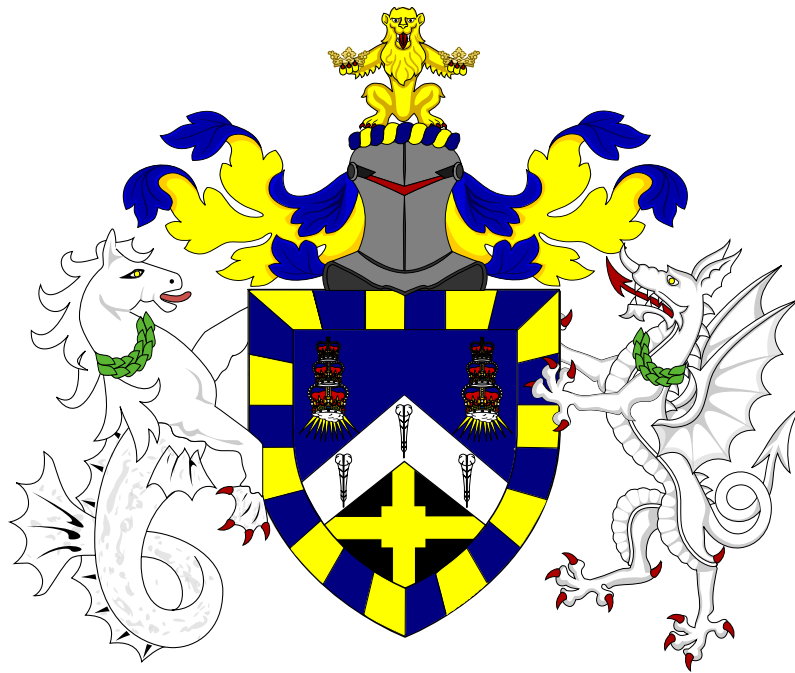


# Measurements of $H \rightarrow b\bar{b}$ decays and $VH$ production

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Doctor of Philosophy June 24, 2020.

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# Chapter 1

## Physics Theory

The following chapter outlines the physics theory that informs and guides the experimental process of searching for new particles or making measurements at a particle collider. Not only are the physics theories described here useful in that context but they also provide an almost complete picture of the universe at certain scales. This chapter was written with the aid of notes taken at the annual STFC High Energy Physics Summer School, and with the aid of several books [2, 3], in which a more detailed description of the theories can be found.

The Standard Model of particle physics is a theoretical framework that describes all elementary particles and three of the fundamental forces of nature. Notably the only force that is not described by the theory is gravity. Particles described by the model are listed in table 1.1 with a white gap separating the matter particles (fermions) from the force carrying particles (bosons). Fermions, which make up solid matter obey Fermi-Dirac statistics [4, 5] whereas bosons obey Bose-Einstein statistics [6]. The Higgs boson is special in that as far as we know it does not carry a force in the conventional sense, instead it is responsible for giving fundamental particles mass, discussed in more detail in section 1.5. In the table of particles quarks (blue) and leptons (red) are ordered in columns by increasing mass, apart from the neutrinos, which are massless in the theory.

As mentioned the model describes forces as being mediated by certain particles, the photon ( $\gamma$ ) mediates the electromagnetic force, particles experiencing electro-

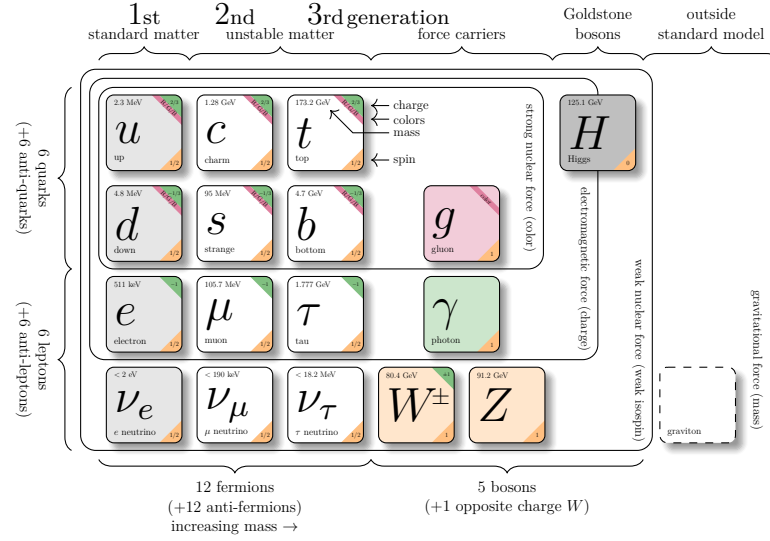


Table 1.1: The particles of the standard model with fermions displayed on the left and bosons on the right. Quarks are in blue, leptons in red, vector bosons in yellow and scalar bosons in green.

