Tutorial 1-1

A proposition is a declarative statement that is true or false, but not both.

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) $2 + 3 = 5$.
d) $5 + 7 = 10$.
e) $x + 2 = 11$.
f) Answer this question.

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c) $2 + 3 = 5$.
d) $5 + 7 = 10$.
e) $x + 2 = 11$.
f) Answer this question.

1. **a)** Yes, T **b)** Yes, F **c)** Yes, T **d)** Yes, F **e)** No **f)** No

Negation of a Proposition ("非")
Let p be a proposition. The statement "It is not the case that p" is another proposition, called the negation of p.
The negation of p is denoted by ¬p.
The proposition "¬p" is read "not p".

3. What is the negation of each of these propositions?
a) Mei has an MP3 player.
b) There is no pollution in New Jersey.
c) $2 + 1 = 3$.
d) The summer in Maine is hot and sunny.

3. What is the negation of each of these propositions?
a) Mei has an MP3 player.
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3. **a)** Mei does not have an MP3 player. **b)** There is pollution in New Jersey. **c)** $2 + 1 \neq 3$. **d)** The summer in Maine is not hot or it is not sunny. 5. **a)** Steve does not have more than

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.
a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$
d) $p \rightarrow \neg q$ **e)** $\neg q \rightarrow p$ **f)** $\neg p \rightarrow \neg q$
g) $p \leftrightarrow \neg q$ **h)** $\neg p \wedge (p \vee \neg q)$

Conjunction of two propositions (“并且”又称”合取”)
Let p and q be propositions. The proposition “p and q”, denoted as “p /\ q” is the proposition that is true when both of them are true and is false otherwise. The proposition “p /\ q” is called the conjunction of p and q.

Disjunction of two propositions (“或者”又称”析”取)
Let p and q be propositions. The proposition “p or q”, denoted as p \/ q, is the proposition that is false when p and q are both false and true otherwise.
The proposition p \/ q is called the disjunction of p and q.

Implication (“蕴含”或称为： Conditional Statement)
Let p and q be propositions. The *implication* p→q is the proposition that is false when p is true and q is false, and is true otherwise.

Biconditional (“当且仅当”又称”等价”)
Let p and q be propositions. The biconditional p↔q is the proposition that is true when p and q have the same truth values, and is false otherwise.

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Operator	key word
¬	not
/\	and
\/	or
→	if
↔	if and only if

9. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$
d) $p \to \neg q$ **e)** $\neg q \to p$ **f)** $\neg p \to \neg q$

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9. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

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d) $p \to \neg q$ **e)** $\neg q \to p$ **f)** $\neg p \to \neg q$

on Sunday 7. **a)** F **b)** T **c)** T **d)** T **e)** T 9. **a)** Sharks have not been spotted near the shore. **b)** Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore. **c)** Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore. **d)** If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore. **e)** If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed. **f)** If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore. **g)** Swimming

Operator	key word
¬	not
/\	and
\/	or
→	if then
↔	if and only if

11. Let p and q be the propositions
p : It is below freezing.
q : It is snowing.
Write these propositions using p and q and logical connectives (including negations).
a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).
e) If it is below freezing, it is also snowing.
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

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f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

sentence.) 11. **a)** $p \wedge q$ **b)** $p \wedge \neg q$ **c)** $\neg p \wedge \neg q$ **d)** $p \vee q$ **e)** $p \to q$ **f)** $(p \vee q) \wedge (p \to \neg q)$ **g)** $q \leftrightarrow p$ 13. **a)** $\neg p$

Let p and q be propositions. The *implication* p→q is the proposition that is false when p is true and q is false, and is true otherwise.

Truth Table

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

17. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.
b) If $1 + 1 = 3$, then $2 + 2 = 4$.
c) If $1 + 1 = 3$, then $2 + 2 = 5$.
d) If monkeys can fly, then $1 + 1 = 3$.

17. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.
b) If $1 + 1 = 3$, then $2 + 2 = 4$.
c) If $1 + 1 = 3$, then $2 + 2 = 5$.
d) If monkeys can fly, then $1 + 1 = 3$.

17. **a)** False **b)** True **c)** True **d)** True

Disjunction of two propositions (“或者”又称”析”取)

Let p and q be propositions. The proposition “p or q”, denoted as p $\vee$ q, is the proposition that is false when p and q are both false and true otherwise.

The proposition p $\vee$ q is called the disjunction of p and q.

