Welcomed Entry in a Model of Multi-Market Contact

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1 Introduction

Oracle, the world's largest database software manufacturer, also has a consulting practice that sets up databases for intra-company or web-based businesses. While Oracle Consulting is a direct competitor to all information technology (IT) firms that consult on database and web applications, Oracle forms partnerships with competing firms. These partnerships include training competing firms' employees on Oracle software, subcontracting parts of large projects that Oracle Consulting wins to competitors, and recommending partners over Oracle Consulting. [1] This decreases profits for Oracle Consulting, yet continues to be standard business practice.

One of our stronger intuitions in industrial organization is that firms with market power fight to keep it. Being a monopoly or an oligopoly grants the firm market power, which allows greater extraction of consumer surplus than is possible in a more competitive environment. By the same reason, a firm who is

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[1] See http://www.oracle.com/consulting for Oracle's consulting subsidiary, and http://oraclepartnernetwork.oracle.com for the partner program run by Oracle. Notice that both sites are sponsored on the Oracle parent site.

not part of this market will fight to enter it, in the hopes that it too can extract profit. Therefore, it is reasonable to think of incumbent and potential entrant firms in an adversarial relationship. Incumbents may even act as a competitive firm in a monopolistic market, thus earning arbitrarily small profits, just to prevent entry. Even if entry is 'accommodated,' the incumbent allows entry because it has no choice. Firms covet market power and are loath to give it up. The qualifier that has not been emphasized, perhaps initially because it was so obvious, was that this analysis is limited to competition in a single market.

Broadening the arena of competition from one to multiple markets offers insights into firms that seem 'too accommodating' to fit with our intuition. This paper attempts to show that under a framework of oligopolistic linear demand competition with potential entrants, entry may be accommodated, and even encouraged, to the point where the incumbent firm(s) would shoulder the fixed cost necessary for the entrant.

This paper builds on the model developed by Bernheim and Whinston (1990, hereafter referred to as B&W). B&W's paper showed that multi-market contact between firms may foster collusion. Their main insight is that even if collusion is not supportable in a market, as long as the aggregate profits from colluding in two or more markets is higher than not colluding across all markets, a sustainable collusive equilibrium can exist. This is because firms have one more instrument with which to punish each other if one decides to deviate from the collusive agreement. [2] Because B&W model a Bertrand game, firm profits are either collusive profits or zero profits. This stark contrast in reward and punishment ensures easier cooperation and makes the problem simpler to analyze. However, modelling a Cournot game allows different levels of punishment depending on the relative sizes and elasticities of the multiple markets, allowing for a richer set of results.

I start with a general linear demand function and quadratic cost curves, and duplicate some of B&W's work using two incumbent firms and one potential

[2] For a more detailed exposition, refer to B&W's article.

entrant firm.[3] Using this framework, I derive additional insights about market structures that would make the incumbent firm actively seek entrants into their market. I then generalize the problem, allowing j incumbent firms and z potential entrant firms to derive additional insights that add to B&W's article.

In section 2, I build a specific case of two incumbent firms and one entrant firm in a world of quantity competition with linear demand curves. I show that the relative size and elasticities of markets plays a role in whether the incumbent would be willing to allow a new firm to enter. I derive a bounding condition on δ , the discount factor, that makes collusion possible in one market, and impossible in the other market. The market in which collusion is possible contains two firms, and the other market, in which collusion is not possible, contains three firms. Two of these three firms also operate in the other market, making them the incumbent firms. The firm that only operates in one market is labelled as a potential entrant. [4]

In section 3, I generalize the model from section 2 to allow more than two incumbent firms and more than one entrant firm. I also force the incumbent firms to either accept all entrant firms into their market or none at all.[5] All the conclusions from section 2 are duplicated. Additionally, I show that as the number of incumbent firms increases, holding the number of entrant firms fixed, incumbent firms would agree more easily to allowing entry until the number of incumbent firms hits a critical value. After this value, firms would find it harder to agree to allow entry. I also show similar results as the number of potential entrants increase, holding the number of incumbent firms constant.

In section 4, I generalize the model from section 2 differently. While I fix the number of incumbent firms to two and the number of firms operating in only one market to two, I only allow one of these uni-market operators the option of

[3] To derive interesting results, The markets under consideration *must* have non-linear structure. It can be in the demand (non-linear demand) or it can be in the supply (non-linear cost). I modelled non-linear cost, but all intuitive results should hold for non-linear demand.

[4] With linear demand and constant marginal cost, there exists only on value of δ that will support collusion in the incumbent market and non-collusion in the other market.

[5] This constraint is relaxed in the next section.

entering the first market. Both the incumbent firms and the entrant must to appease the 'left-over' firm, the lone firm not allowed to enter the incumbents' market, to sustain collusion. I allow the incumbent firms and entrant to pay (receive) side payments to (from) left-over firms. I show that a stable collusion requires the incumbent and entrant firms to make side payments to the left-over firm.

2 Simple Linear Demand Model Specification

B&W specified that firms compete under Bertrand competition. I explore the entry problem using quantity competition with linear demand and quadratic cost. Let there be two markets: A and B.

$$\begin{aligned} P_A &= a - b \cdot Q_A && (1) \\ P_B &= c - d \cdot Q_B && (2) \\ \pi_A &= (a - b \cdot Q_A) \cdot q_A - \frac{q_A^2}{2} && (3) \\ \pi_B &= (c - d \cdot Q_B) \cdot q_B - \frac{q_B^2}{2} && (4) \end{aligned}$$

where q_A is production in market A by one firm, and Q_A is aggregate production in market A by all firms. All firms markets A and B produce homogenous goods. Market A has two firms that are competing as an oligopoly in a repeated quantity game. Collusion is a stable equilibrium in this market.