magnetic repulsion or attraction are described as exchanging photons. The strength and direction of the force experienced is proportional to the electromagnetic charge of particles involved. The Standard Model is a theory of quantum fields in which the strength of interactions between fields, or particles which are described as excitations in the fields, is parametrised by something known as a coupling constant. It is natural to assume that the strength of the interaction between photons and charged particles is related to the electromagnetic charge of the particles involved. Indeed this is the case consider the Coulomb force between two protons,

$$F = \frac{e^2}{4\pi\epsilon_0 r}, \quad (1.1)$$

where  $e$  is the elementary charge,  $\epsilon_0$  is the electric constant and  $r$  is the distance between the two protons in question and also the energy of a photon given by

$$E = \frac{hc}{\lambda}, \quad (1.2)$$

where  $h$  is Planck's constant,  $c$  is the speed of light and  $\lambda$  is the wavelength of the

photon. The value of the ratio of these two quantities

$$\alpha = \frac{e^2 \lambda}{4\pi\epsilon_0 r h c}, \quad (1.3)$$

known as the fine structure constant, is the coupling constant that describes the strength of the interactions between the photon field and fields of particles with electromagnetic charge. So as is now clear this coupling constant does indeed depend on the electromagnetic charge of the objects involved and so it is not in fact constant.

As well as the electromagnetic force the Standard Model describes the strong nuclear force and the weak nuclear force, shortened to just the strong and weak forces respectively. Like the electromagnetic force they too are mediated by the exchange of particles, the gluons ( $g$ ) carry the strong force and the  $W^\pm$  and  $Z^0$  bosons carry the weak force.

The charge associated with the strong force is known as colour which can take values that are mapped onto colours in the visible spectrum (red, green, blue) for ease of description. For each of these colours an anti-colour is also allowed (anti-red, anti-green, anti-blue). Unlike with the electromagnetic charge, particles with colour charge are not found freely in nature. Instead we find particles known as hadrons which are bound states of quarks and anti-quarks (e.g. the proton). The phenomenon of coloured particles being bound in such a manner is known as colour confinement, and the bound states are described by the quantum numbers isospin ( $I$ ) and hypercharge ( $Y_c$ ). It is commonly assumed that all free particles in nature are colour singlets e.g. for a hadron the state could be written as

$$\frac{(r\bar{r} + b\bar{b} + g\bar{g})}{\sqrt{3}}, \quad (1.4)$$

where  $r$ ,  $b$  and  $g$  represent red, blue and green charges respectively. This phenomenon is known as quark confinement. Gluons carry colour and anti-colour indicating that there should be nine possible quantum mechanical states for the gluon given the available number of colour/anti-colour combinations, however when one

considers that the strong force is exclusively short range, and therefore that there should be no free gluons (disallowing colour singlet gluons) the number of possible states is reduced to eight. The state of a particle, as far as its description with respect to the strong force is concerned, is given by a vector which lives in a vector space, in which elements of the Lie group  $SU(3)_C$  act as unitary operators, where the  $C$  denotes that the group is associated with the colour charge. The  $SU(3)$  group is the group of  $3 \times 3$  unitary matrices whose determinant is one. These correspond to the eight generators of  $SU(3)$  where in general for a group  $SU(N)$  the number of generators is given by  $N^2 - 1$ .

Describing the weak force requires introducing further quantum numbers weak isospin  $T$  and weak hypercharge  $Y_W$ . The state of a particle with regards to the weak force is given by a vector which lives in a vector space in which elements of  $SU(2)_L \times U(1)_{Y_W}$  act as unitary operators where the  $L$  denotes that only particles in left-handed chiral states interact with the weak force<sup>1</sup>. Left-handed fermions are represented as doublets in the theory with weak isospin  $T = 1/2$  whilst right-handed fermions are singlets with weak isospin  $T = 0$ .

