## Tutorial 1-1

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A proposition is a declarative statement that is true or false, but not both.
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- 1. Which of these sentences are propositions? What are the truth values of those that are propositions?
- a) Boston is the capital of Massachusetts.b) Miami is the capital of Florida.

- c) 2 + 3 = 5.
   d) 5 + 7 = 10.
   e) x + 2 = 11.
   f) Answer this question.

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Negation of a Proposition ("\#") Let p be a proposition. The statement "It is not the case that p" is another proposition, called the negation of p. The negation of p is denoted by \neg p. The proposition "\neg p" is read "not p".
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- 3. What is the negation of each of these propositions?

  a) Mei has an MP3 player.

  b) There is no pollution in New Jersey.

  c) 2+1 = 3.

  d) The summer in Maine is hot and sunny.

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3. What is the negation of each of these propositions?
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- a) Mei has an MP3 player.
  b) There is no pollution in New Jersey.
  c) 2+1=3.
  d) The summer in Maine is hot and sunny.

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3. a) Mei does not have an MP3 player. b) There is pollution
in New Jersey. c) 2 + 1 \neq 3. d) The summer in Maine is not
hot or it is not sunny. 5. a) Steve does not have more than
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    Which of these sentences are propositions? What are the truth values of those that are propositions?
    Boston is the capital of Massachusetts.
      b) Miami is the capital of Florida.

c) 2 + 3 = 5.
d) 5 + 7 = 10.
e) x + 2 = 11.

     f) Answer this question.
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1. a) Yes, T b) Yes, F c) Yes, T d) Yes, F e) No f) No

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    Let p and q be the propositions "Swimming at the New
Jersey shore is allowed" and "Sharks have been spotted
near the shore," respectively. Express each of these com-
pound propositions as an English sentence.
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a)  $\neg q$  b)  $p \wedge q$  c)  $\neg p \vee q$ d)  $p \rightarrow \neg q$  e)  $\neg q \rightarrow p$  f)  $\neg p \rightarrow \neg q$ g)  $p \leftrightarrow \neg q$  h)  $\neg p \wedge (p \vee \neg q)$ 

Conjunction of two propositions ("并且"又称"合歌") Let p and q be propositions. The proposition "p and q", denoted as "p / q" is the proposition that is true when both of them are true and is false otherwise. The proposition "p / q" is called the conjunction of p and q.

Disjunction of two propositions (" $\|\hat{q}\|_{\infty}^{2}$ ";  $\|\hat{q}\|_{\infty}^{2}\|\|_{\Omega}^{2}$ ) Let p and p be propositions. The projection "p or q", denoted as  $p \bigvee q$ , is the proposition that is false when p and q are both false and true otherwise. The proposition p q is called the disjunction of p and q.

Implication ("蕴含"或称为:Conditional Statement) Let p and g be propositions. The *implication* p→q is the proposition that is false when p is true and q is false, and is true otherwise.

Biconditional ("当且仅当"又称"等价") Let p and q be propositions. The biconditional pcm is the proposition that is true when p and q have the same truth values, and is false otherwise.

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these com-pound propositions as an English sentence.

a)  $\neg q$  b)  $p \wedge q$  c)  $\neg p \vee q$ d)  $p \rightarrow \neg q$  e)  $\neg q \rightarrow p$  f)  $\neg p \rightarrow \neg q$ 

on Sunday 7, a) F b) T c) T d) T e) T 9, a) Sharks have not been spotted near the shore. b) Swimming at the New Jercys shore is allowed, and sharks have been spotted near the shore. G) Swimming at the New Jercy shore is not allowed, as the shore of the shore is allowed, then sharks have not been spotted near the shore, then summing at the New Jercys shore is allowed, then shore, then shored, then sharks have not been spotted near the shore, then summing at the New Jercys shore is not allowed, then sharks have not been spotted near the shore. 2) Swimming sharks have not been spotted near the shore.

Conjunction of two propositions ("并且"又称"合取") Let p and q be proposition. The proposition that is true when both of them are true and is false otherwise. The proposition " $p \land q$ " is called the conjunction of p and q. Implication ("蓋含"或称为:Conditional Statement) Let p and q be propositions. The *implication* p→q is the proposition that is false when p is true and q is false, and is true otherwise.

Biconditional ("当且仅当"又称"等价") Let p and q be propositions. The biconditional page is the proposition that is true when p and q have the same truth values, and is falso otherwise.