Collusion in this paper is defined as each firm producing an equal share of the monopoly quantity, and sharing monopoly profits equally.[6] The punishment is a trigger strategy of Nash quantity production forever. The collusive equilibrium is stable if the stream of the share of the monopoly profit is greater than a one period deviation profit plus a stream of Nash profits. A firm can deviate by choosing its quantity different from the monopoly quantity. The deviating firm will act as if it were a Stackleberg leader, taking the cooperating firm(s) quantity as given and known. This is specified below for market A:

[6]The solution is equivalent to a monopolist with multiple plants, each with quadratic cost.

$$\frac{1}{1-\delta}\left(\frac{a^2}{2+8b}\right) \geq \frac{a^2(1+3b)^2}{2(1+2b)(1+4b)^2} + \frac{\delta}{1-\delta}\left(\frac{a^2(1+2b)}{2(1+3b)^2}\right) \tag{5}$$

Solving for δ that will allow collusion, I obtain:

$$\delta \geq \frac{(1+3b)^2}{2+b(12+17b)}$$

Market B has three firms that are competing competitively as a repeated Cournot game, because collusion cannot be supported. Note that the two firms operating in Market A are also operating in Market B.

$$\frac{1}{1-\delta}\left(\frac{c^2}{2+12d}\right) \leq \frac{c^2(1+4d)^2}{2(1+2d)(1+6d)^2} + \frac{\delta}{1-\delta}\left(\frac{c^2(1+2d)}{2(1+4d)^2}\right) \tag{6}$$

Solving for δ that will not allow collusion, I obtain:

$$\delta \leq \frac{1+8d+16d^2}{2+16d+28d^2}$$

I attempt to show, under what conditions, the incumbent firms in Market A would be willing to allow the third firm to enter into market A and compete with them.

If the discount factor of the firm is between $\frac{(1+3b)^2}{2+b(12+17b)} \leq \delta \leq \frac{1+8d+16d^2}{2+16d+28d^2}$ then collusion will hold in market A, and collusion will not hold in market B. These conditions are necessary to make the problem interesting. As B&W showed, if both markets do not allow collusion, than the sum of the two market will not yield collusion either. If both markets yield collusion, then the incumbent firms have no reason to allow entry into market A.[7]

Next, I allow the firm operating only in market B to enter market A. To simplify the discussion, assume that the cost of entry for the entrant is infinity until the incumbents grant entry, at which point the cost becomes arbitrarily small.

[7] I assume that firms in market A have sole discretion on whether to allow entry by a new firm.

This precludes the possibility of the traditional analysis of accommodated entry. Incumbent firms never need fear competition from outsiders, unless they want it.

Intuitively, incumbent firms will allow a firm operating only in market B to enter if and only if then entry will result in higher overall profits in markets A and B.

Equilibrium Condition 1:

Four constraints must be met for entry to take place.

1. *It must be optimal for the entrant to enter.*
2. *It must be optimal for the entrant to cooperate once it has entered.*
3. *It must be optimal for the incumbents to allow entry.*
4. *It must be optimal for the incumbents to cooperate once entry has been permitted.*

The four constraints specify conditions under what both incumbents and the entrant have no desire to deviate from the collusive outcome.

In the context of the model, it's clear that constraint 1 must hold if the other three hold. If constraint 3 holds, the incumbent firms must be earning higher profits in market B then previously. Since the entrant firm is identical to incumbents, it must also be earning higher profits in market B after entry. Therefore, even if entrant is making zero profits in market A, it is still optimal to enter.

As will be shown below, size of market and slope of the demand curve become of paramount importance when analyzing the equilibrium condition. By size of market, I refer to the profit potential in the market. Because we are dealing with linear demand functions, the monopoly profit level nicely uses all parameters of the demand function, and serves as a good proxy for market size.

Constraint 2 is specified as below:

$$\frac{1}{1-\delta}(\pi_A^{monopoly} + \pi_B^{monopoly}) \geq \pi_A^{deviate} + \pi_B^{deviate} + \frac{\delta}{1-\delta}(\pi_A^{Cournot} + \pi_B^{Cournot}) \quad (7)$$

After plugging in explicitly for the profit terms, I derive:

$$\frac{1}{1-\delta}\left(\frac{a^2}{2+8b} + \frac{c^2}{2+12d}\right) \geq \frac{a^2(2+3b)^2}{8(1+2b)^3} + \frac{c^2(3+4d)^2}{18(1+2d)^3} + \frac{\delta}{1-\delta}\left(\frac{a^2(1+2b)}{2(1+3b)^2} + \frac{c^2(1+2d)}{2(1+4d)^2}\right) \quad (8)$$

s.t. $\frac{(1+3b)^2}{2+b(12+17b)} \leq \delta \leq \frac{1+8d+16d^2}{2+16d+28d^2}$

I solve for the critical δ value that will support constraint 2. The range of δ that satisfies constraint 2 is:

$$\delta \geq \frac{t_1(1+2b) + t_2(1+2d)}{2(t_1(1+2b(4+7b))(1+2d) + t_2(1+2d(4+7d))(1+2b)}$$

where

$$\begin{aligned} t_1 &= a^2b^2(1+6d)^2(1+4d)^2 \\ t_2 &= c^2d^2(1+6b)^2(1+4b)^2 \end{aligned}$$

I show in the appendix that this range of δ, for some values of the demand functions in markets A and B, satisfy collusion in market A and no collusion in market B.[8] In addition, I show that constraint 2 becomes easier to satisfy as d increases. As d increases, market B becomes more inelastic, and there is more profit to be extracted from market B. Then, sustained collusion becomes a more attractive option for potential entrants because deviation will result in the loss of monopoly profits in market B.

Notice that constraint 4 is identical to constraint 2, because all firms are identical. Once all firms are in all markets, each firm can deviate in exactly the same way as any other firm.

Constraint 3 is as specified as below:

$$\frac{1}{1-\delta}\left(\frac{a^2}{2+8b} + \frac{c^2(1+2d)}{2(1+4d)^2}\right) \leq \frac{1}{1-\delta}\left(\frac{a^2}{2+12b} + \frac{c^2}{2+12d}\right) \quad (9)$$

[8] From here on, for all illustrative examples, I set $a = 30$, $b = 10$, and $c = 40$.

After some simple manipulation, we arrive at a final expression for satisfying constraint 3:

$$\frac{2c^2d^2}{(1+4d)^2(1+6d)} \geq \frac{a^2b}{1+10b+24b^2}$$

Constraint 3 contains parameters of from both demand functions. One way to look at this inequality is to break up the fraction into functions of monopoly profits in market A and market B:

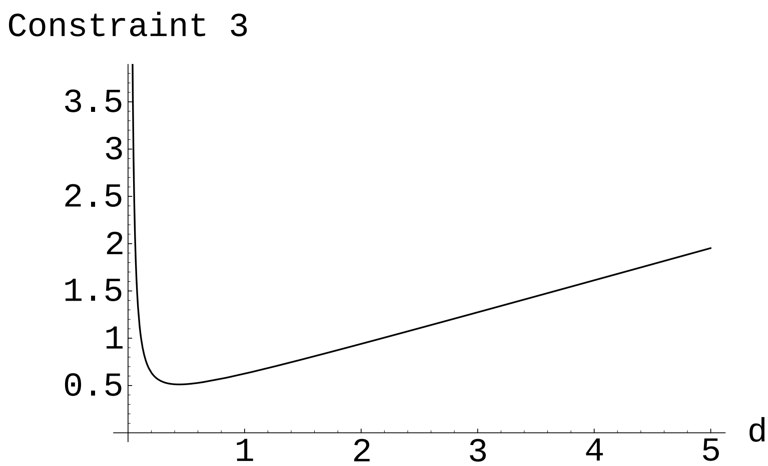
$$\frac{\sum \pi_A^{monopoly}}{\sum \pi_B^{monopoly}} \cdot \frac{b(1+4d)^2}{2(1+6b)d^2} \leq 1 \tag{10}$$

Constraint 3 is an attempt to satisfy the status quo for the incumbent firms. Incumbent firms have market power in market A but is forced to compete to its detriment in market B. Constraint 3 can be interpreted as the incumbent firm trading off a portion of its market power in market A, for market power in market B. In order to give up some of their power in market A, it must be that market B is larger than market A by a factor of $\frac{b(1+4d)^2}{2(1+6b)d^2}$.

The inequality shows that the incumbent firms will be examining the aggregate monopoly profits in market A before entry, and the aggregate monopoly profits in market B after entry. If we think of the monopoly profits (produced by collusion) as a measure of market size, we see that incumbent firms will only allow entry if the portion of the market size gained in market B is larger than the portion of the market size lost in market A.

In addition, it's clear why this inequality will depend critically on b and d, the slopes of the demand in markets A and B. From figure 1 (given $\frac{b(1+4d)^2}{2(1+6b)d^2} > 0$ and b fixed), equation (10) first becomes easier, then more difficult to satisfy as d increases. That is, as market B becomes more inelastic compared to market A, it first becomes easier, then more difficult to satisfy the incumbent firms. Initially, for very low values of d, market B is elastic enough that colluding profits in market B is not sufficiently higher than Cournot profits to compensate incumbent firms for the loss in market share in market A. An increase in d increases colluding profits in market B, which induces incumbent firms to want

Figure 1: Change in relative market size to satisfy constraint 3 as d changes



to collude in order to capture higher profits in market B. Then, as d continues to increase, the market becomes so inelastic that the difference between Cournot and colluding profits decreases, and the incumbent firms have less incentive to collude.

It is clear why the potential entrant firm will want to collude with the incumbents. The entrant firm is never really giving up any portion of market B, because that market only allowed for Cournot competition. This is why the entrant will easily agree to enter market A. In essence, the entrant firm is getting a free ride.

If market B is larger than market A, by some constant that is dependent on the slopes of demand in markets A and B, incumbent firms would agree to allow entry, and collusion among the three firms would be a stable equilibrium.

3 Extended Linear Demand Model Specification

In this section, I extend the original model, to allow for more than two firms in market A, and more than three firms in market B. The incumbent firms attempt to bring in all firms from market B. I show in the next section that bringing in a subset of the firms greatly complicates the analysis.

It is a well-known result from B&W that as the number of firms increases in a market, collusive outcomes are more difficult to support. The δ factor approaches one as the number of firms increase in the market. Intuitively, as more and more firms are in the market, and gains from cooperating become smaller and smaller, firms must value the future income stream highly to refuse to deviate in the short run.

It's simple to show, that δ approaches one for both bounds on δ in markets A and B.[9] As the number of firms increase in a market, the bounds on δ become smaller. As the number of firms approaches infinity, the bounds on δ must collapse to a single value. Therefore, without doing any further analysis, it's already clear that it will be more difficult to imagine a situation where the incumbent firms will agree to allow entrants to come into market A. However, in contrast to B&W results, I show that for low numbers of incumbents and potential entrants, an increase in either is actually *beneficial* for collusion.

The bounds on δ is as follows:

$$\frac{(1+(j+1)b)^2}{2+4(j+1)b+(1+6j+j^2)b^2} \leq \delta \leq \frac{(1+(j+z+1)d)^2}{2+4(j+z+1)d+(1+6(j+z)+(j+z)^2)d^2} \tag{11}$$

The j term represents the number of incumbent firms, and the z term represents the number of potential entrants.

Assuming that the bounds hold, once again the four constraints specified above must hold. By the same reasoning as in section 2, constraints 1, 2, and 4 hold.

The δ value from constraint 2 (derived as in section 2) falls between the range defined in equation (11) using the same logic in section 2. Constraint 2 will be more difficult to satisfy as the number of firms increase in both markets. As the total number of firms increases, the bound for δ approaches one.[10] δ approaches one because the gains to colluding decreases for the entrants as the number of firms increases. Because the monopoly profit is divided equally

[9] See appendix.

[10] This is difficult to see because the δ term is very complex. In the appendix, I fix the parameters of the demand functions, and plot the generated δ values as j and z change.

between all firms under collusion, the share that each firm gains from collusion becomes smaller and smaller. Therefore, the difference in profits between Cournot equilibrium and monopoly equilibrium is decreasing as well. The colluding firms must value future stream of income a lot, in order to not deviate and capture Stackleberg leader profits for one period.

Proposition 1. *Up to a certain critical number of firms that is dependent on the relative sizes and slopes of the markets, as the number of incumbent firms increases, it becomes easier for these firms to reach an agreement that allows entry. However, past this critical number of firms, collusion becomes more difficult.*

Constraint 3 is specified below:

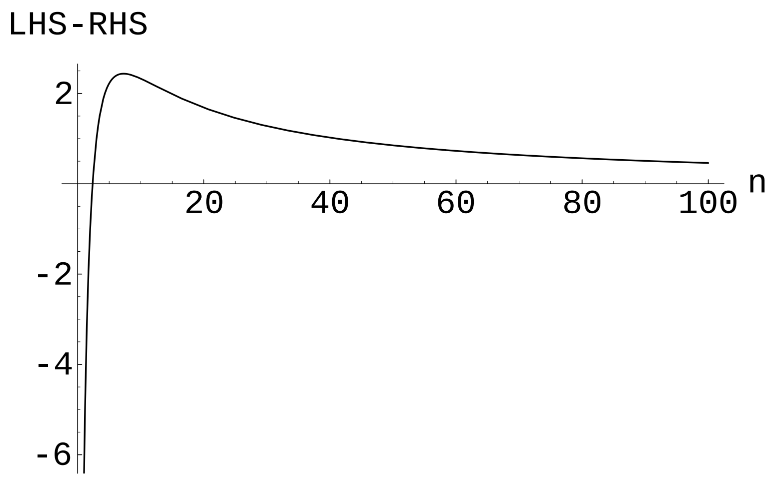
$$\frac{1}{1-\delta}\left(\frac{a^2}{2+4jb}+\frac{c^2(1+2d)}{2(1+(j+z+1)d)^2}\right)\leq\frac{1}{1-\delta}\left(\frac{a^2}{2+4(j+z)b}+\frac{c^2}{2+4(j+z)d}\right)\quad(12)$$