Along the way we have described particle states with respect to particular forces as vectors living in some vector space where the action of the element of a group has been as a unitary operator. If we are to describe a particle state taking into account the full model, the group whose elements should act as unitary operators on the particle state (the gauge group) is  $SU(3)_C \times SU(2)_L \times U(1)_{Y_W}$ . For each of the groups in the direct product we have established a (gauge) symmetry and therefore due to Noether's theorem [7] there should be an associated conserved quantity. The conserved quantities in this case are the electric charge, the weak hypercharge and isospin and the colour charge.

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<sup>1</sup>More specifically only left-handed chiral particles participate in weak charged current interactions.



## 1.1 Historical Aside

This section provides some historical context surrounding the Dirac equation which will later be used as the starting point in the discussion of quantum electrodynamics, which is the sector of the Standard Model that describes electromagnetic interactions.

In 1905 Albert Einstein first proposed the idea of special relativity [8]. The aim of the idea was to unify the then inconsistent theories of Maxwell's electromagnetism and Newtonian mechanics. The result of Einstein's work was a theory of motion which agreed with the predictions of Newtonian mechanics at velocities much smaller than the speed of light but whose predictions were accurate also at much higher velocities (for which Newtonian predictions fail). Arguably, the most far reaching consequence of special relativity is that it demands that any equation of motion must be invariant under Lorentz transformations, at least in terms of the formulation of new theories is concerned. Many physical phenomena predicted by special relativity could be considered of higher consequence in general, for example the phenomena of length contraction, time dilation, energy-mass equivalence and the universal speed limit (equal to the speed of light in vacuum), all of which are extensively scrutinised experimentally [9–15]. It is the Lorentz transformation however that should be kept in mind for the following discussion, the transformation may be written as

$$\begin{aligned} t' &= \gamma(t - vx/c^2) \\ x' &= \gamma(x - vt) \\ \text{with } \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}}, \end{aligned} \tag{1.5}$$

in a single dimension of space  $x$  and one of time  $t$  where  $v$  represents the velocity of the system described by the primed coordinates relative to the unprimed coordinates and  $c$  is the speed of light in vacuum.

Twenty years after Einstein introduced the ideas of special relativity Erwin Schrödinger postulated new ideas regarding the motion of quantum mechanical systems [16].

Though he knew his new equation was not invariant under Lorentz transformations, and therefore incomplete, Schrödinger's formulation of quantum mechanics changed the way physicists thought about the universe for ever. His famous equation

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{x}, t) = \left( \frac{-\hbar^2}{2m} \nabla^2 + V(\vec{x}, t) \right) \Psi(\vec{x}, t), \quad (1.6)$$

describes the states of particles as wave-functions  $\Psi$  which can only be interpreted in a probabilistic manner and contains Planck's constant the quantum of action. This work had many consequences including the quantisation of the values of measured observables (meaning they can only take discrete values) and the descriptions of particles as waves.

It was the aim of Paul Dirac to make the Schrödinger equation Lorentz invariant and thus provide a more complete description of quantum systems. Along the way he came to the realisation that in order for his equation to satisfy his needs the wave-function had to be replaced with a four component spinor ( $\psi$ ) and the introduction of matrices known now as the Dirac matrices (labeled  $\gamma^\mu$  with  $\mu = 0, 1, 2, 3$ ) was required. Though not the form he originally wrote down Dirac's Lagrangian density takes the form

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi, \quad (1.7)$$

where the repeated up and down indices are implicitly summed over ( $\partial_\mu$  represents the partial derivative taken with respect to a spatial coordinate  $\mu = 1, 2, 3$  or time  $\mu = 0$ ).