Exclusive or of two propositions (“异或”)

Let p and q be propositions. The exclusive *or* of p and q, denoted by p $\oplus$ q, is the proposition that is true when exactly one of p and q is true and is false otherwise

21. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a) To take discrete mathematics, you must have taken calculus or a course in computer science.
b) When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.

21. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a) To take discrete mathematics, you must have taken calculus or a course in computer science.
b) When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.

not allow that. 21. **a)** Inclusive or: It is allowable to take discrete mathematics if you have had calculus or computer science, or both. Exclusive or: It is allowable to take discrete mathematics if you have had calculus or computer science, but not if you have had both. Most likely the inclusive or is intended. **b)** Inclusive or: You can take the rebate, or you can get a low-interest loan, or you can get both the rebate and a low-interest loan. Exclusive or: You can take the rebate, or you can get a low-interest loan, but you cannot get both the rebate and a low-interest loan. Most likely the exclusive or is intended. **c)** Inclusive or: You can order two items from col-

23. Write each of these statements in the form “if *p*, then *q*” in English. [*Hint:* Refer to the list of common ways to express conditional statements.]

a) It snows whenever the wind blows from the northeast.
b) The apple trees will bloom if it stays warm for a week.
c) That the Pistons win the championship implies that they beat the Lakers.
d) It is necessary to walk 8 miles to get to the top of Long’s Peak.

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school. Certainly the inclusive or is intended. 23. **a)** If the wind blows from the northeast, then it snows. **b)** If it stays warm for a week, then the apple trees will bloom. **c)** If the Pistons win the championship, then they beat the Lakers. **d)** If you get to the top of Long’s Peak, then you must have walked 8 miles. **c)** If you are world-famous, then you will get tenure

q→p is called the converse of p→q (逆命题).
¬q→¬p is called contrapositive of p→q (逆否命题).
¬p→¬q is called inverse of p→q (否命题).

27. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.

27. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.

take the train. **27. a)** Converse: "I will ski tomorrow only if it snows today." Contrapositive: "If I do not ski tomorrow, then it will not have snowed today." Inverse: "If it does not snow today, then I will not ski tomorrow." **b)** Converse: "If I come to class, then there will be a quiz." Contrapositive: "If I do not come to class, then there will not be a quiz." Inverse: "If there is not going to be a quiz, then I don't come to class."

29. How many rows appear in a truth table for each of these compound propositions?

a) $p \to \neg p$
b) $(p \vee \neg r) \wedge (q \vee \neg s)$

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a) $p \to \neg p$
b) $(p \vee \neg r) \wedge (q \vee \neg s)$

29. a) 2 **b)** 16

Truth table p ⋀ q

p	q	p ⋀ q
T	T	T
T	F	F
F	T	F
F	F	F

Truth table p ⋁ q

p	q	p ⋁ q
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table p→q

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table ¬p

p	¬p
T	F
F	T

31. Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$ **b)** $p \vee \neg p$
c) $(p \vee \neg q) \to q$ **d)** $(p \vee q) \to (p \wedge q)$

31. Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$ **b)** $p \vee \neg p$
c) $(p \vee \neg q) \to q$ **d)** $(p \vee q) \to (p \wedge q)$

31. a)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

b)

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

c)

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Truth table

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

39. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

39. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

39.

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Truth table "OR"

p	q	p ⋁ q
T	T	T
T	F	T
F	T	T
F	F	F

Truth table "AND"

p	q	p ⋀ q
T	T	T
T	F	F
F	T	F
F	F	F

Truth table "XOR"

p	q	p ⊕ q
T	T	*F*
T	F	T
F	T	T
F	F	*F*

43. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001
b) 1111 0000, 1010 1010
c) 00 0111 0001, 10 0100 1000
d) 11 1111 1111, 00 0000 0000

43. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001
b) 1111 0000, 1010 1010
c) 00 0111 0001, 10 0100 1000
d) 11 1111 1111, 00 0000 0000

have the same truth value. **43. a)** Bitwise *OR* is 111 1111; bitwise *AND* is 000 0000; bitwise *XOR* is 111 1111. **b)** Bitwise *OR* is 1111 1010; bitwise *AND* is 1010 0000; bitwise *XOR* is 0101 1010. **c)** Bitwise *OR* is 10 0111 1001; bitwise *AND* is 00 0100 0000; bitwise *XOR* is 10 0011 1001. **d)** Bitwise *OR* is 11 1111 1111; bitwise *AND* is 00 0000 0000; bitwise *XOR* is 11 1111 1111. **45.** 0.2, 0.6 **47.** 0.8, 0.6 **49. a)** The

Tutorial 1-3

Logical Equivalence

(1) Example 2 (page 22)

Show that ¬(p ⋁q) and ¬p ⋀ ¬q are logically equivalent.