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11. Let p and q be the propositions p: It is below freezing.q: It is snowing. Write these propositions using p and q and logical con-nectives (including negations). a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing. c) It is not below freezing and it is not snowing.
 d) It is either snowing or below freezing (or both).
 e) If it is below freezing, it is also snowing.
 f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
 f) That it is below freezing is necessary and sufficient for it to be snowing. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these com-pound propositions as an English sentence.

a)  $\neg q$  b)  $p \wedge q$  c)  $\neg p \vee q$ d)  $p \rightarrow \neg q$  e)  $\neg q \rightarrow p$  f)  $\neg p \rightarrow \neg q$ 

key word Operator → ... if and only if

11. Let p and q be the propositions p: It is below freezing. q: It is snowing. Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing. b) It is below freezing but not snowing.
 c) It is not below freezing and it is not snowing. d) It is either snowing or below freezing (or both).

(a) It is either snowing or below freezing (or both).
 (b) If it is below freezing, it is also snowing.
 (f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
 (g) That it is below freezing is necessary and sufficient for it to be snowing.

sentence.) 11. a)  $p \wedge q$  b)  $p \wedge \neg q$  c)  $\neg p \wedge \neg q$  d)  $p \vee q$ e)  $p \rightarrow q$  f)  $(p \lor q) \land (p \rightarrow \neg q)$  g)  $q \leftrightarrow p$  13. a)  $\neg p$ 

## Let p and q be propositions. The *implication* $p \rightarrow q$ is the proposition that is false when p is true and q is false, and is true otherwise.

Truth Table

17. Determine whether each of these conditional statements is true or false.

a) If 1 + 1 = 2, then 2 + 2 = 5. a) If 1+1 = 2, then 2+2 = 5.
b) If 1+1 = 3, then 2+2 = 4.
c) If 1+1 = 3, then 2+2 = 5.
d) If monkeys can fly, then 1+1 = 3.

- 21. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a dis-junction) versus an exclusive or. Which of these meanings of or do you think is intended?
- a) To take discrete mathematics, you must have taken
- b) When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.

not allow that. 21. a) Inclusive or: It is allowable to take discrete mathematics if you have had calculus or computer describes the computer of the computer of the computer of the computer of the computer science, but not if you have had calculus or computer science, but not if you have had both. Most fikely the inclusive or is intended, b) Inclusive or Tivo can arise the relatic, or you can restantiate the computer of the co

17. Determine whether each of these conditional statements

a) If 1 + 1 = 2, then 2 + 2 = 5. b) If 1 + 1 = 3, then 2 + 2 = 4. c) If 1 + 1 = 3, then 2 + 2 = 5. d) If monkeys can fly, then 1 + 1 = 3.

17. a) False b) True c) True d) True

- 23. Write each of these statements in the form "if p, then q" in English. [Hint: Refer to the list of common ways to express conditional statements.]
  a) It snows whenever the wind blows from the northeast.
  b) The apple trees will bloom if it stays warm for a week.

- That the Pistons win other in studys warm for a week.
   That the Pistons win the championship implies that they beat the Lakers.
   It is necessary to walk 8 miles to get to the top of Long's Peak.

Disjunction of two propositions ("或者"又称"析"取) Let p and q be propositions. The proposition "p or q", denoted as p y \( \) q, is the proposition that is false when p and q are both false and true otherwise.

1. For each of these sentences, state what the sentence means if the looical connective or is an inclusive or (that is, a distinct or inclusive or that is, a distinct or inclusive or that is, a distinct or inclusive or in The proposition p \/ q is called the disjunction of p and q.

Exclusive or of two propositions ("异或")
Let p and q be propositions. The exclusive *or* of p and q, denoted by p **@** q, is the proposition that is true when exactly one of p and q is true and is false

- if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?
- a) To take discrete mathematics, you must have taken calculus or a course in computer science.
- When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.

23. Write each of these statements in the form "if p, then q" in English. [Hint: Refer to the list of common ways to express conditional statements.]
 a) It snows whenever the wind blows from the northeast.

b) The apple trees will bloom if it stays warm for a week.
 c) That the Pistons win the championship implies that they beat the Lakers.

d) It is necessary to walk 8 miles to get to the top of Long's Peak.

school. Certainly the inclusive or is intended.

23. a) If the wind blows from the northeast, then it snows. b) If it stays warm for a week, then the apple trees will bloom. c) If the Pistons win the championship, then they beat the Lakers. d) If you get to the top of Long's Peak, then you must have walked 8 miles. c) If you are world-lamous, then you will get tenure

## q→p is called the converse of p→q (逆命題). ¬q→¬p is called contrapositive of p→q (逆杏命題). ¬p→¬q is called inverse of p→q (否命題).

- State the converse, contrapositive, and inverse of each of these conditional statements.
- a) If it snows today, I will ski tomorrow.
   b) I come to class whenever there is going to be a quiz.

