to sorting on unobservable characteristics may be of some concern if the unobservable (positive) teacher characteristics are not homogeneously distributed across all schools along the bonus CDF. Schools with more teachers with desirable unobservable characteristics may have the ability to more effectively match teachers to students to maximize education production. For instance, if the proportion of teachers with desirable unobservable characteristics is positively correlated with the school's location on the bonus CDF, a policy change that moves schools on the left and right tails to the middle of the distribution may yield larger than expected gains for schools that were at the right tail, and smaller than expected gains for schools that were at the left tail, provided that both types of schools sort to take advantage of the unobservable teacher characteristics.⁵⁴ The literature has been agnostic about *why* sorting occurs, but many studies show that teachers with more seniority are often in classrooms that have a larger proportion of higher-achieving students. If most sorting occurs because of

⁵³There is some evidence that students and teachers paired by race (for instance) tend to do better. See Dee (2004). See Clotfelter, Ladd, and Vigdor (2006) among others for examples of non-random sorting.

⁵⁴It should be noted that schools located toward the left tail of the CDF are not necessarily 'undesirable schools' in the traditional sense (low test scores, urban, poor, etc.). In fact, the correlation between a school's ABC status (whether it qualifies for the bonus) and whether the school made AYP is relatively small. Even if there is high correlation between positive unobservable teacher characteristics and the desirability of the school (as measured by AYP), the correlation between the positive unobservable teacher characteristics and her school's place on the probability CDF may not be as high.

seniority rules or personal and professional relationships with the principal and not due to accountability pressure, the bias is not expected to be large.⁵⁵

Over the ‘medium-term’ (across one or two years), teachers may respond to accountability pressure by attempting to transfer to schools that offer a higher likelihood of qualifying for the bonus. Alternatively, schools under higher pressure may seek out teachers who can increase achievement.⁵⁶ The decision to transfer next year or the near future should not impact a teacher’s effort in the current academic year, since the probability of bonus receipt is determined year to year. It is possible to argue that a teacher may seek to maximize effort output this year as an ‘audition’ to transfer to a preferred school next year. In this case, the incentive effect may be underestimated, since teachers in schools that have low probability of qualifying for the bonus may exert additional effort in order to facilitate a transfer to a better school in the near future.

Over the long-term (across several years), if the bonus is perceived as a pay increase, the state may be able to attract a larger number of higher-ability teachers. As Hanushek (2009) demonstrates, replacing even a small fraction of low ability teachers with average teachers may have large positive long-term impact. If retiring teachers are continually replaced with higher quality teaching candidates on average, state-wide academic achievement may increase. The ABC bonus program began paying out in 1996/97. The starting year of the data set used in this study is 1999/00. It is possible that some on the fence about entering the teaching profession could have been enticed to do so due to the bonus. If better teachers enter the labor market, the ‘true’ impact of the bonus may be underestimated. The true impact would be the incentive effect plus the increase in the ‘base-line’ academic achievement of students due to the increase in the average quality of teachers.

⁵⁵To test for possible sorting of observable characteristics in response to accountability pressure, I ran the following regression:

$$DevMin = \beta_0 + \beta_1 \cdot pdf + \beta_2 \cdot tchexp + \beta_3 \cdot [tchexp * pdf] + z + \epsilon$$

Where *DevMin* is classroom minority % divided by school minority % (to capture overt redistribution of students), *pdf* is the measure of accountability strength, *tchexp* is the teacher experience variable, and *[tchexp*pdf]* is the interaction term. The vector *z* contains all other variables, including teacher demographic characteristics and indicators for year, grade, and district. ϵ is the error term. The β_2 term is negative and significant, confirming that sorting on seniority does occur. The regression results are insignificant for β_3 , lending weight to the idea that the sorting is not occurring due to accountability pressure. The intuition is that if the classroom composition is a policy tool (and principals want to use this tool by matching effective teachers with either high or low minority-heavy classes to maximize education production), as pressure increases, we should see a positive β_3 if effective teacher-high minority class is attempted, and negative β_3 if effective teacher-low minority class is attempted. Note that this type of test can only check for sorting on *observable* teacher characteristics.

⁵⁶See Ahn (2013), where teachers with higher fixed-effect values tend to match more frequently with schools that are under more accountability pressure.

Students sorting across schools in response to accountability pressure may also distort the accountability system. Students moving to (or away from) schools under pressure will *not* necessarily raise or lower a school's bonus receipt probability. Because previous year test scores are attached to the student and not the school, the test score growth requirement will remain stable even with student transfers. However, students may impact classmates' test scores through peer effects, and movements of a large number of students in and out of schools may be disruptive, not only for the transferring students but for their old and new classmates as well. In the data, the transfer rate of students is not strongly correlated with accountability pressure. If students with higher test score growth potential are more likely to transfer, this may lead to incentive effects being overestimated.⁵⁷

While the ability to measure teacher effort and its effect on student achievement is interesting in its own right, the more important question is how to design policy to effectively induce effort. The next section presents two possible policy changes. The first experiment changes the criterion from a school-wide performance to a classroom-level performance measure, attempting to eliminate the incentive dilution problem. The second experiment changes the current standard to a grade-level performance measure.

7 Policy Simulations

An incentive system that rewards based on group performance may suffer from free-rider problems. Therefore, targeting the bonus at the individual-level may improve performance. I present two possible policy alterations to the current school-level rewards to examine possible effects. First, I change accountability criterion to target bonus at the classroom-level, completely eliminating the free-rider effect. Second, I change the criterion to target at the grade-level, an intermediate level of grouping between school-level and classroom-level.⁵⁸

Computationally, I change the bonus rule (equation (4)) to reward teachers at either the grade-level or the classroom-level, by changing the level at which student scores are aggregated. This generates a new probability of bonus receipt for each teacher. The new probability and the appropriate count of teachers (one for classroom-level or the appropriate

⁵⁷Student take-up rate for transfers when schools failed to make adequate yearly progress under No Child Left Behind in North Carolina was also very low during the sample years.

