Chapter 2 Basic Structures: Sets, Functions, Sequences, Sum and Matrices

§ 2.1 Sets
§ 2.2 Set Operations
§ 2.3 Functions
§ 2.5 Cardinality of Sets
§ 2.6 Matrices

§ 2.1 Sets

1. Introduction

(1) Definition 1 (page 116)

A set is an unordered collection of objects.

(2) Definition 1 (page 116)

The objects in a set are also called the elements or members, of the set. A set is said to contain its elements

(3) How to describe a set?

The first way

-----listing all the members of a set

Examples (page 116)

(a) The set V of all vowels (元音字母) in the English alphabet

V={a, e, i, o, u}

(b) The set of odd positive integers less than 10

O={1, 3, 5, 7, 9}

(c) A set can contain some unrelated elements

{a, 2, Fred, New Jersey}

(d) The set of positive integers less than 100

{1, 2, 3, ……, 99}

(e) Some important sets (page 116)

N, Z, Z^+, Q, R

(4) How to describe a set?

The second way

-----Using set builder notation (page 116)

Example: the set of all odd positive integers less than 10

O={x | x is an odd positive integer less than 10}

(5) The equation of two sets

Definition 2 (page 117)

Two sets are equal if and only if they have the same elements.

Example 6 (page 117)

{1, 3, 5}

= {3, 5, 1}

= {1, 3, 3, 3, 5, 5, 5, 5} but we usually do write this way (do not repeat elements)

(6) Empty set (空集, page 118)

{ } or ∅

How about { ∅ }?

(7) Venn diagram (文氏图)

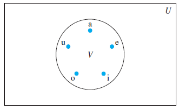
Universal set U (全集)-----containing all the objects under consideration

Example: Venn diagram for the set of vowels (page 118)

Solution: See blackboard.

(7) Venn diagram (文氏图)

Example: Venn diagram for the set of vowels (page 118)



Universal set U (全集)-----containing all the objects under consideration

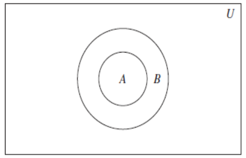
(8) Subset (子集)

Definition 3 (page 119)

The set A is said to be a subset of B if and only if every element of A is also an element of B. We use the notation ⊆ to indicate subset relation.

$A \subseteq B$

iff $\forall x\,(x \in A \rightarrow x \in B)$



(9) Theorem 1 (page 120)

For any set S,

(a) $\emptyset \subseteq S$ (b) $S \subseteq S$

(10) Proper subset (真子集, page 120)

A is a proper subset of B if and only if A is a subset of B but that A≠B.

The notation-----$A \subset B$

(11) One way to show that two sets are equal

$A=B$ iff $A \subseteq B$ and $B \subseteq A$

(12) The Size of a Set (基数)

Definition 4 (page 121)

Let S be a set. If there are exactly n distinct elements in S where n is nonnegative integer, we say that S is a finite set and that n is the cardinality (基数) of S. The cardinality of S is denoted by |S|.

Example 10 (page 121)

Let A be the set of odd positive integers less than 10. Then |A|=5.

Definition 5 (page 121)

A set is said to be infinite if it is not finite.

Example13 (page 121)

The set of positive integers is infinite.

2. The Power Set (幂集)

(1) Definition 6 (page 121)

Given a set S, the power set of S is the set of all subsets of the set S. The power set of S is denoted by *P*(S).

(2) Example 14

What is the power set of {0, 1, 2}?

Answer:

$P(\{0, 1, 2\}) = \{\varnothing, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$.

(3) What is power set of the empty set?

What is the power set of the set {∅}?

Answer:

$P(\varnothing) = \{\varnothing\}$

$P(\{\varnothing\}) = \{\varnothing, \{\varnothing\}\}$

3. Cartesian product (笛卡儿乘积)

(1) Ordered n-tuple $(a_1, a_2, \ldots\ldots, a_n)$

Definition 7 (page 122)

The *ordered n-tuple* $(a_1, a_2, \ldots\ldots, a_n)$ (有序n元组) is the ordered collection that has a_1 as its first element, a_2 as its second element, … , a_n as its nth element.

(2) Equality of two ordered n-tuples

$(a_1, a_2, \ldots\ldots, a_n) = (b_1, b_2, \ldots\ldots, b_n)$ iff

$a_i = b_i$ for $i=1,2,\ldots\ldots,n$

(3) Cartesian product of two sets

$A \times B = \{ (a,b) \mid a \in A \wedge b \in B \}$

Example 17 (page 123)

A={1, 2} and B={a, b, c}

Answer:

$A \times B = \{ (1, a), (1, b), (1, c), (2, a), (2, b), (2, c) \}$

$B \times A = \{ (a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$

(4) Cartesian product of $A_1, A_2, \ldots\ldots, A_n$

$A_1 \times A_2 \times \ldots\ldots \times A_n$

$=\{ (a_1, a_2, \ldots\ldots, a_n) \mid a_i \in A_i \text{ for } i=1, 2,\ldots, n\}$

Example 19 (page 124)

A={0,1}

B={1,2}

C={0,1,2}

What is A×B×C?

4 Using set notation with quantifiers

$\forall x \in S\ P(x)$ ------- $\forall x\ (x \in S \rightarrow P(x))$

$\exists x \in S\ P(x)$ ------- $\exists x\ (x \in S \wedge P(x))$

Example 19 (page 119)

What do the statements $\forall x \in R\ (x^2 \geq 0)$ and $\exists x \in Z\ (x^2 \geq 1)$ mean?

Discussion

7,11,25,39