## 1.2 Quantum Electrodynamics

In order to take Dirac's Lagrangian (eq. 1.7) and turn it into something that appropriately describes quantum electrodynamics (QED), we should consider a  $U(1)$

gauge transformation of the Dirac spinor and it's adjoint

$$\begin{aligned}\psi \rightarrow \psi' &= e^{i\alpha(x)}\psi, \\ \bar{\psi} \rightarrow \bar{\psi}' &= e^{-i\alpha(x)}\bar{\psi},\end{aligned}\tag{1.8}$$

with  $\bar{\psi} \equiv \psi^\dagger \gamma^0$  and where  $\alpha(x)$  is a local phase. Under this transformation the Lagrangian transforms as

$$\mathcal{L}_{Dirac} \rightarrow \mathcal{L}'_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \bar{\psi}\gamma^\mu \alpha(x)\psi\tag{1.9}$$

which is not equivalent to the original due to the factor resulting from the derivative of the transformed spinor. Instead let us change the derivative to the gauge covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu\tag{1.10}$$

where we interpret  $A_\mu$  as the photon field, with coupling constant  $e$ , parametrising the interaction strength. The field is also referred to as the electromagnetic gauge field since it arrives during the process of making the Lagrangian invariant under the  $U(1)$  group, the gauge group of electromagnetism. Note that here what we have labeled  $e$  is nothing more than the fine structure constant previously denoted  $\alpha$  in eq. 1.3. The transformation of the new field under the action of the gauge is defined as

$$A_\mu \rightarrow A'_\mu \equiv A_\mu - \frac{1}{e}\partial_\mu \alpha(x).\tag{1.11}$$

This means that the action of the gauge covariant derivative on the spinor transforms as

$$D_\mu \psi \rightarrow D'_\mu \psi' = e^{i\alpha(x)} D_\mu \psi\tag{1.12}$$

which means that the new Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi\tag{1.13}$$

is invariant under the action of the gauge as desired. What remains in order to write a description of QED is to write down a kinetic term for the photon field. An appropriately gauge and Lorentz invariant term is

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (1.14)$$

where the electromagnetic tensor is defined as

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (1.15)$$

Putting everything together we can define the Lagrangian for QED as

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (1.16)$$

### 1.3 Quantum Chromodynamics

Quantum Chromodynamics (QCD) is the theory of the strong force. Its mathematical formulation is similar to that of QED. Except the gauge group for QCD is  $SU(3)_C$  where the  $C$  denotes that the force is associated with colour charge. As previously discussed the eight generators of the group are associated with the eight gluons of the Standard Model. The generators are present in the form of the transformation of a fermion field under an element of  $SU(3)$

$$\psi \rightarrow \psi' = \exp\left(i\alpha_a(x) \cdot \frac{\lambda_a}{2}\right)\psi, \quad (1.17)$$

where the  $\lambda_a$  are the Gell-Mann matrices, generators of  $SU(3)$ . A key difference between the strong force and the other forces described by the Standard Model is that it increases in strength with range. This property leads to a phenomena known as quark confinement which has been discussed previously. Quark confinement is the reason for many of the complications that arise when trying to detect certain particles in a particle detector such as ATLAS. Specifically, quarks that are produced

in collisions undergo a process called hadronisation whereby they transition from their coloured states to colour singlets. Excess energy present in this process results in the creation of lots of different states, some which decay to leptons, with the overall process producing a conical shower of particles known as a jet.

## 1.4 Electroweak theory

The Glashow-Salam-Weinberg model of electroweak interactions [17–19] describes the weak force and electromagnetism as a quantum field theory, which is gauge invariant under transformations that are elements of  $SU(2)_L \times U(1)_{Y_W}$ . As previously mentioned the  $L$  and  $Y_W$  subscripts denote that the gauge groups in the direct product that are associated with left-handed chiral particles and weak hypercharge respectively. The association with weak hypercharge distinguishes this  $U(1)$  group with the  $U(1)$  group from QED. The transformation of the fermion fields under  $SU(2)$  is given by

$$\psi \rightarrow \psi' = \exp\left(i\vec{\alpha}(x) \cdot \frac{\vec{\sigma}}{2}\right)\psi, \quad (1.18)$$

where  $\vec{\sigma}$  is a vector of the Pauli matrices  $\sigma_i$  with  $i = 1, 2, 3$ , a familiar representation of  $SU(2)$  generators. Constructing a gauge covariant derivative for the full transformation under  $SU(2) \times U(1)$  requires the addition of new fields analogous to the photon field from QED, the new derivative takes the form