⁵⁸It is worth noting that a continuously increasing individual-level bonus payment in student-level achievement growth (assuming revenue neutrality) would almost surely be the most efficient policy change. The simulations conducted in this section are more modest alterations in line with the types of policy changes that a legislative body may be willing to consider. For example in 2005/06, the rule set for high school bonus calculations was amended to take into account two previous years test scores while keeping the same pay-out structure.

number of teacher for grade-level) are plugged into equation (5), keeping the parameter values fixed. This generates a new effort level (absence), which is then plugged into the student achievement equation (equation (6)), again, with the estimated parameter values fixed. The newly generated test score outcomes are summed and averaged according to the new bonus rule and plugged into equation (4), generating a new probability of bonus receipt. The system is iterated until the average difference in each teacher’s absence from one iteration to the next is less than 1%.

Classroom-level incentives simulation are presented in the middle column of Table 7.⁵⁹ The results show that the expected bonus per teacher increases by about \$150. Since there are roughly 45,000 teacher/year observations over the five-year sample period, and assuming that there is an equal number of kindergarten, first, and second grade teachers in the qualifying schools, gross bonus pay out would increase by roughly \$13.5 million each year. This is approximately equivalent to a 13% increase in the state budget for bonus pay out. Yet at the same time, average predicted effort decreases by about 5.6% and variance of expected bonus (and by extension variance of effort) increases.

These seemingly contradictory results can be explained by examining teacher incentives. With the regime switch from school-level to classroom-level, teachers would be split sharply into those who can achieve the bonus standard, and those who cannot. Going from a school-wide effort to an individual effort will starkly separate low and high ability teachers.⁶⁰ A teacher with a low chance of success will have little incentive to exert effort, while a teacher with a fair chance of success will only increase her effort until the marginal benefit of doing so matches marginal effort cost. In addition, teachers with very high ability and/or very high achieving students will decrease effort, since the probability of bonus receipt is no longer dragged downward by lower achieving classes. With the right and left tail of the expected bonus distribution pushing effort downward, average effort decreases.

Therefore, moving from school-wide to classroom standards eliminates the direct negative effect of free-ridership but is offset by locating more teachers at the tails of the bonus probability pdf. The histograms in Figures 4, 5, and 6 demonstrate this indirect effect. Figure 4 shows that switching the bonus criterion to classroom performance increases the weight of the tails of the expected bonus distribution. The right tail represents classrooms which were outperforming the rest of the school, yet were being dragged down as part of the school-wide

⁵⁹It is possible that as incentive structures change, classroom allocations of students may also change. For instance, a principal may optimally reallocate students to get all teachers to the middle of the bonus probability to induce greater effort. See Barlevy and Neal (2012). The simulations do not account for this possibility.

⁶⁰‘Low’ ability does not necessarily indicate poor quality teachers, but teachers with low probability of success. This could be the result of the composition of students in her class or school, for example.

criterion. The left tail represents classrooms which were under performing compared to the rest of the school, yet were buoyed by the being part of the school-wide criterion. Figure 5 shows that as the bonus criterion changes, teachers adjust their efforts accordingly.⁶¹ While there is little change in the high effort (low absence) part of the distribution, the percentage of teachers exerting low effort (high absence) increases. This is directly attributable to the increase in the tails of the histogram in Figure 4. Highly under/over-achieving teachers are no longer pushed toward the middle of the distribution, resulting in lower effort exertion for these teachers. The decrease in average teacher effort is reflected in the increase in the fraction of students with lower standardized scores and the slight decrease in students with higher standardized scores as illustrated in Figure 6.

If attempting to eliminate the free-rider effect through individual incentives results in negative achievement growth due to loss of motivation at the tails of the pdf, an intermediate step of targeting at the grade-level may mitigate the free-rider effect, yet retain the desirable cooperative properties. The last column in Table 7 demonstrates this to be the case. Average teacher effort and student achievement increase. The differences in variance in expected bonus across the three regimes show that targeting at the grade-level lowers the number of teachers pushed out to the tails of the probability distribution compared to classroom-level targeting. At the same time, the reduction in the number of teachers to approximately a third, compared to the school-level targeting, raises the base effort level of all teachers. The combination of these two effects manages to hit a ‘sweet-spot,’ where it is possible to increase average teacher effort and student achievement.⁶² Expected bonus amount for each teacher increases by an amount similar to classroom incentives.

As seen in Table 8, the distributional impacts of classroom and grade-level targeting differ for advantaged and disadvantaged students. When the state evaluates bonuses based on classroom averages, the impact on test scores for white students is slightly negative. For students with parents with high levels of education, there is a small positive impact. However, for minority students and students with parents who have high school education or less, the impact on test scores is large and negative. The last four rows show that roughly one third of students achieve higher scores under the classroom regime, with students in the more advantaged subgroups having a slight advantage. The benefits of the regime change are concentrated in the middle of the student achievement distribution, while those at the lower

⁶¹The mass at 30 days for classroom regime is an imposed upper limit. Removing the upper limit does not qualitatively change the results.

⁶²This is not meant to imply that grade-level targeting is a first-best solution. There are other alternatives, such as explicitly tying the performance of remedial classes with advanced track classes, to induce academic gains.

end (more heavily concentrated in minority and low parental education students) and the top end (more heavily concentrated in white and high parental education students) are deprived. When the regime is changed to grade-level targeting, the gains accrue more evenly across all students.