Truth Table

p	q	p ⋁ q	¬(p ⋁ q)	¬p	¬q	¬p ⋀ ¬q
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

1. Use truth tables to verify these equivalences.

a) $p \wedge \mathbf{T} \equiv p$
b) $p \vee \mathbf{F} \equiv p$
c) $p \wedge \mathbf{F} \equiv \mathbf{F}$
d) $p \vee \mathbf{T} \equiv \mathbf{T}$
e) $p \vee p \equiv p$
f) $p \wedge p \equiv p$

1. Use truth tables to verify these equivalences.

a) $p \wedge \mathbf{T} \equiv p$ **b)** $p \vee \mathbf{F} \equiv p$
c) $p \wedge \mathbf{F} \equiv \mathbf{F}$ **d)** $p \vee \mathbf{T} \equiv \mathbf{T}$
e) $p \vee p \equiv p$ **f)** $p \wedge p \equiv p$

1. The equivalences follow by showing that the appropriate pairs of columns of this table agree.

p	$p \wedge \mathbf{T}$	$p \vee \mathbf{F}$	$p \wedge \mathbf{F}$	$p \vee \mathbf{T}$	$p \vee p$	$p \wedge p$
T	T	T	F	T	T	T
F	F	F	F	T	F	F

$p \vee q \equiv q \vee p$ **Commutative laws** **(or)**
$p \wedge q \equiv q \wedge P$ **(交换律).** **(and)**

3. Use truth tables to verify the commutative laws
a) $p \vee q \equiv q \vee p.$ **b)** $p \wedge q \equiv q \wedge p.$

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3. Use truth tables to verify the commutative laws
a) $p \vee q \equiv q \vee p.$ **b)** $p \wedge q \equiv q \wedge p.$

3. **a)**

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

b)

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ **De Morgan's laws**
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ **(德摩根律)**

7. Use De Morgan's laws to find the negation of each of the following statements.
a) Jan is rich and happy.
b) Carlos will bicycle or run tomorrow.

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7. Use De Morgan's laws to find the negation of each of the following statements.
a) Jan is rich and happy.
b) Carlos will bicycle or run tomorrow.

7. **a)** Jan is not rich, or Jan is not happy. **b)** Carlos will not bicycle tomorrow, and Carlos will not run tomorrow. **c)** Mei does not walk to class, and Mei does not take the bus to class. **d)** Ibrahim is not smart, or Ibrahim is not hard working.

$p \vee \neg p$ is always true. It is a tautology. (永真公式)

☞ **9.** Show that each of these conditional statements is a tautology by using truth tables.
a) $(p \wedge q) \rightarrow p$ **b)** $p \rightarrow (p \vee q)$

9. Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$ b) $p \rightarrow (p \vee q)$

9. a)

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b)

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$

Absorption laws
(吸收律)

13. Use truth tables to verify the absorption laws.

a) $p \vee (p \wedge q) \equiv p$ b) $p \wedge (p \vee q) \equiv p$

13. Use truth tables to verify the absorption laws.

a) $p \vee (p \wedge q) \equiv p$ b) $p \wedge (p \vee q) \equiv p$

false. Hence, the conclusion $\neg q$ is true. 13. That the fourth column of the truth table shown is identical to the first column proves part (a), and that the sixth column is identical to the first column proves part (b).

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	F	F	F

Logical Equivalence

(1) Example 2 (page 22)

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Table

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

17. Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

17. Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

15. It is a tautology. 17. Each of these is true precisely when p and q have opposite truth values. 19. The proposition

$p \vee \neg p$ is always true. It is a tautology. (永真公式)

29. Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.

29. Show that $(p \to q) \land (q \to r) \to (p \to r)$ is a tautology.

same truth values. **29.** The last column is all Ts.

p	q	r	$p \to q$	$q \to r$	$(p \to q) \land (q \to r)$	$p \to r$	$(p \to q) \land (q \to r) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Logical Equivalence

(1) Example 2 (page 22)

Show that $\neg(p \lor q)$ and $\neg p \land \neg q$ are logically equivalent.