$$D_\mu = \partial_\mu - i\frac{g_1}{2}Y_W B_\mu - i\frac{g_2}{2}\sigma_i W_\mu^i, \quad (1.19)$$

where coupling constants  $g_1$  and  $g_2$  parametrise the strength of interactions with each field. The index  $i$  runs over the three Pauli matrices and three new fields  $W_\mu^i$  with  $i = 1, 2, 3$  which are associated with the  $SU(2)$  gauge. The  $B_\mu$  field is associated with the  $U(1)_{Y_W}$  gauge and is obtained in the same way as the photon field in QED but is given a new symbol as it is *not* the photon field.

In fact none of the fields added here are the physical fields that we have access to in nature associated with the electromagnetic force or the weak currents. In order

to obtain the physical fields for the weak charged current one can simply take the linear superposition

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2). \quad (1.20)$$

In order to recover the photon field and reveal the field for the weak neutral current the idea of weak mixing must be introduced. Weak mixing was introduced to theory after the discovery of parity violation [20]. Parity is equivalent to chirality for massless particles, however for particles with mass a Lorentz boost can always be appear to flip the chirality of the particles state whereas parity is a fundamental property of a particle. A fermion field with left or right handed chirality can be obtained by multiplication with one of two corresponding projection operators defined as

$$\begin{aligned} P_L &= (1 - \gamma^5)/2, \\ P_R &= (1 + \gamma^5)/2, \end{aligned} \quad (1.21)$$

with  $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ , where  $\gamma^\mu$  are the Dirac matrices, like so

$$\begin{aligned} \psi_L &= P_L\psi, \\ \psi_R &= P_R\psi. \end{aligned} \quad (1.22)$$

It is known that the weak neutral current and indeed the electromagnetic force both interact with particles of left and right handed chirality. Spontaneous symmetry breaking, theorised to have occurred due to an electroweak phase transition in the early universe, has the effect of rotating the plane defined by the  $B_\mu$  and  $W_\mu^3$  fields into the physical fields we see in nature today. The mixing of the fields due to this rotation takes the form

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \quad (1.23)$$

where  $\theta_W$  the Weinberg angle parametrises the amount of mixing. This picture shows that unification of QED and a description of the weak force has been achieved.

Although it at first seems like the  $U(1)_{EM}$  gauge group is not present in  $SU(2)_L \times U(1)_{Y_W}$  gauge group of electroweak theory it has been shown that the QED gauge symmetry is recovered by spontaneous symmetry breaking. Also the  $Y_W$  subscript in the gauge group represents weak hypercharge which is related to electric charge  $Q$  by the following relationship

$$Y_W = 2(Q - T^3), \quad (1.24)$$

where  $T^3$  is the third component of isospin, the component that is conserved.

The particles associated with the weak neutral and charged currents are observed to have masses in nature [21–24] therefore one would naively like to write mass terms of the form

$$\mathcal{L}_{mass} \propto M_B^2 B^\mu B_\mu \quad (1.25)$$

$$+ M_W^2 W_a^\mu W_\mu^a. \quad (1.26)$$

The above mass terms are however not gauge invariant therefore another solution is required, one which will be discussed in the next section.

## 1.5 The Brout-Englert-Higgs Mechanism

The Brout-Englert-Higgs mechanism was made complete almost simultaneously by R. Brout and F. Englert [25], P. Higgs [26] and, G. Guralnik, C. R. Hagen and T. Kibble [27]. The underlying mechanism was proposed prior to this work by P. Anderson [28], though this initial theory was not relativistic invariant. It was initially proposed as a means to give the vector bosons mass terms that were gauge invariant. The theory predicts a complex scalar field (the Higgs field) that undergoes spontaneous symmetry breaking. Interactions with this field are predicted to be mediated by a massive spin-1 scalar particle that is now known to be the Higgs boson. This particle also gives mass to the fermions via a different mechanism. In

general spontaneous symmetry breaking is a process by which a symmetry breaks once conditions meet some threshold. An example of this is a hot sphere of ferromagnetic material whose spins are isotropically oriented. As the sphere cools the ferromagnetic property of the material will align the spins. In the hot scenario the sphere had symmetry in all spatial directions, by this it is meant that the changes to the sphere's orientation were indistinguishable. Once the spins have aligned however this is no longer the case, the fact that the spins point in a specific direction means that direction is special and so some of the symmetry was spontaneously broken. It can be noted though that a preserved symmetry still exists as rotations about the axis defined by the direction of the spins would leave the sphere invariant. In the Standard Model the symmetry that breaks is that of the complex scalar Higgs field. Consider a Lagrangian involving the field  $\phi$  of the form

$$\mathcal{L} = T - V(\phi) = \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (1.27)$$

$$\text{with } \phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2). \quad (1.28)$$