Ignoring budgetary concerns, the policy simulation results would seem to suggest that the state should switch to a grade-level criterion. However, several problems arise in switching to the proposed standard. There could be large, negative general equilibrium effects. Non-random sorting of students in grades and classrooms may make it difficult to gauge the teacher's true effectiveness and create tension among teachers.⁶³ Teachers may compete to avoid grades least likely to make year-over-year improvements. With experienced teachers having seniority, we may see the students who require the most help ending up with the least experienced teachers. Another possibility is that teachers who are perpetually stuck with under-achieving students may seek to transfer or exit the profession altogether. Even if the principal could commit to random sorting of students, some teachers may win or lose out on the bonus due to pure chance. In addition, teachers in non-tested grades would need to be compensated in some alternative manner. Currently, kindergarten, first, and second grades students are not administered an EOG exam. Teachers in charge of these students are paid the bonus according to school-wide performance. In evaluating a move to an alternative criterion, these negative factors must be considered carefully, along with the positive results from the incentives.

8 Conclusion

This study measured the impact of teacher effort on student test scores and examined the effectiveness of accountability legislation using a principal-agent model and the North Carolina administrative education data set. Going beyond reduced form estimates of the impact of accountability systems on education production, the theoretical and econometric models connect academic achievement to the North Carolina accountability policy by inserting in between the true causal mechanism, teacher effort. Teacher effort is captured by measuring the teacher absence response to changes in incentive pressure, which reduces the noise in raw absence data.

Explicitly modeling the accountability system in this way and estimating a structural econometric model reveals that even a relatively simple and modest accountability program like the North Carolina system can yield complex and unintended consequences. The theoret-

⁶³See Rothstein (2010).

ical model for a pay-for-performance program where a bonus is paid to teachers for the school passing a strict threshold for gains in test scores shows that: 1) the free-rider problem increases in the number of teachers, 2) the non-linearity of the probability of bonus receipt leads to higher effort exertions by teachers when the bonus outcome is in doubt, and 3) the free-rider and the incentive effects from the probability of bonus-receipt often pull teacher effort in opposite directions. The theoretical model points to the possibility that policy alterations aimed at minimizing the free-rider effect by changing the bonus system from school-level to classroom-level incentives may not be effective in raising teacher effort or student achievement. The parameter estimates from the empirical model confirm that teacher effort (predicted absence) is an important component of student achievement, teachers respond to cash incentives, free-rider effect exists, and the non-linearity of the bonus probability is important in determining teacher effort.

I performed two policy simulations to gauge the possible effects of accountability reform. The first experiment changed the criterion from school to class-level performance, attempting to raise achievement by eliminating the free-rider effect. The second experiment evaluated bonus at the grade-level, attempting balance the free-rider effect and the incentive pressure.

Classroom-level incentives result in lower teacher effort and test score performance compared to school-level incentives. While the free-rider effect is completely gone, separating teachers into individual classes sharply divides many teachers into two extreme groups: those who have almost no chance of qualifying for the bonus, and those who are almost assured of the bonus. The rational response of both groups of teachers is to exert less effort. This change in incentive pressure overwhelms the gains in achievement from the elimination of free-rider effects.

Grade-level incentives divide teachers into groups roughly one-third in size. The reduced number of teachers mitigates the impact of free-rider effects, while the grouping still forces the high and low ability teachers to exert effort in an attempt to qualify for the bonus. Grade-level incentives yield higher teacher effort and test scores compared to classroom-level or school-level incentives, managing to balance the two forces better than the accountability policies at the two extremes.

The findings in this study hold generally for games in which managers must set all-or-nothing rewards for employees in a cooperative project with noisy output and signals. The North Carolina system is comparable to rewarding the team as a whole on the success of a project. The first simulation moves from team reward to individual contribution. The second simulation adjusts the size of the team to foster collaboration and to mitigate free-riding. Ultimately, the research shows that, as is the case with most well-meaning legislation or

incentive systems, there are always unanticipated responses from the targets of the legislation such that the end result may be quite different from what was originally intended.

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9 Tables and Figures

Table 1: Sample Statistics[†]

	Variable	Mean (Std. Dev.) [Min, Max (where appropriate)]
Student	Male	0.504
	Minority	0.346
	Parent HS or less	0.724
	Gifted	0.137
	Disabled	0.114
	Limited English	0.022
	Observations	897,471
Teacher	Male	0.077
	Minority	0.153
	Experienced	0.942
	Certified	0.056
	Absent days/Acad. Yr.	9.562 (6.107) [0, 30]
	Observations	45,011
Class	% Male	0.511 (0.093) [0, 1]
	% Minority	0.353 (0.280) [0, 1]
	% Parent HS or less	0.703 (0.263) [0, 1]
	Class size	22.01 (4.151) [5, 36]
School	School size	517.43 (196.199) [46, 1,423]
	Number of teachers	34.43 (11.944) [4, 77]
	Rural school	0.468

[†]NCERDC dataset. Years 1999/00 - 2003/04. All 3rd to 5th grade public elementary students and their teachers, excluding charter and alternative schools. Teachers with more than 30 days of absence excluded. Students with 0 or 1 exam record excluded. ‘Experienced’ equals 1 for teachers with more than 1 year of experience, due to limitations of the administrative data set in tracking total years of employment.