Rearranging the inequality slightly, we obtain this expression:

$$\frac{a^2}{2+4jb}-\frac{a^2}{2+4(j+z)b}\geq\frac{c^2}{2+4(j+z)d}-\frac{c^2(1+2d)}{2(1+(j+z+1)d)^2}$$

As the j term increases, the constraint becomes easier to satisfy. On the left hand side, the denominators both grow at a linear rate, and therefore the difference becomes smaller. On the right hand side, the denominator of the first term is growing linearly and the denominator of the second term is growing geometrically, and therefore the difference is increasing. Since the j term represents the number of incumbent firms, we see that as the number of incumbent firms increases, the share of the monopoly profits each firm has to give up decreases. Also, because each incumbent has little power in market A, they are more willing to trade it to gain power in market B. Therefore, it's easier to agree to allow entry of new firms. However, as the number of incumbent firms increases past a critical value, it becomes more difficult to agree to allow entry. This is because the absolute gains to colluding decreases in the number

Figure 2: difference of LHS and RHS of constraint 3 holding z fixed.



of firms that are colluding. As the number of firms increases toward infinity, the competitive equilibrium solution results. This can be seen in figure 2.

Proposition 2. *Up to a certain critical number of firms that is dependent on the relative sizes and slopes of the markets, as the number of potential entrant firms increases, it becomes easier for incumbent firms to reach an agreement that allows entry. However, past this critical number of firms, collusion becomes more difficult.*

A similar condition holds as the z term, the number of potential entrants increases, as can be seen in figure 3. The intuition for the downward trend is that as the number of potential entrants increase, the share of profits that must be given up in market A by incumbent firms increases, and the share of profits to be gained from market B decreases. Therefore, it is more difficult to agree to allow entry of new firms. The upward trend is a bit more perplexing, but a comparison of constant marginal cost and quadratic cost provides the solution. When similar analysis is done with constant marginal cost, the upward trend is not observed. Therefore, there must be something about the non-linearity of the cost curve that is generating this upward trend.

Imagine the size of markets A and B are fixed, with one incumbent firm.

Table 1: Aggregate production in market B under different cost structures and numbers of potential entrants.

	one entrant	two entrants
$C = q$	$\frac{c}{2d}$	$\frac{c}{2d}$
$C = \frac{q^2}{2}$	$\frac{3c}{1+6d}$	$\frac{4c}{1+8d}$

Two situations under two cost structures are compared. In one instance, there is one potential entrant, and in the other there are two potential entrants. The cost curves are linear or quadratic. The difference in aggregate production when there are two entrant firms and when there is one entrant firm under linear cost is zero. The difference when cost is quadratic is positive. This is because the the quadratic cost function curtails increased production by firms, in addition to oligopolistic concerns. See table 1.

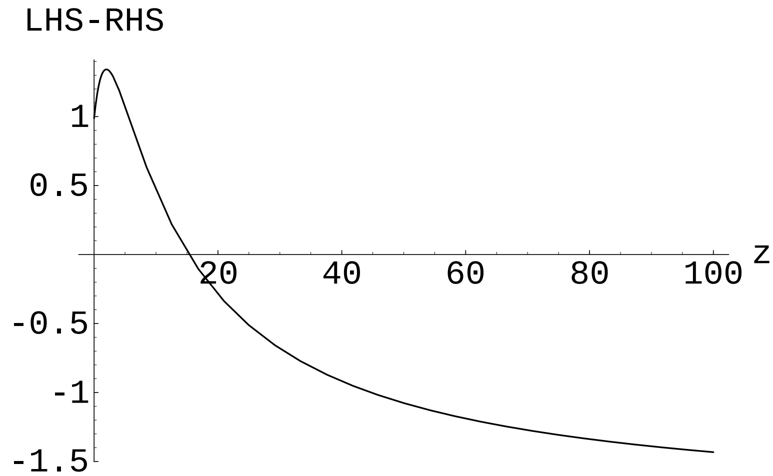
From the perspective of incumbents, if there is a small number of firms operating in market B, these firms are already forced to cut production due to the quadratic nature of cost curves. There is little to be gained from allowing these firms to enter market A. On the other hand, if there are more firms in market B, production is significantly higher, and prices are significantly lower. Then, there may be enough incentive to bring these firms into market A to encourage collusion in market B. Once again, this holds only up to a certain critical number of potential entrants, after which the absolute share of profits to be gained from market B begins to decrease.

Rearranging the inequality, this expression results:

$$\frac{\sum \pi_A^{monopoly}}{\sum \pi_B^{monopoly}} \cdot \frac{2bz(1+d(j+z+1))^2}{d^2(j+z-1)^2(1+2b(j+z))} \leq 1 \tag{13}$$

This is the generalized statement of equation (10). This expression again shows that relative sizes of markets A and B play a critical role in allowing collusion. Because the entrant firms are essentially free-riding on the greed of the incumbents, the incumbent firms' constraints are naturally more difficult to

Figure 3: difference of LHS and RHS of constraint 3 holding j fixed



satisfy. this shows that as the numbers of incumbents and/or entrants increase, market B must be that much larger than market A. Market B must be 'worth it' for the incumbent firms to open up the gates to market A.

4 Extension to Moving a Subset of Market B Firms to Market A

4.1 Extension to Moving a Subset of Market B Firms to Market A

The last natural extension to this model is to allow to firms in market A to let only a subset of firms in market B to enter. Intuitively, incumbent firms may wish to do this because they want to increase market power in market B, but do not wish to give up too much market power in market A.

Allowing a subset of firms from market B to move into market A does not, by itself, give a collusive outcome. Unless all firms in market B are willing to collude on some level, market B will continue to yield a Cournot equilibrium. Unless all firms are willing to collude, the deviating firm will just capture full Stackleberg leader profits in the short run.