Invariance under global phase transformations of the form  $\phi \rightarrow e^{i\theta}\phi$  depends on the parameters of the potential  $\mu$  and  $\lambda$ . Figure 1.1 shows two sketches of the potential for the scenarios where  $\mu^2 > 0$ ,  $\lambda < 0$  (left) and  $\mu^2 < 0$ ,  $\lambda < 0$  (right). To suggest

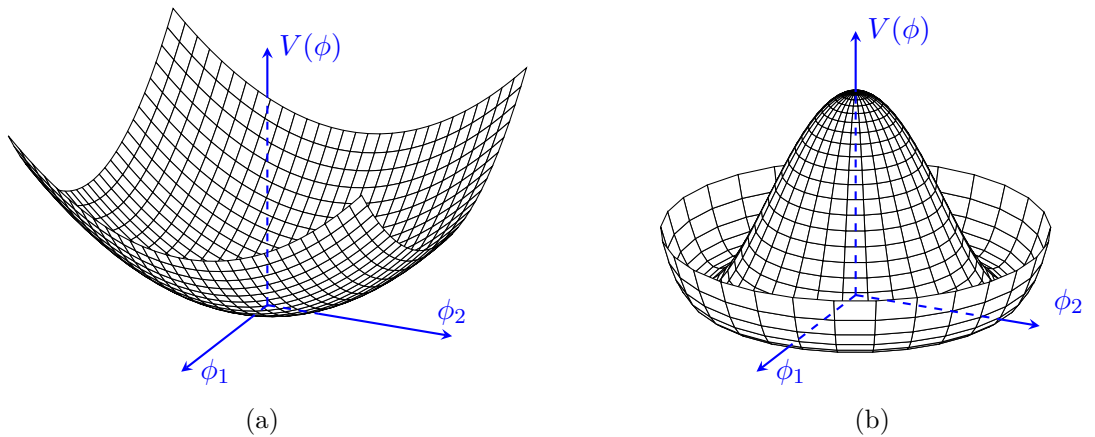


Figure 1.1: The Higgs potential in its fully and broken symmetric forms.

that in our universe this symmetry is spontaneously broken is to suggest that the values of these parameters evolved over time from the full to the broken state. This



ends up leading to masses for the vector bosons that are dependent on  $\mu^2$ .

## 1.6 Higgs bosons at the LHC

Higgs bosons are produced at the LHC in a number of different ways, the four most common of which are shown in figure 1.2. The prevalence of these processes

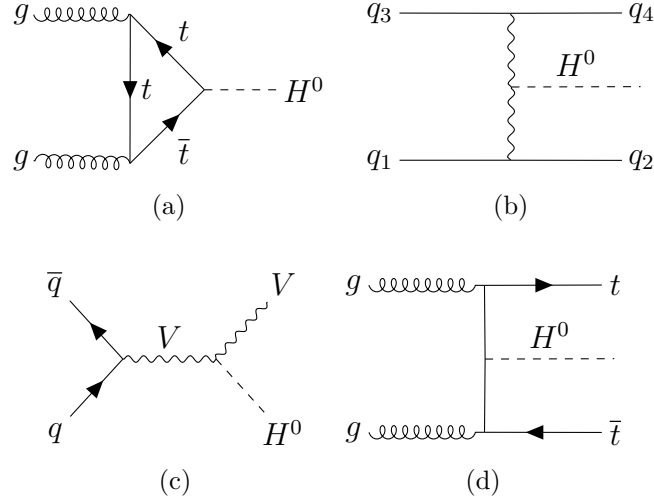


Figure 1.2: The four most common Higgs boson production methods from proton-proton collisions at the LHC.