Table 2: OLS Estimates of the Impact of Absence on Academic Achievement[†]

Variable	Read. Coeff. (Std. Dev.)	Math Coeff. (Std. Dev.)
Absence	-0.0010 (0.0001)	-0.0022 (0.0001)
Absence X Minority	0.0001 (0.0002)	0.0002 (0.0002)
Last year subject score	0.7085 (0.0007)	0.7454(0.0007)
Male	-0.0378 (0.0013)	-0.0060 (0.0012)
Minority	-0.1693 (0.0027)	-0.1437 (0.0025)
Parent HS or less	-0.1887 (0.0017)	-0.1737 (0.0016)
Teacher male	-0.0259 (0.0025)	-0.0204 (0.0023)
Teacher minority	-0.0053 (0.0020)	-0.0338 (0.0019)
Teacher certified	0.0290 (0.0028)	0.0500 (0.0020)
Teacher experienced	0.0774 (0.0029)	0.1134 (0.0027)
% Class w/ parent HS or less	-0.0078 (0.0049)	-0.0331 (0.0046)
% Class minority	-0.1125 (0.0074)	-0.1306 (0.0070)
% School w/ parent HS or less	-0.1190 (0.0066)	-0.0405 (0.0062)
% School minority	0.0555 (0.0083)	0.0690 (0.0073)
Class size	0.0036 (0.0004)	0.0035 (0.0004)
School size/1000	-0.0267 (0.0169)	0.0119 (0.0158)

[†]Estimation included district, year, and grade fixed effects. Dependent variables are standardized EOG reading and mathematics exam scores.

Table 3: Structural Estimates of the Impact of Effort on Academic Achievement[†]

Variable	Read. Coeff. (Std. Dev.)	Math Coeff. (Std. Dev.)
Absence	-0.0250 (0.0052)	-0.0342 (0.0048)
Absence X Minority	0.0023 (0.0007)	0.0018 (0.0007)
Last year subject score	0.7086 (0.0007)	0.7456 (0.0007)
Male	-0.0378 (0.0013)	-0.0063 (0.0012)
Minority	-0.1900 (0.0074)	-0.1599 (0.0069)
Parent HS or less	-0.1884 (0.0018)	-0.1734 (0.0016)
Teacher male	-0.0555 (0.0071)	-0.0604 (0.0066)
Teacher minority	0.0044 (0.0030)	-0.0208 (0.0028)
Teacher certified	0.0667 (0.0088)	0.1006 (0.0082)
Teacher experienced	0.1803 (0.0231)	0.2535 (0.0216)
% Class w/ parent HS or less	-0.0065 (0.0049)	-0.0308 (0.0046)
% Class minority	-0.1125 (0.0075)	-0.1307 (0.0070)
% School w/ parent HS or less	-0.1091 (0.0069)	-0.0267 (0.0065)
% School minority	0.0705 (0.0090)	0.0901 (0.0084)
Class size	0.0033 (0.0004)	0.0032 (0.0004)
School size/1000	-0.0846 (0.0216)	-0.0655 (0.0020)

[†]Estimation included district, year, and grade fixed effects. Dependent variables are standardized EOG reading and mathematics exam scores.

Table 4: Teacher Effort Decision Estimates[†]

Variable	Coefficient (Std. Dev.)
Log Number of Teachers	0.3858 (0.1561)
Log ϕ	-0.2613 (0.1067)
Male	-1.2871 (0.1561)
Minority	0.4215 (0.0843)
Experienced	4.4572 (0.1254)
Certified	1.6172 (0.1188)
Class size	-0.0825 (0.0351)
School size/1000	-3.7188 (0.8637)
% Class w/ parent HS or less	0.63765 (0.4948)
% Class minority	0.1411 (0.2798)
% School w/ parent HS or less	-0.0068 (0.5327)
% School minority	0.4109 (0.3189)

[†]Dependent variable is days of teacher absence. Estimation included district, year, and grade fixed effects.

Table 5: Testing for Direct Effect of Teacher Exposure on Achievement[†]

Variable	Coefficient (Std. Dev.)
Log Absence	-0.2452 (0.0090)
Log Absence X Minority	0.0148 (0.0057)
Log Number of Teachers X Pr(Bonus) \cong 0 or 1	-0.0029 (0.0035)
Last year reading score	0.7104 (0.0007)
Male	-0.0372 (0.0013)
Minority	-0.1866 (0.0130)
Parent HS or less	-0.1882 (0.0018)
Teacher certified	0.0700 (0.0031)
Teacher experienced	0.2434 (0.0066)
% Class w/ parent HS or less	-0.0415 (0.0040)
% Class minority	-0.0901 (0.0040)
Class size	-0.0010 (0.0002)
School size/1000	-0.0036 (0.0006)

[†]Estimation included district, year, and grade fixed effects. Dependent variable is standardized EOG reading exam scores.

Table 6: Testing for Direct Effect of Bonus on Achievement[†]

Variable	Coefficient (Std. Dev.)
Log Absence	-0.0165 (0.0067)
Log Absence X Minority	0.0028 (0.0007)
I(< 5% Pr. of Bonus receipt)	0.0028 (0.0048)
I(> 95% Pr. of Bonus receipt)	-0.0067 (0.0044)
Last year reading score	0.7035 (0.0007)
Male	-0.0384 (0.0013)
Minority	-0.1998 (0.0071)
Parent HS or less	-0.1905 (0.0017)
Teacher certified	0.0487 (0.0112)
Teacher experienced	0.1352 (0.0298)
% Class w/ parent HS or less	0.0256 (0.0046)
% Class minority	-0.1755 (0.0075)
Class size	-0.0006 (0.0002)
School size/1000	-0.0008 (0.0002)

[†]Estimation included district, year, and grade fixed effects. Dependent variable is standardized EOG reading exam scores.

Table 7: Classroom and Grade-level Targeting

	School	Classroom	Grade
Expected bonus	737.69 (149.69)	889.56 (562.96)	858.64 (456.11)
Effort Increase		-5.62%	+3.70%
Δ Reading score [†]		-1.04%	+1.32%
Δ Math score [†]		-1.38%	+1.72%
Teachers w/ higher effort		35.0%	92.7%
Schools w/ higher scores		46.5%	88.0%
Students w/ higher scores		33.0%	93.2%

[†]Change in percent of one standard deviation in test scores.