B&W touched on this briefly when they stated their assumption for multi-market collusion. Their assumption was that while collusion is unsustainable with N firms, it would be with N-2 firms. B&W support their equilibrium by having the collusive firms cut back their market share in market B, to allow the remaining firms or left-over firms to have more market power. This allows strictly positive profits in market B. All firms in market B, including the firms that cut back their market share, earn positive profits in market B. Because B&W specified Bertrand competition, punishment for deviation is zero profits, which greatly simplifies the analysis. Firms operating only in market B do not deviate from this equilibrium because the alternative is earning zero profits forever, and firms that operate in both markets do not deviate because this maximizes joint profits across markets.

With quantity competition, the analysis becomes more complicated. I will bring in a subset of firms from market B to market A. Following B&W's intuition, the incumbent and entrant firms will try to appease the left-over firm in market B to try to support a collusive outcome.

I will perform this part of the analysis with a specific setup of two firms in market A and four firms in market B. Only one of the firms in market B will be allowed entry into market A. This can be generalized as in the previous section, allowing for j firms in market A, z firms in market B, and r firms left over in market B after entry. For this paper, the analysis is restricted to the specific case to better illustrate some of the theoretical results.

The set up of the game is unchanged from section 1, except for the mechanism by which collusion with the left-over firm is maintained. I allow incumbent and entrant firms to make equal side payments to the left-over firm *before* production commences each period. Because there is no enforcement mechanism to force the other firms to pay the left-over firm once production is complete, I assume this for simplification.

The incumbents in market A can only bring in one of the two firms in market B. This can be an assumption, or can be enforced by making the size of market A small enough to be unable to support a collusive equilibrium with four firms.

Notice that there are now more than four constraints that must be satisfied, because there are three separate groups of firms: the incumbents, the entrants, and the left-over firms.

Equilibrium Condition 2:

In addition to the four constraints specified above, the following constraint must also hold.

5. It must be optimal for the left-over firms to maintain collusion.

The bounds on δ is calculated as in the previous sections, and is found to be:

$$\frac{(1+3b)^2}{2+12b+17b^2} \leq \delta \leq \frac{(1+5d)^2}{2+20d+41d^2}$$

Once again, following the reasoning used above, constraint 1 is trivially satisfied if constraint 2 and 3 are satisfied.

Constraint 2 is specified below:

$$\begin{aligned} \frac{1}{1-\delta}\left(\frac{a^2}{2+12b} + \pi_B^{cooperate}\right) \quad &\geq \quad \frac{a^2(1+4b)^2}{2(1+2b)(1+6b)^2} + \pi_B^{deviate} \\ &\quad + \frac{\delta}{1-\delta}\left(\frac{a^2(1+2b)}{2(1+4b)^2} + \frac{c^2(1+2d)}{2(1+5d)^2}\right) \end{aligned} \tag{14}$$

If the entrant finds it optimal to not deviate conditional on entry, it will always enter. This argument is similar to the explanation offered in previous sections. If the entrant firm decides to deviate after entry, in market B, it will gain Stackleberg leader profits in the first period and Cournot profits in all subsequent periods. Since the entrant was earning Cournot profits in market B in the first place, it's unequivocally better off than before being allowed entry.

Constraint 3 is specified below:

$$\frac{1}{1-\delta}\left(\frac{a^2}{2+12b} + \pi_B^{cooperate}\right) \geq \frac{1}{1-\delta}\left(\frac{a^2}{2+8b} + \frac{c^2(1+2d)}{2(1+5d)^2}\right) \tag{15}$$

Rearranging the inequality:

$$\pi_B^{cooperate} \geq \frac{a^2 b}{1 + 10b + 24b^2} + \frac{c^2(1+2d)}{2(1+5d)^2}$$

Profit in market B by the colluding firms is left as a general expression for now, because we cannot simply have a division of monopoly profits equally as we had in the previous sections. Because the incumbents and entrants may have to cater to the left over firm, they may earn less than equal shares of the monopoly profit. The inequality shows a lower limit for new profits in market B.

Constraint 4 is identical to constraint 2.

The left over firm causes complications in this problem because all three groups of firms (incumbents, entrant, left-over) must be willing to collude. The motivations for the incumbents and the entrant were explored in the previous section. The left over firm can only be induced to collude, and continue to collude if it is gaining some market power as well. Since it will not be operating in market A, all its gains must come in market B.

Constraint 5 is specified below:

$$\frac{1}{1-\delta}(\pi_{B-leftover}^{cooperate}) \geq \pi_{B-leftover}^{deviate} + \frac{\delta}{1-\delta}\left(\frac{c^2(1+2d)}{2(1+5d)^2}\right) \tag{16}$$

I rigorously spell out what the profits look like for the three groups under deviation and cooperation. I continue to assume that the four firms in market B act as a single firm with four production facilities, but allow side payments from the colluding firms (incumbents and potential entrant) to/from the left-over firm to foster collusion. If this strategy works, and the left-over firm is willing to collude as well, the profits derived in market B should look like this:

$$3\pi_B^{cooperate} + \pi_{B-leftover}^{cooperate} = \sum \pi_B^{cooperate} \tag{17}$$

or:

$$3x\frac{c^2}{2+16d}+(4-3x)\frac{c^2}{2+16d}=\frac{2c^2}{1+8d} \tag{18}$$

The x term represents share of monopoly profit kept by the incumbent and entrant firms. The $(1-x)$ term is the side payment that each of the colluding firms must pay to the left-over firm before production begins. Therefore, in addition to its full share of monopoly profits, the left-over firm also collect $3(1-x)$ shares as well.

Given this expression, I solve for profits of the left over firm when it decides to deviate.

$$\pi_{B-leftover}^{deviate} \leq \frac{c^2\left(\frac{4-3x}{1+8d}-\frac{(1+2d)\delta}{(1+5d)^2}\right)}{2(1-\delta)} \tag{19}$$

I know exactly how the left over firm will deviate, if it chooses to do so: as a Stackleberg leader. Solving for left over firm's maximum deviation profits, I obtain this expression[11]:

$$\pi_{B-leftover}^{deviate}=\frac{c^2(-2+d(-16+3cx))^2}{8(1+2d)(1+8d)^2} \tag{20}$$

Plugging this expression into equation (19), and solving for x (fraction of monopoly profits kept by each colluding firm), I find a upper bound for x.

$$x \leq \frac{(1+5d)^2(4+d(36+d(109+104d)))+d(1+3d)(1+8d)(4+d(27+41d))\delta}{4(2-\delta)(1+2d)(1+4d)^2(1+5d)^2}$$

Following B&W's logic, unless the incumbent and entrant firms sacrifice enough of their market power in market B, the left over firm will not collude. The market share given up by the colluding firms are is a double-edged sword. If the share is large enough for the left over firm, it will agree to collude, but if the share given up is large, but not large enough, the left over firm will have greater incentive to deviate, because it can capture a bigger piece of the market B in

[11] See appendix for rigorous derivation

the first period by playing Stackleberg leader. (One can see that interesting problems will arise once we think of generalizing the number of incumbents, entrants, and left over firms in the context of this problem)

The x limit described above does not depend on a or b, the parameters of market A. This is intuitive, because from the perspective of the left-over firm, the profit potential in market A does not matter at all.