Truth Table

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

33. Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

33. Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

true. **33.** Many answers are possible. If we let r be true and p, q, and s be false, then $(p \to q) \to (r \to s)$ will be false, but $(p \to r) \to (q \to s)$ will be true. **35. a)** $p \lor \neg q \lor \neg r$

(4) Some Important Equivalences

(a) Table 5 (Logical Equivalence)

$p \land T \equiv p$ **Identity laws**
$p \lor F \equiv p$ **(同一律)**

$p \lor T \equiv T$ **Domination laws**
$p \land F \equiv F$ **(零律)**

$p \lor p \equiv p$ **Idempotent laws**
$p \land p \equiv p$ **(幂等律)**

$\neg(\neg p) \equiv p$ **Double negation law**
(双重否定律)

$p \lor q \equiv q \lor p$ **Commutative laws** **(or)**
$p \land q \equiv q \land P$ **(交换律).** **(and)**

$(p \lor q) \lor r \equiv p \lor (q \lor r)$ **Associative laws**
$(p \land q) \land r \equiv p \land (q \land r)$ **(结合律)**

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ **Distributive laws**
$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ **(分配律)**

$\neg(p \land q) \equiv \neg p \lor \neg q$ **De Morgan's laws**
$\neg(p \lor q) \equiv \neg p \land \neg q$ **(德摩根律)**

$p \lor (p \land q) \equiv p$ **Absorption laws**
$p \land (p \lor q) \equiv p$ **(吸收律)**

$p \lor \neg p \equiv T$ **(排中律)** **Negation laws**
$p \land \neg p \equiv F$ **(矛盾律)**

(b) Table 6 (Logical Equivalence Involving Implication)

$p \to q \equiv \neg p \lor q$

$p \to q \equiv \neg q \to \neg p$

$p \lor q \equiv \neg p \to q$

$p \land q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \land \neg q$

$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$

$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$

$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

$p \to q \equiv \neg q \to \neg p$

- $p \to q$
  $\equiv \neg p \lor q$
  $\equiv q \lor \neg p$
  $\equiv \neg(\neg q) \lor \neg p$
  $\equiv \neg q \to \neg p$

$p \to q \equiv \neg p \lor q$
$p \lor q \equiv q \lor p$
$\neg(\neg p) \equiv p$
$\neg p \lor q \equiv p \to q$

-
  $p \lor q \equiv q \lor p$ Commutative laws (交换律).
  $\neg(\neg p) \equiv p$ Double negation law(双重否定律)
  $p \to q \equiv \neg p \lor q$
  $\neg p \lor q \equiv p \to q$

$p \lor q \equiv \neg p \to q$

- $p \lor q$
  $\equiv \neg(\neg p) \lor q$
  $\equiv \neg p \to q$

$\neg(\neg p) \equiv p$
$\neg p \lor q \equiv p \to q$

-
  $\neg(\neg p) \equiv p$ Double negation law(双重否定律)
  $\neg p \lor q \equiv p \to q$

$p \land q \equiv \neg(p \to \neg q)$

- $p \land q$
  $\equiv \neg(\neg(p \land q))$
  $\equiv \neg(\neg p \lor \neg q)$
  $\equiv \neg(p \to \neg q)$

$\neg(\neg p) \equiv p$
$\neg(p \land q) \equiv \neg p \lor \neg q$
$\neg p \lor q \equiv p \to q$

-
  $\neg(\neg p) \equiv p$. Double negation law(双重否定律)
  $\neg(p \land q) \equiv \neg p \lor \neg q$. De Morgan's laws (德摩根律)
  $\neg p \lor q \equiv p \to q$

$\neg(p \to q) \equiv p \land \neg q$

- $\neg(p \to q)$
  $\equiv \neg(\neg p \lor q)$
  $\equiv \neg\neg p \land \neg q$
  $\equiv p \land \neg q$

$p \to q \equiv \neg p \lor q$
$\neg(p \lor q) \equiv \neg p \land \neg q$

-
  $p \to q \equiv \neg p \lor q$
  $\neg(p \lor q) \equiv \neg p \land \neg q$. De Morgan's laws (德摩根律)

$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

- $(p \to q) \land (p \to r)$
  $\equiv (\neg p \lor q) \land (\neg p \lor r)$
  $\equiv \neg p \lor (q \land r)$
  $\equiv p \to (q \land r)$

$p \to q \equiv \neg p \lor q$
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
$\neg p \lor q \equiv p \to q$