Table 8: Classroom and Grade-level Targeting Distributional Effects

	Classroom	Grade
<i>Mean Reading Scores Change for:</i>		
Students w/ low parent ed.	-1.61%	+1.02%
Minority students	-3.00%	+0.40%
Students w/ high parent ed.	+0.03%	+2.71%
White students	-0.26%	+1.85%
<i>Mean Math Scores Change for:</i>		
Students w/ low parent ed.	-2.13%	+1.34%
Minority students	-3.94%	+0.54%
Students w/ high parent ed.	+0.04%	+3.54%
White students	-0.35%	+2.47%
<i>Percent of sub-group with higher scores[†]:</i>		
Students w/ low parent ed.	31.2%	97.0%
Minority students	31.7%	94.3%
Students w/ high parent ed.	33.7%	91.8%
White students	35.5%	75.1%

[†] Number of students in sub-group whose scores increased as a result of policy divided by number of students in sub-group. Results are identical for reading and math scores.

Figure 1: Non-cooperative Regime

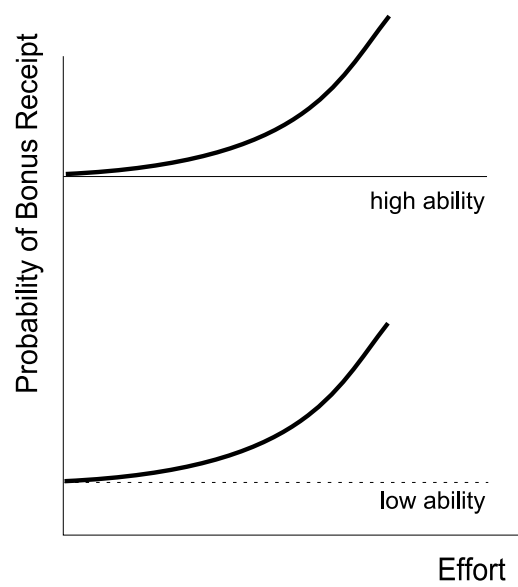


Figure 2: Cooperative Regime

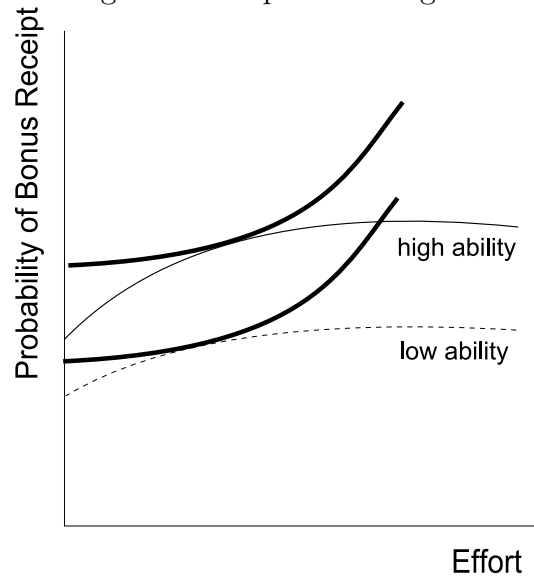


Figure 3: The Effect of ϕ on Effort

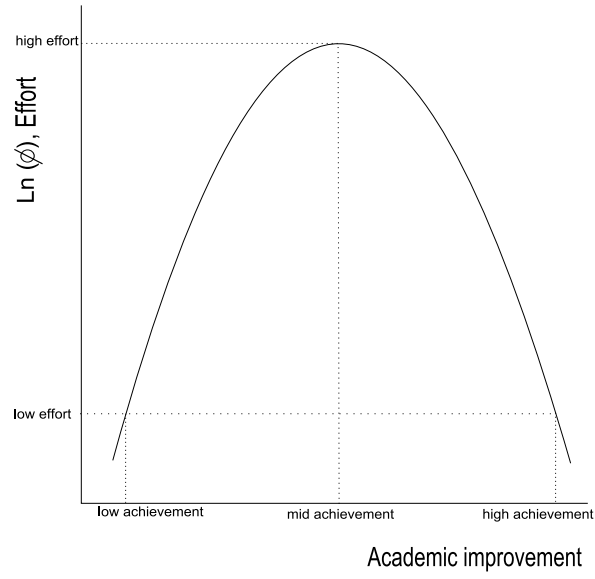
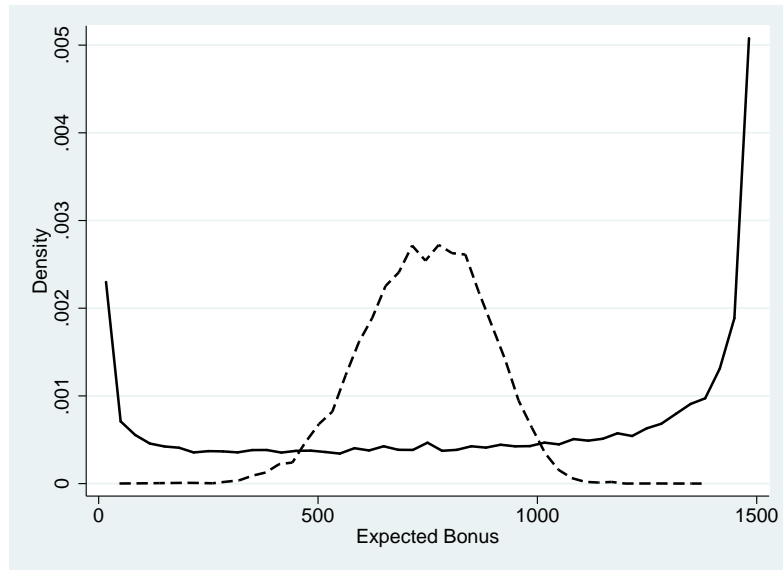
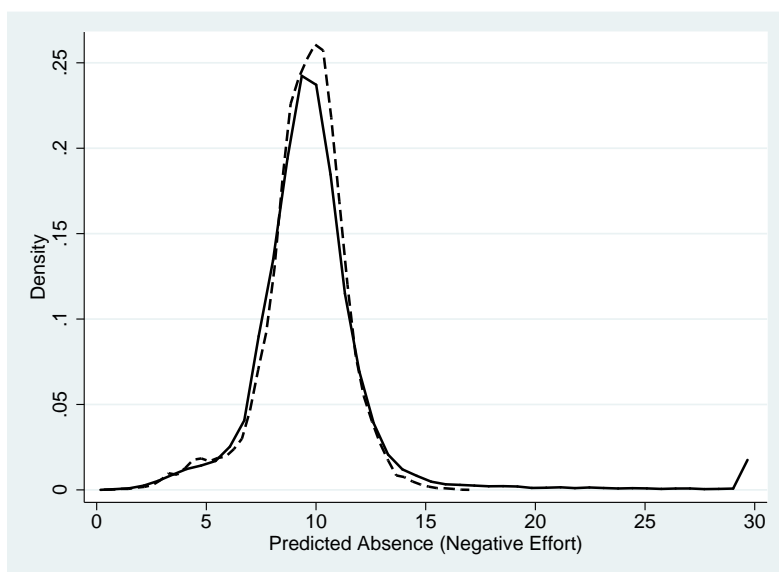