Unfortunately, the expression for the upper bound of x is complicated in its raw form. Therefore, I fix demand parameters and plot how x changes as d, the slope of market B demand changes from 0 to 10. See figure 4 top panel.

Proposition 3. *Left-over firms must be paid by the incumbent and entrant firms to sustain a stable collusive equilibrium.*

We see from the top panel of figure 4 that x is relatively close to one. This is equivalent to saying that it is very easy to induce the left over firm to collude. Essentially, the left over firm is now being offered a free ride. As d increases, the x bound increases. This is because as market B becomes more inelastic, the profit gains from the quantity reduction due to collusion becomes larger compared to the side payments. Therefore, left-over firms are willing to accept smaller and smaller side payments in order to foster collusion.

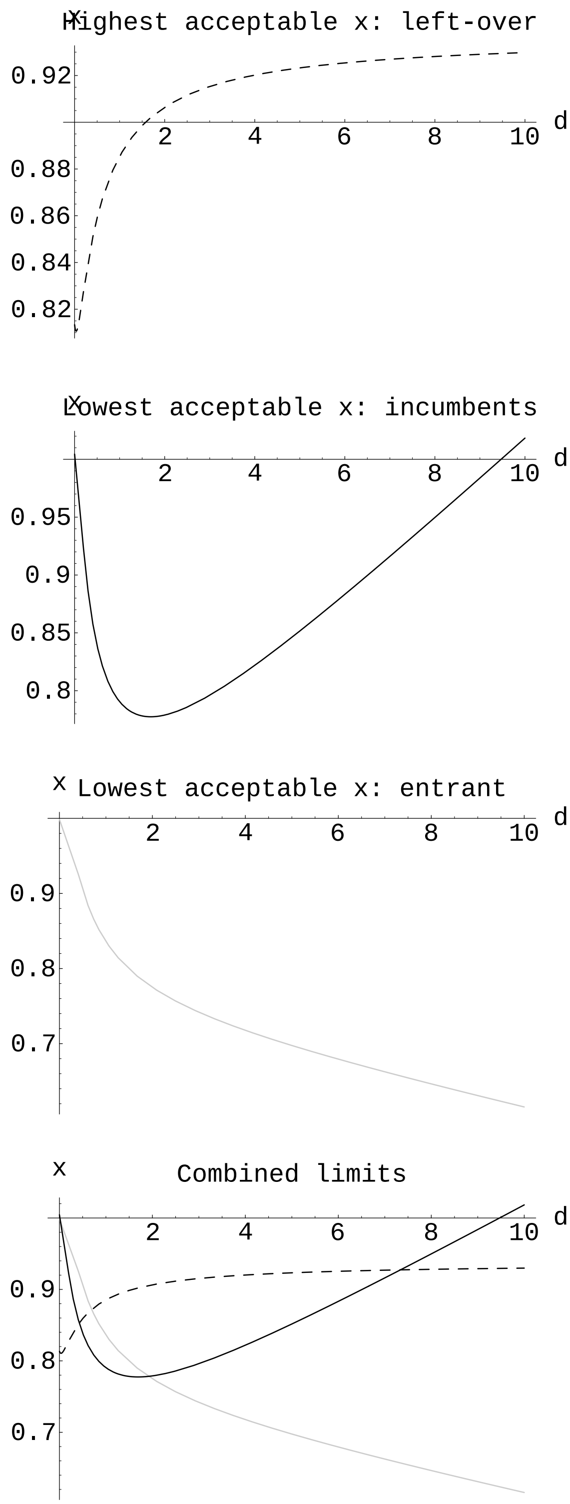
The incumbents' constraint in terms of x, after simple algebra is:

$$x \geq \frac{2b}{1+6b} \cdot \frac{\sum \pi_A^{monopoly}}{\sum \pi_B^{monopoly}} + \frac{(1+2d)(1+8d)}{(1+5d)^2} \tag{21}$$

The incumbent firms would not agree to cut back their market power in market B and allow entry of a firm into market A, if they could not keep at least this share of monopoly profits. Note that this lower bound depends on both b and d. From the incumbents' perspective, the size of b directly impacts how much profit they must give up when entry is allowed, and the size of d impacts how much they have to gain when the stable collusion in market B is achieved.

See the second panel in figure 4 for an illustration of the lower bound of x for incumbent firms. Notice that when market B is very elastic, incumbents must

Figure 4: Limits of x as d changes.



be compensated (in addition to any gains from collusion) to collude. As market B becomes more inelastic, the gains to collusion is large enough to induce these firms to provide side payments. However, the bounds turn up as d increases even further, because at this point, the market is inelastic enough to support something close to monopoly production without collusion.

A upper and a lower bound for x has been defined, in which the incumbents and the left-over firms would agree to collude. However, it is still unclear if collusion is possible for the entrants.

I can also solve for the entrant's constraint from constraint 2 as:

$$\frac{1}{1-\delta}\left(\frac{a^2}{2+12b}+x\frac{c^2}{2+16d}\right) \geq \frac{a^2(1+4b)^2}{2(1+2b)(1+6b)^2}+\frac{c^2}{8}\frac{3d(4+d(19+24d))+4(1+2d)(1+4d)^2x}{(1+2d)(1+4d)^2(1+8d)} + \frac{\delta}{1-\delta}\left(\frac{a^2(1+2b)}{2(1+4b)^2}+\frac{c^2(1+2d)}{2(1+5d)^2}\right) \tag{22}$$

Note that the profits for the entrant deviating were solved as was done for the left over firm.[12]

Solving for the δ term yields a large, unintuitive expression. The actual value is shown in the appendix. In contrast to the δ values that were seen in the two previous sections, this bound will depend on the size of the markets as well as market share retained in market B, x. While this term is essentially impossible to interpret, we can see there exists an x value that will make it possible to satisfy the constraints for δ.

