§ 2.2 Set Operations

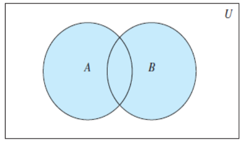
1. Introduction

(1) Definition 1 (page 127)

Let A and B be sets. The *union* of the sets A and B, denoted by A∪B, is the set that contains those elements that either in A or in B, or in both.

$$A \cup B = \{ x \mid x \in A \lor x \in B \}$$

$$\{1,3,5\} \cup \{1,2,3\} = \{1,2,3,5\}$$



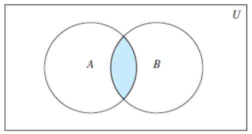
$A \cup B$ is shaded.

$$A \cup B = \{ x \mid x \in A \lor x \in B \}$$

$$\{1,3,5\} \cup \{1,2,3\} = \{1,2,3,5\}$$

(2) Definition 2 (intersection)

Let A and B be sets. The *intersection* of the sets A and B, denoted by A∩B, is the set containing those elements in both A and B.



$A \cap B$ is shaded.

$$A \cap B = \{ x \mid x \in A \land x \in B \}$$

$$\{1,3,5\} \cap \{1,2,3\} = \{1,3\}$$

(3) Definition 3 (page 128)

Two sets are called *disjoint* if their intersection is the empty set.

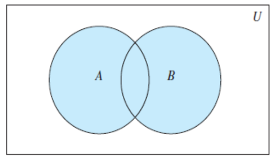
Example 5 (page 128)

Let A={1,3,5,7,9} and B={2,4,6,8,10}. Since A∩B=∅ , A and B are disjoint.

(5) For two finite sets A and B, we have:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Venn Diagram



$A \cup B$ is shaded.

(5) Definition 4 (the difference of two sets, 差集)

Let A and B be sets. The *difference* of A and B, denoted by A-B, is the set that containing those elements that are in A but not in B.

The difference of A and B is also called the *complement of B with respect to* A (关于A的集合B的补集)

A-B={x | x∈A ∧ x not in B }

Explain it via Venn Diagram (page 129).

Example 6 (page 128)

{1,3,5}-{1,2,3} = ?

{1,2,3}-{1,3,5} = ?

(6) Definition 5 (complement of a set, 补集)

Let U be universal set (全集). The *complement* of the set A, denoted by ~A, is the complement of A with respect to U.

~A = U-A

~A = { x | x not in A}

Explain it via Venn Diagram (page 129).

Example 8 (page 129)

A={a,e,i,o,u}

U-----the set of letters of the English alphabet

~A = ?

Example 9 (page 129)

A----the set of positive integers greater than 10

U---all positive integers

~A = ?

2. Set identities (集合的恒等式)

(1) Introduction

Table 1 (page 130)

Identity law (同一律)
Domination Law (零律)
Idempotent law (幂等律)
Complementation law (双重否定律)
Commutative law (交换律)
Associative law (结合律)
Distributive law (分配律)
De Morgan's law (德摩根律)
Absorption laws (吸收律)
A∪~A=U (排中律)
A ∩~A= ∅ (矛盾律)

(2) Example 10 (page 130)

Prove that ~(A∩B) = ~A ∪~B.

Solution:

(a) Left ⊆ Right

(b) Right ⊆ Left

(3) Use set builder notation and logical equivalence to show that

$\sim(A \cap B) = \sim A \cup \sim B$

(However, the instructor prefers method before, that is, using basic set definitions)

(4) Example 12 (page 131)

Prove that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

for all sets A, B, C.

(5) Let A, B, and C be sets. Show that

$\sim(A \cup (B \cap C)) = (\sim C \cup \sim B) \cap \sim A$

Proof:

By using the set identities proved previously.

See book (page 132)

3. Generalized unions and intersections

(1) Introduction

The well-definedness of

$A \cup B \cup C$ **and** $A \cap B \cap C$

Why?

Reason:

the associative law of ∪ and ∪

$(A \cup B) \cup C = A \cup (B \cup C)$

$A \cap (B \cap C) = (A \cap B) \cap C$

(2) Example 15 (page 133)

Let A={0,2,4,6,8}, B={0,1,2,3,4}, and C={0,3,6,9}.

What are $A \cup B \cup C$ **and** $A \cap B \cap C$**?**

$A \cup B \cup C =$

$A \cap B \cap C =$

(3) Definition 6

The *union* of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

The notation:

$A_1 \cup A_2 \ldots \cup A_n = \cup_{i=1}^{n} A_i$

(4) Definition 7

The *intersection* of a collection of sets is the set that contains those elements that are members of *all the sets in the collection.*

The notation:

$A_1 \cap A_2 \ldots \cap A_n = \cap_{i=1}^{n} A_i$

(5) Let $A_i=\{i, i+1, i+2, \ldots\ldots\}$.

What are $\cup_{i=1}^{n} A_i$ and $\cap_{i=1}^{n} A_i$?

Discussion in theory class

7,27