Figure 4: Histogram of Expected Bonus under School-level and Classroom-level Incentives[†]



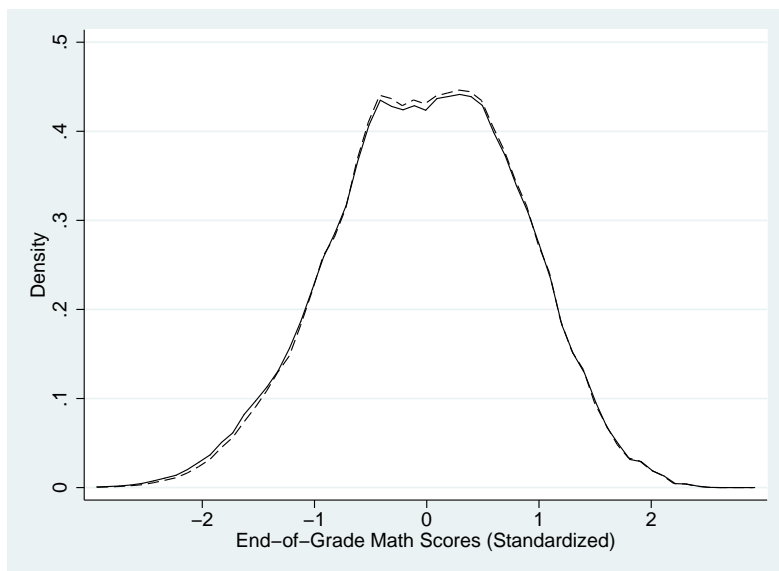
[†]The dashed line represents school-level incentives. The solid line represents classroom-level incentives.

Figure 5: Histogram of Predicted Absence under School-level and Classroom-level Incentives[†]



[†]The dashed line represents school-level incentives. The solid line represents classroom-level incentives.

Figure 6: Histogram of Predicted Test Scores under School-level and Classroom-level Incentives[†]



[†]The dashed line represents school-level incentives. The solid line represents classroom-level incentives.

10 Appendix: Proofs

Proof of existence of NE Since the utility function is concave in effort, the best response functions can be obtained by solving for first order conditions. Solving for first order conditions, J equations:

$$B \cdot F'(\cdot) \frac{w_j}{J} \frac{\partial y_j}{\partial e_j} - C'(e_j) = 0$$

By assumption, a teacher's utility is strictly concave in her own effort. Re-write the first order conditions for teacher j as:

$$B \cdot F' \left(\frac{w_j \cdot y_j}{J} + Y_{-j} \right) \frac{w_j}{J} \frac{\partial y_j}{\partial e_j} = C'(e_j)$$

Note that $Y_{-j} \in [\underline{Y}, \bar{Y}]$ maps into $e_j \in [\underline{e}, \bar{e}]$ for all j .

Assume $F''(\cdot) \geq 0$, as we perturb Y from its minimal to maximal value, e_j must increase. That is, the reaction function of e is positively sloped when $F'' \geq 0$. When the reaction function is positively sloped, e is a strategic complement, and it is well established that games in strategic complements have a unique pure strategy Nash equilibrium. See Milgrom and Shannon (1994) for details.

Assume $F''(\cdot) < 0$, as we perturb Y from its minimal to maximal value, e_j must decrease. In this case, the reaction function is negatively sloped when $F'' < 0$, and effort is a strategic substitute.

If $Y^L < Y_{-j} < Y^H$ for $i = 1, 2, \dots, J$, all agents must exert some effort, and there exists a unique pure strategy Nash equilibrium. See Bramoulle and Kranton (2007) for details. QED.

Proof of Free Rider Problem

The condition on x and Y implies an interior solution. Since we assume that all teachers/classes are identical, $y_j = y_{-j}$ for all j . Let x_{J+1} , the additional class/teacher, also be identical to the other classes. Then, Y_{-j} does not change. The first order condition changes to:

$$B \cdot F' \left(\frac{w_j \cdot y_j}{J+1} + Y_{-j} \right) \frac{1}{J+1} \frac{\partial y_j}{\partial e_j} = C'(e_j)$$

This requires e to decrease in order for the FOC to be met. Since teachers are identical,

all teachers reduce e . The result follows that the incentive system suffers from a free rider problem. Note here that one cannot take derivatives to test for $\frac{\partial e_j}{\partial J}$ because increases in J are *discrete*, and an increase in J also implies a new x_{J+1} and e_{J+1} . QED.