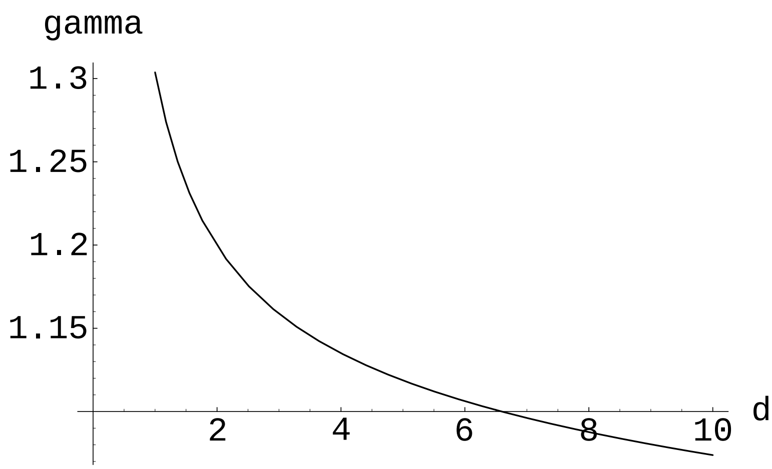
I set $x = 0.9$, fix demand parameters, and factor out the upper bound, $\frac{(1+5d)^2}{2+20d+41d^2}$, from δ. I call this new term γ, and plot values of γ as d changes, in figure 5.

It is clear that this value is less than one given some d, which means the δ that satisfies constraint 2 is lower than the upper bound on δ.

So far, I have solved for the bounds on x for incumbents and left over firms. Finally, I solve the bounds for potential entrants. Intuitively, the entrant firm would have no problems with capturing higher and higher shares of market B. Therefore, we must find a lower limit of x that will satisfy the entrant. Solving equation (22) for x setting $\delta = \frac{(1+5d)^2}{2+20d+41d^2}$ and simplifying:

[12] See appendix.

Figure 5: Change in γ as d changes.



$$x \geq \frac{\sum \pi_A^{monopoly}}{\sum \pi_B^{monopoly}} \frac{\sigma}{12(1+2b)(1+4b)^2(1+6b)(1+4d)^2(1+d(10+19d))} \quad (23)$$

where

$$\begin{aligned} \sigma &= (1+2b)(1+4b)^2(1+6b)(1+8d)(8+d(92+d(313+328d)))\frac{\sum \pi_B^{monopoly}}{\sum \pi_A^{monopoly}} \\ &\quad -16(2b(d-1)+3d)(b+4bd)^2(3d+b(2+22d)) \end{aligned}$$

This lower bound of x for potential entrants is plotted in the third panel of figure 4. Note that as d increases, the potential entrant firm is willing to accept smaller shares of the collusive profit. Again, this is because the entrant is essentially free-riding. Comparing the bounds introduced by the entrant firm to those of the incumbent, it's easy to see the incumbents' bound will be more difficult to satisfy when d is large.[13] The reason for this is clear:

The incumbent unequivocally loses profit in market A, and must make up for it in market B. The entrant is free-riding. Entry into market A guarantees an increase in profit.

The incentive constraints for incumbents, potential entrants and left-over firms are combined on the last panel of figure 4. The range of x that will

[13]See second and third panels of figure 4.

support the collusive equilibrium is bracketed by the dotted line, which is what the left-over firms demand to collude, the thick solid line, which is what the incumbents are willing to provide to collude, and the grey line, which is what the entrants are willing to provide to collude. For the most part, x must be between 0.8 and 0.9 for there to be a stable collusive equilibrium, and collusive equilibrium is more likely (there is a wider range of acceptable x) when market B is relatively elastic.

4.2 Entering Market B as a Left-over Firm

While the analysis would be similar to section 4.1, it may be instructive to think about a new entry deterrent mechanism that is created because of the analysis of multi-market contact. All assumptions made in section 3 hold, such that all firms in markets A and B collude. There is a fixed cost of entry that a potential left-over firm is considering paying to enter market B. If market B is observed in isolation, firms in the market are making abnormally high profits. The profit stream in market B is such that if the new firm enters, incumbent firms would just earn its share of monopoly collusive profits instead of devolving into Cournot competition. Assuming that incumbent firms in market B do not fight entry, the maximum fixed cost that would encourage entry would be $\frac{1}{(1-\delta)}\pi_B^{cooperate}$.

However, if market A is also considered, it is easy to see that the deviation holds greater short term rewards from market A. Therefore, it is entirely possible to have a situation in which an entrant into market B destroys the collusion of all firms, and all firms devolve into Cournot competition. Therefore, in this case, the maximum fixed cost of entry a potential left-over firm is willing to pay should be the stream of Cournot profits from market B.

By contrast, the colluding firms have an active interest in deterring the entrant. If the fixed cost is higher than the stream of Cournot profits, the incumbent firms would inform the potential entrant of the situation. If the entrant is fully aware, it would not enter and entry deterrence would be costless. If the fixed cost is lower than the stream of Cournot profits, the incumbent firms would have an incentive to inform the potential entrant and make side payments to the potential entrant of Cournot profit + ε to deter entry. Of course, the reduction in profits of incumbents due to this payment may also destabilize collusion, in which case, the only stable equilibrium is Cournot profits in both markets.

5 Conclusion

This paper attempted to show under what conditions a set of incumbent firms who have market power and can maintain non-competitive profits would collectively agree to allow entrants. The problem for the incumbents is two fold, in that not only will the incumbents have to share the profit pie with the new entrant, but the new entrants may make the collusive equilibrium unstable, forcing all firms to compete in a Cournot game.

Using B&W's insight that multi-market contact could generate collusion across multiple markets, I showed that under certain conditions, incumbent firms would be willing and eager to allow entry. In fact, as was shown in section 3, entrants actually free-ride on the incumbent firms' desire for power in market B, gaining higher profits in both markets. In section 4, I showed that if there is a firm left out of the collusive agreement, the incubment firms and potential entrant firm *must* compensate this left-over firm to maintain a stable collusive equilibrium. Therefore, not only must the incubment firms give up power in market A to appease the incumbent firms, but is also must appease the left-over firm in market B.

I have also shown that the relative sizes of the markets are crucial in fostering collusion, where the firms giving up its market power must be entering into a situation that allows them market power in a much larger market.

Using the insights generated in this paper, the actions of Oracle Consulting can be explained as an attempt to draw firms in the IT consulting sector into tacit collusion in the database software sector. It must be that the aggregate profits generated in the database software market and the consulting market with collusion is higher for Oracle than competing separately in each market.