with respect to the centre of mass energy of the proton-proton collision is shown in figure 1.3 (a). It can be seen the gluon-gluon fusion (fig 1.2 a) is by far the dominate contributor occurring over an order of magnitude more than the next highest process which is quark associated production (fig 1.2 b). The next highest production channel with respect to cross section is vector boson associated (fig 1.2 c) which will be the focus of the rest of this report. Finally top quark associated production (fig 1.2 d) has the smallest cross section of these processes. The Higgs boson is predicted by the Standard Model to decay in a number of different ways depending on its mass, a free parameter of the model. In figure 1.3 (b) the branching ratios of the Higgs can be seen, plotted with respect to Higgs mass. The decay that will be focused on for the rest of this report is  $H \rightarrow b\bar{b}$ . Given that the focus here is on vector boson associated production of a Higgs boson it is also important to consider the decay of the vector boson. Three possible scenarios are represented in

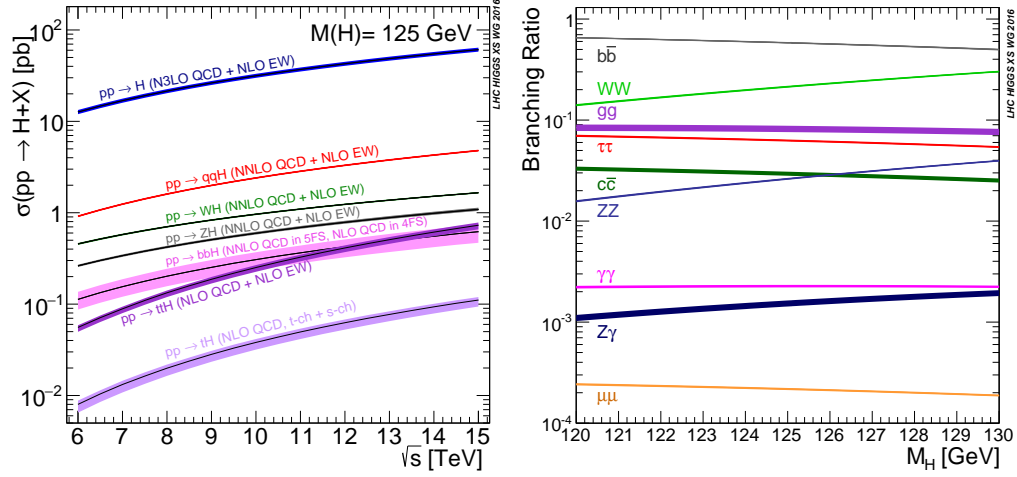


Figure 1.3: Higgs production cross-sections (left), and branching ratios (right) for a range of centre of mass energies and Higgs boson masses respectively [1].

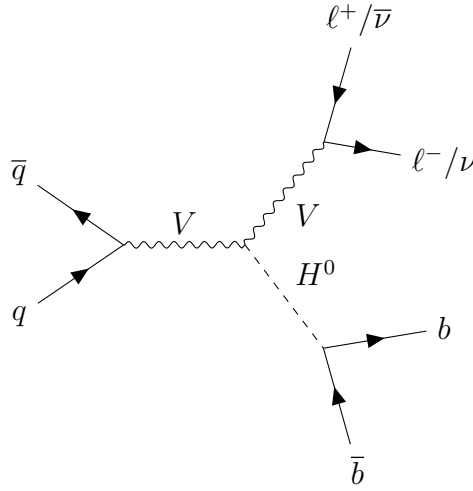


Figure 1.4: A diagram showing a Higgs boson (decaying to a pair of b quarks) produced in association with a vector boson (decaying to 0, 1, or 2 charged leptons denoted  $\ell^{+/-}$ ).

figure 1.4, namely the situations where the vector boson decays to 1, 2 or 3 charged leptons and the appropriate number of neutrinos. It is in fact these leptonic decay modes that motivate the reason for studying this production mechanism as opposed to one of the more common ones. The issue with looking at the other production modes is that very large QCD generated backgrounds are present due to initial state radiation. Whilst these backgrounds are also present when looking at the vector associated channel they can be partially suppressed by triggering on a lepton.

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