Proof of Individual Criterion not Always Increasing Effort

Let $J = 2$. Assuming identical class size, in the individual criterion, the FOCs are:

$$\begin{aligned} B \cdot F'((e_1 \exp(x_1))) \exp(x_1) - C'(e_1) &= 0 \\ B \cdot F'((e_2 \exp(x_2))) \exp(x_2) - C'(e_2) &= 0 \end{aligned}$$

Let $x_1 = x^H$ and $x_2 = x^L$. By Assumption (5), $\frac{\partial F'}{\partial e_i} \cong 0$ for $i = 1, 2$. The solution is at a corner, and $e_1^* = e_2^* = 0$, where $\underline{e} = 0$.

In school-wide criterion, the FOCs are:

$$\begin{aligned} B \cdot F'((\frac{1}{2}(e_1 \exp(x_1) + e_2 \exp(x_2)))) \frac{1}{2} \exp(x_1) - C'(e_1) &= 0 \\ B \cdot F'((\frac{1}{2}(e_1 \exp(x_1) + e_2 \exp(x_2)))) \frac{1}{2} \exp(x_2) - C'(e_2) &= 0 \end{aligned}$$

$\frac{\partial F'}{\partial e_i} > 0$. The FOCs are jointly satisfied if and only if $e_1^* > 0$ and $e_2^* > 0$. While $e_1^* = e_2^* = 0$ is still a possible solution, the positive effort solution dominates, as $U_i|(e_i = 0) = 0$ for $i = 1, 2$, but $U_i|(e_i \geq 0) \geq 0$ for $i = 1, 2$. QED.

Illustration of the Negative Relationship between Absence and Effort

Assume teachers get a daily ‘potential effort’ draw from some distribution $G(\delta) \in [0, \bar{\delta}]$, such that teacher i has $\{\delta_{i1}, \delta_{i2}, \dots, \delta_{id}, \dots, \delta_{iD}\}$ where D is the total number of days in an academic year. Define μ_i as the mean value of effort for teacher i . There exists some $\underline{\delta}$ such that if $\delta_{id} < \underline{\delta}$, teacher i is absent on day d , $A_{id} = 1$. Therefore, the probability of teacher i being absent on any given day is:

$$Pr(A_{id} = 1) = Pr(\delta_{id} < \underline{\delta})$$

Then, the number of absent days for teacher i in year t is: $A_{it} = D \cdot Pr(\delta_{id} < \underline{\delta})$. Assuming that effort is not transferable, potential effort on day d , δ_d is equivalent to actual effort e_d . For two teachers i and j where $\mu_i \geq \mu_j$, and $E(A_{it}) \leq E(A_{jt})$.

Now assume effort can be stored but decays at rate λ . For illustration purposes, I focus on the last two days of the academic year before the EOG exam, $D - 1$ and D . Education production during the two days is $\exp(x)e_{D-1} + \exp(x)e_D$ if the teacher teaches both days. If the teacher opts to take one day off and teach on the last day, education production is

$\exp(x)0 + \exp(x)(\lambda e_{D-1} + e_D)$. If $\lambda < 1$, education production declines if effort is stored up. Therefore, if there is any effort decay, education production is maximized by teaching both days.

Now assume there is education decays from day-to-day as students forget material learned. Assume that education decays at rate ξ . Again, focusing on the last two days, education production if the teacher teaches both days is $\exp(x)\xi e_{D-1} + \exp(x)e_D$. If the teacher takes a day off, education production is $\exp(x)\xi 0 + \exp(x)(\lambda e_{D-1} + e_D)$. Teachers have no incentive to store up effort if $\lambda \geq \xi$.

To sum up, if there is no education or effort decay, the assumption $E(A_i) \geq E(A_j)$ if and only if $e_i \leq e_j$ always holds. If effort can be stored, the condition holds if there is *any* decay in effort from day-to-day. If effort can be stored and education decays from day-to-day, the condition holds if the rate of decay of effort is greater than the rate of decay of education. The separability of the daily education production function is not critical to the analysis. In fact, if there is complementarity between days, it is possible to simply define production across both days as equalling $\exp(x)e_{D-1} + \sigma \exp(x)e_D$ if both days are taught, and $\exp(x)(e_{D-1} + e_D)$ if only one day of teaching occurs, with $\sigma > 1$. Then the above analysis holds, with σ always pulling the inequality towards teaching both days yielding higher education production.

Decreasing Class Size by Increasing J

Let class achievement of teacher k at time t be redefined as:

$$y_{kt} = y(x_{kt}, e_{kt}, sz_{kt})$$

where sz_{kt} is the class size. For simplicity, class size is the school population divided by J_t , the number of teachers in the schools. For this example, I assume the school can add ‘fractional’ numbers of teachers, perhaps by adding teaching assistants, substitutes, or team teachers. Then, the probability of bonus receipt is:

$$Pr_t = \Phi \left(\frac{\sum_1^J y(x_{kt}, e_{kt}, sz_{kt})}{J_t} - \bar{Y}_{t-1} > Cr \right)$$

If a principal has control over J , for any given vector of effort by teachers:

$$\begin{aligned}
\frac{\partial Pr}{\partial J_t} &= \Phi'(\cdot) \left\{ \frac{1}{J} \left(\sum_1^J \frac{\partial y_{kt}}{\partial J_t} \right) + \left(\sum_1^J y_{kt} \right) \left(\frac{-1}{J^2} \right) \right\} \\
&= \frac{1}{J} \Phi'(\cdot) \left(\sum_1^J \frac{\partial y_{kt}}{\partial J_t} - \frac{Y_t}{J_t} \right)
\end{aligned}$$

Assuming that class achievement increase is weakly concave with class size reduction, such that $\frac{\partial y}{\partial J} \geq 0$ and $\frac{\partial^2 y}{\partial J^2} \leq 0$, for different values of J_t and a given level of effort, $\frac{\partial Pr_t}{\partial J_t}$ may be greater than, equal to, or less than 0.