It is also clear why Microsoft would not use this principle in competition with its internet browser rival Netscape. Because market size for operating system is so much larger than the market size for browsers, Microsoft realizes that the gains from collusion in the browser sector would never offset the loss in market power due to allowing an entrant into the OS market. In fact, because the OS market is by definition the largest software market in existence (every computer must run some sort of OS) Microsoft can be guaranteed never to allow entry into the OS market.

However, the model does point to an interesting observation on the relationship between Apple and Microsoft. Apple, besides their computers, produces MacOS, which is a proprietary OS that runs only on Macintoshes. Ignoring the obvious distinction between PCs and Macs, the 'charity' that Microsoft showed by bailing out Apple from its financial woes a few years ago is put in a whole new light when we realize that Mac has granted a virtual monopoly to Microsoft in its word-processor, spreadsheet, and database softwares markets via Microsoft Office. Microsoft, by tacitly colluding with Apple (letting it survive) gains market power in another market.

6 Appendix

Solution for Collusive Profit Per-firm profit (with j firms):

$$\begin{aligned} \pi &= \frac{1}{j} \cdot \left((a - b \cdot q^* \cdot j) \cdot (q^* \cdot j) - \frac{(q^*)^2}{2} \right) \\ q^* &= \frac{a}{1 + 2bj} \\ \pi^* &= \frac{a^2}{2 + 4bj} \end{aligned}$$

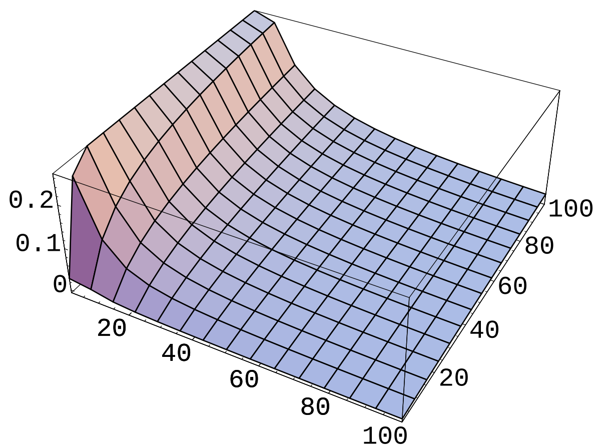
Solution for Nash Profit Per-firm profit (with j firms):

$$\begin{aligned} \pi &= (a - b \cdot q^* \cdot j) \cdot q^* - \frac{(q^*)^2}{2} \\ q^* &= \frac{a}{1 + (j+1)b} \\ \pi^* &= \frac{a^2(1 + 2b)}{2(1 + (j+1)b} \end{aligned}$$

Solution for deviation profits Per-firm profit (with j firms):

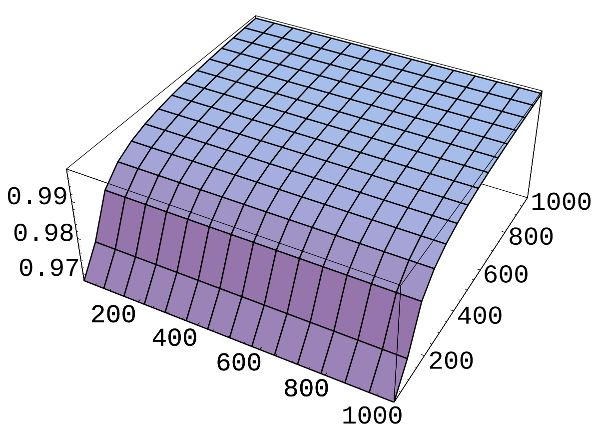
$$\begin{aligned}
\pi &= \left(a - b \cdot \left(q^* + (j-1) \cdot \frac{a}{1+2bj}\right)\right) q^* - \frac{(q^*)^2}{2} \\
q^* &= \frac{a(1+(j+1)b}{(1+2b)(1+2bj)} \\
pi^* &= \frac{a^2(1+(j+1)b)^2}{2(1+2b)(1+2bj)^2}
\end{aligned}$$

Proof that bound on δ from equation (11) hold Fix demand parameters. Then, subtracting the upper bound from the lower bound, and plotting in j and z, we have:

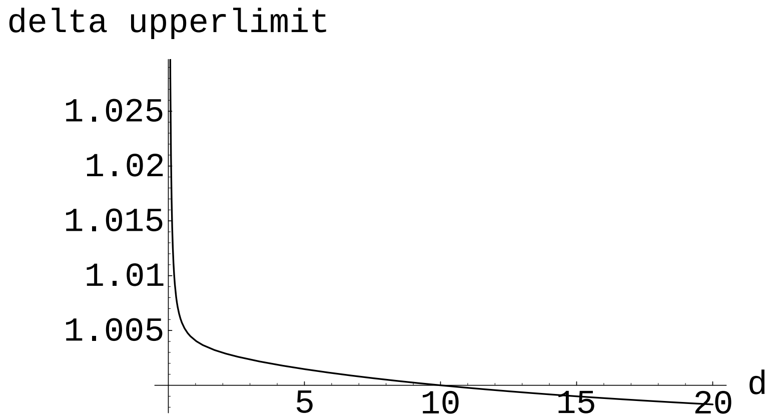


Plot for constraint 2

x-axis is j, y axis is z, and vertical axis is constraint2.



Proof that constraint 2 holds I divide the lower limit of *delta* from constraint 2 and divide by the absolute upper limit of δ (no collusion in market B). Setting the demand parameters as fixed, I plot the change in this new term as d changes.



Rigorous derivation of equation (20) If the left-over firm seeks to deviate, its profits will be as follows:

$$\pi_{B-leftover}^{deviate} = 3x\pi_B^{cooperate} + \text{Stackleberg profit}$$

That is, the left-over firm receives the side payments and reneges and produces Stackelberg amounts, knowing that the other firms will produce thge collusive monopoly quantities. The first term is $3x\frac{c^2}{2+16d}$. Realizing that this term is exogenous to control variables, find the quantity to produce without using this term.

$$\begin{aligned}
\frac{\partial \pi_{B-leftover}^{deviate}}{\partial q} &= c - (1+d)q - d\left(\frac{3c}{2+8d} + q\right) = 0 \\
q^* &= \frac{c(2+5d)}{2(1+6d+8d^2)} \\
\pi^* &= \frac{c^2}{8}\left(\frac{(2+5d)^2}{(1+2d)(1+4d)^2} + \frac{12x}{1+8d}\right)
\end{aligned}$$

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