29. How many rows appear in a truth table for each of these compound propositions?

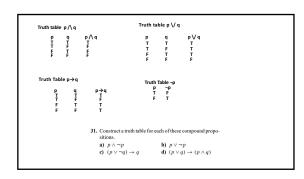
a)  $p \rightarrow \neg p$ b)  $(p \vee \neg r) \wedge (q \vee \neg s)$ 

**29. a)** 2 **b)** 16

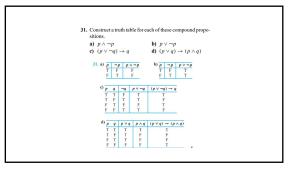
State the converse, contrapositive, and inverse of each of these conditional statements.

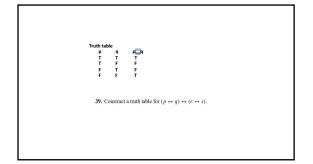
a) If it snows today, I will ski tomorrow.
 b) I come to class whenever there is going to be a quiz.

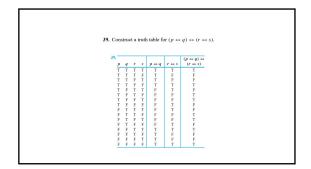
take the train. 27. a) Converse: "I will ski tomorrow only if it stows today." Contrapositive: "If I do not ski tomorrow, then it will not have snowed today." Inverse: "If it does not snow today, then I will not ak it tomorrow." De Converse: "If I come to class, then there will be a quia." Contrapositive: "If I do not come to class, then there will not a quiz." Inverse: "If there is not going to be a quiz, then I don't come to class."

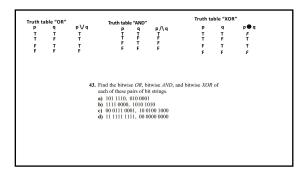


29. How many rows appear in a truth table for each of these compound propositions? a)  $p \rightarrow \neg p$ b)  $(p \vee \neg r) \wedge (q \vee \neg s)$ 

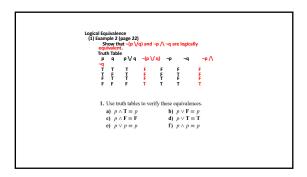








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Tutorial 1-3
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1. Use truth tables to verify these equivalences.

 a) p ∧ T ≡ p
 c) p ∧ F ≡ F
 e) p ∨ p ≡ p b) p ∨ F ≡ p
 d) p ∨ T ≡ T
 f) p ∧ p ≡ p

The equivalences follow by showing that the appropriate pairs of columns of this table agree.

p	$p \wedge T$	$p \vee F$	$p \wedge F$	$p \vee T$	$p \lor p$	$p \wedge p$
T	T	T	F	T	T	T
F	F	F	F	T	F	F

¬(p \ q) = ¬p \ ¬q De Morgan's laws ¬(p \ q) = ¬p \ ¬q (德摩根律)

Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy.b) Carlos will bicycle or run tomorrow.

p ∨ q ≡ q ∨ p Commutative laws p ∧ q ≡ q ∧ P (交换律).

3. Use truth tables to verify the commutative laws a)  $p \lor q = q \lor p$ . b)  $p \land q = q \land p$ .

Use De Morgan's laws to find the negation of each of the following statements.
 Jan is rich and happy.

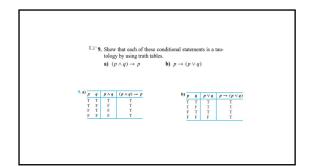
b) Carlos will bicycle or run tomorrow.

7. a) Jan is not rich, or Jan is not happy. b) Carlos will not bicycle tomorrow, and Carlos will not run tomorrow. c) Mei does not walk to class, and Mei does not take the bus to class. d) Ibrahim is not smart, or Ibrahim is not hard working.

3. Use truth tables to verify the commutative laws a)  $p \lor q \equiv q \lor p$ . b)  $p \land q \equiv q \land p$ .

p V ¬p is always true. It is a tautology. (永真公式)

Let  $\mathbf{9}$ . Show that each of these conditional statements is a tautology by using truth tables. a)  $(p \land q) \rightarrow p$  b)  $p \rightarrow (p \lor q)$ 





Logical Equivalence (1) Example 2 (page 22) Show that  $-(p \setminus q)$  and  $-p \land \neg q$  are logically controlled by the point  $-(p \setminus q)$  and  $-p \land \neg q$  are logically equivalent.

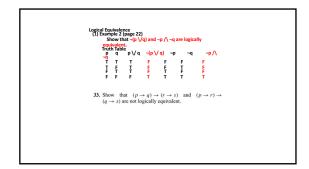
Logical Equivalence (1)  $-(p \setminus q)$  and  $-(p \setminus q)$  are logically equivalent.