While the increase in J increases classroom achievement by decreasing classroom size, it also increases the free-rider effect, which decreases the probability of bonus receipt. It is important to note here that all of these changes to Pr_t may increase or decrease achievement, and this change may increase or decrease teacher effort, depending on the situation.

If a school is toward the left-tail of the probability distribution (the school is essentially out of the bonus), and if $\frac{\partial Pr_t}{\partial J_t} > 0$, then the principal should hire more teachers if the opportunity is available. The increase in student achievement from reduced class size will move the school toward the middle of the distribution. The response of teachers will be then to exert higher effort, resulting in yet higher achievement. If $\frac{\partial Pr_t}{\partial J_t} < 0$ for these schools, the free-rider effect overwhelms positive impact from class size reduction. As the probability decreases even further, teachers have less incentive to exert effort. The optimal response of the principal is to *decrease* the number of teachers (and increase class size).

On the other hand, if a school is toward the right-tail of the probability distribution (bonus is virtually assured), and if $\frac{\partial Pr_t}{\partial J_t} < 0$, the principal's optimal response is less clear cut. The free-rider effect is larger than the class size reduction effect, which pulls down test scores (thus decreasing the probability of bonus receipt). The movement to the left will induce more effort on the part of teachers. On the other hand, if $\frac{\partial Pr_t}{\partial J_t} > 0$ for these schools, again, the principal's best response is unclear. The increase in the number of teachers increases the test score (thus increasing the probability of bonus receipt). However, this increase in probability will elicit less effort from teachers, which will pull achievement down. In both cases, it is unclear if achievement will end up higher or lower than the original score.

Altering Class Size While Leaving J Unchanged

To simplify the analysis, I assume $J = 2$, the probability CDF is linearized, and class average achievement is linear in effort and class size. Let teacher 1 be the 'high ability'

teacher, and let teacher 2 be the ‘low ability’ teacher. Teacher j ’s utility function is:

$$U_j = B\Phi(Y) - C(e_j)$$

The effort cost function is simply $C(e_j) = e_j^2$. The school-wide average achievement Y is defined as:

$$\begin{aligned} Y &= f_1 \cdot y_1(f_1, e_1) + f_2 \cdot y_2(f_2, e_2) \\ &= f_1 \cdot y_1(f_1, e_1) + (1 - f_1) \cdot y_2(1 - f_1, e_2) \end{aligned}$$

where f_j is the fraction of the school population assigned to teacher j . Define the linearized CDF such that:

$$\Phi = \begin{cases} 0 & \text{if } Y \leq \underline{Y} \\ a + bY & \text{if } \underline{Y} < Y < \bar{Y} \\ 1 & \text{if } Y \geq \bar{Y} \end{cases}$$

Teacher 1’s education production function is:

$$y_1 = \alpha_1 + \alpha_2 f_1 + \alpha_3 e_1$$

Teacher 2’s education production function is:

$$y_2 = \beta_1 + \beta_2(1 - f_1) + \beta_3 e_2$$

To impose that teacher 1 has higher ability, I assume that:

$$\begin{aligned} \alpha_1 &> \beta_1 > 0 \\ 0 &> \alpha_2 \geq \beta_2 \\ \alpha_3 &\geq \beta_3 > 0 \end{aligned}$$

α_1 and β_1 represent base ability of teachers in the absence of additional effort exertion from incentive pressure. The principal attempts to maximize Y by changing the fraction of students taught by teachers 1 and 2 (f_1). Teacher j attempts to maximize U_j by adjusting effort, e_j . Solving for each FOC, it is easy to see (with the simplifications allowing for closed form

solutions):

$$\begin{aligned} e_1^* &= B \cdot b(f_1 \alpha_3)^2 / 2 \\ e_2^* &= B \cdot b((1 - f_1) \beta_3)^2 / 2 \\ f_1^* &= \frac{\beta_1 - \alpha_1 + 2\beta_2 + \beta_3 e_2 - \alpha_3 e_1}{2(\alpha_2 + \beta_2)} \end{aligned}$$

Noting that α_2 and β_2 are less than zero, the principal's optimal response is to increase f_1 as teacher 1 becomes more productive (compared to teacher 2). Teacher 1's optimal effort increases as f_1 or α_3 increases, and teacher 2's optimal effort increases as f_1 decreases or β_3 increases.

Because teacher 1 has higher ability, it is reasonable to guess that setting $f_1 = 1$ is optimal. From the FOC:

$$f_1^* = 1 \iff \beta_1 - \alpha_1 + 2\beta_2 + \beta_3 e_2 - \alpha_3 e_1 \leq 2(\alpha_2 + \beta_2) < 0$$

With $f_1 = 1$, we use $e_2^* = 0$, plug in for Y and simplify to:

$$f_1^* = 1 \iff 2 \cdot \beta_1 - Y \leq \alpha_2 < 0$$

If β_1 is large then it is possible that $\alpha_2 < 2 \cdot \beta_1 - Y$, and the inequality defined above does not hold. β_1 (and α_1) being large (relative to α_2 , α_3 , β_2 , and β_3) is a statement about the relative strength of the incentives compared to the base effort of teachers. In this case, if incentives are comparatively weak, principals may want to distribute students more evenly across the two teachers. If the incentive pressure is strong, the principal may be tempted to have teacher 1 teach a higher proportion of (if not all) students.