$p \to q \equiv \neg p \lor q$
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$. Distributive laws(分配律)
$\neg p \lor q \equiv p \to q$

$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$

- $(p \to r) \land (q \to r)$
  $\equiv (\neg p \lor r) \land (\neg q \lor r)$
  $\equiv (r \lor \neg p) \land (r \lor \neg q)$
  $\equiv r \lor (\neg p \land \neg q)$
  $\equiv (\neg p \land \neg q) \lor r$
  $\equiv \neg(p \lor q) \lor r$
  $\equiv (p \lor q) \to r$

$p \to q \equiv \neg p \lor q$
$p \lor q \equiv q \lor p$
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
$p \lor q \equiv q \lor p$
$\neg(p \lor q) \equiv \neg p \land \neg q$
$p \to q \equiv \neg p \lor q$

-
  $p \to q \equiv \neg p \lor q$
  $p \lor q \equiv q \lor p$. Commutative laws (交换律).
  $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$. Distributive laws (分配律)
  $\neg(p \lor q) \equiv \neg p \land \neg q$. De Morgan's laws (德摩根律)

$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$

- $(p \to q) \lor (p \to r)$
  $\equiv (\neg p \lor r) \lor (\neg q \lor r)$ — $p \to q \equiv \neg p \lor q$
  $\equiv \neg p \lor \neg q \lor r$ — $p \lor p \equiv p$
  $\equiv \neg p \lor (\neg q \lor r)$ — $(p \lor q) \lor r \equiv p \lor (q \lor r)$
  $\equiv \neg p \lor (p \to r)$ — $\neg p \lor q \equiv p \to q$
  $\equiv p \to (q \lor r)$ — $\neg p \lor q \equiv p \to q$

-
  $p \to q \equiv \neg p \lor q$
  $p \lor p \equiv p$ Idempotent laws (幂等律)
  $(p \lor q) \lor r \equiv p \lor (q \lor r)$ Associative laws(结合律)
  $\neg p \lor q \equiv p \to q$

$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

- $(p \to q) \lor (p \to r)$
  $\equiv (\neg p \lor r) \lor (\neg q \lor r)$ — $p \to q \equiv \neg p \lor q$
  $\equiv \neg p \lor \neg q \lor r$
  $\equiv (\neg p \lor \neg q) \lor r$ — $(p \lor q) \lor r \equiv p \lor (q \lor r)$
  $\equiv \neg(p \land q) \lor r$ — $\neg p \lor \neg q \equiv \neg(p \land q)$
  $\equiv (p \land q) \to r$ — $p \to q \equiv \neg p \lor q$

-
  $p \to q \equiv \neg p \lor q$
  $\neg p \lor \neg q \equiv \neg(p \land q)$. De Morgan's laws (德摩根律)
  $(p \lor q) \lor r \equiv p \lor (q \lor r)$ Associative laws(结合律)

- $p \to q \equiv \neg p \lor q$
  $\neg p \lor q \equiv p \to q$
  $p \lor q \equiv q \lor p$. Commutative laws (交换律).
  $\neg(\neg p) \equiv p$ Double negation law(双重否定律)
  $\neg(p \land q) \equiv \neg p \lor \neg q$. De Morgan's laws (德摩根律)
  $\neg p \lor \neg q \equiv \neg(p \land q)$. De Morgan's laws (德摩根律)
  $p \lor p \equiv p$ Idempotent laws (幂等律)
  $(p \lor q) \lor r \equiv p \lor (q \lor r)$ Associative laws(结合律)
  $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ Distributive laws (分配律)

(c) Table 6 (Logical Equivalence Involving Biconditionals)

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$
$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

(d) Questions:
How to verify these equivalences?
Answer: One way is by constructing the truth table.

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \land (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$

p	q	$p \leftrightarrow q$	$\neg q$	$\neg p$	$\neg q \leftrightarrow \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	F	F	T	F
F	F	T	T	T	T

$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

p	q	$p \leftrightarrow q$	$p \land q$	$\neg p$	$\neg q$	$(\neg p \land \neg q)$	$(p \land q) \lor (\neg p \land \neg q)$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg q$	$p \leftrightarrow \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

assignment

- Chapter1.1 Exercises (Page 12) Question Number:
- 2, 8, 12, 16, 30

- Chapter1.3(Page 34) Question Number:
- 2, 10, 16, 32
- Please take your homework a photo and transfer it into pdf or scan it.
- Only pdf is accepted.