17. Show that  $\neg(p \leftrightarrow q)$  and  $p \leftrightarrow \neg q$  are logically equivalent.

18. It is a totalology. 17. Each of these is true precisely when  $\rho$  and q have apposite truth values. 19. The proposition

p V ¬p is always true. It is a tautology. (永真公式)

**29.** Show that  $(p \to q) \land (q \to r) \to (p \to r)$  is a tautol-



(4) Some Important Equivalences
(a) Table 5 (Logical Equivalence)

pハT sp Identity laws

pソF sp (同一律)

pソT sT Domination laws

pハF sF (零律)

pソp sp Idempotent laws

pハp sp (悪等性)

¬(¬p) sp Double negation law

(双重否定律)

33. Show that  $(p \to q) \to (r \to s)$  and  $(p \to r) \to (q \to s)$  are not logically equivalent.

true. 33. Many answers are possible. If we let r be true and p,q, and s be false, then  $(p \to q) = (p \to s)$  will be false, but  $(p \to r) \to (q \to s)$  will be take.  $(p \to r) \to (q \to s)$  will be true. 36. 3)  $p \lor \neg q \lor \neg q$ 

(b) Table 6 (Logical Equivalence Involving Implication)

 $\begin{aligned} p &\rightarrow q \stackrel{=}{=} \neg p \bigvee q \\ p &\rightarrow q \stackrel{=}{=} \neg q \rightarrow \neg p \\ p &\lor q \stackrel{=}{=} \neg q \rightarrow \neg q \\ p &\land q \stackrel{=}{=} (p \rightarrow \neg q) \\ \neg (p \rightarrow q) \stackrel{=}{=} p \bigwedge \neg q \\ (p \rightarrow q) \bigwedge (p \rightarrow r) \stackrel{=}{=} p \rightarrow (q \bigwedge r) \\ (p \rightarrow r) \bigwedge (q \rightarrow r) \stackrel{=}{=} (p \rightarrow q) \rightarrow r \\ (p \rightarrow q) \bigvee (p \rightarrow r) \stackrel{=}{=} p \rightarrow (q \bigvee r) \\ (p \rightarrow r) \bigvee (q \rightarrow r) \stackrel{=}{=} (p \bigwedge q) \rightarrow r \end{aligned}$ 

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p V q ■ ¬p → q

· p ∨ q
≡ ¬(¬p) ∨ q
≡ ¬(¬p) ∨ q
≡ ¬p ∨ q ≡ p → q

· ¬(¬p) ≡ p
¬p ∨ q ≡ p → q

Double negation law(双重否定律)
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p ∧ q ≡ ¬(p → ¬q)

· p ∧ q
≡¬(¬p) = p
=¬(¬p) √¬q)
=¬(¬p) √¬q)
=¬(¬p) ¬q)
¬¬p ∨ q ≡ p → q

· ¬(¬p) = p
¬(p ∧ q) ≡ ¬p ∨ ¬q
¬p ∨ q ≡ p → q

Double negation law(双重否定律)
¬p ∨ q ≡ p → q

De Morgan's laws (德摩根律)
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 (p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r 
 \cdot (p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r 
 \cdot (p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee r) \rightarrow r 
 = (r \vee p) \wedge (r \vee q) \qquad p \vee q = q \vee p 
 = r \vee (r \vee p) \wedge q) \vee r \qquad p \vee q = q \vee p 
 = (p \vee q) \vee r \qquad p \vee q = q \vee p 
 = (p \vee q) \vee r \qquad r \wedge p \vee q = r \wedge p \wedge q 
 = (p \vee q) \vee r \qquad r \wedge p \vee q = r \wedge p \wedge q 
 p \vee q = q \vee p \qquad Commutative laws (交換律) \qquad p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \qquad Distributive laws (分配律) 
 \sim p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \qquad Distributive laws (分配律) 
 \sim (p \vee q) = r \wedge p \wedge q \qquad De Morgan's laws (德摩根律)
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p \bigotimes q \equiv (p \rightarrow q) \land (q \rightarrow p)
p \quad q \quad p \boxtimes q \quad p \rightarrow q \quad q \rightarrow p \quad (p \rightarrow q) \land (q \rightarrow p)
T \quad T \quad T \quad T \quad T
T \quad F \quad F \quad F \quad T \quad F
F \quad T \quad F \quad T \quad F \quad F
F \quad T \quad T \quad T \quad T
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・p → q = ¬p ∨ q

¬q / q = p > q

¬q / q = p > q

(¬p) / q = p ∨ q.

¬q / ¬q = ¬p / ¬q.

¬q / ¬q = ¬p / ¬q.

p ∨ q ∨ r = p ∨ (q) · q.

p ∨ (q) ∨ r = (p ∨ q) / (p ∨ r) Distributive laws (分配律)

p ∨ (q) ∨ r = (p ∨ q) / (p ∨ r) Distributive laws (分配律)
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## assignment

- Chapter1.1 Exercises (Page 12) Question Number:
- 2, 8, 12, 16, 30
- Chapter1.3(Page 34) Question Number:
- 2, 10, 16, 32
- Please take your homework a photo and transfer it into pdf or scan it.
- Only pdf is accepted.