mathematical diary

entry #1 - 4.8.20
IMO 1979 problem 3 reads $\sim$ two circles intersect as A. two points, starting simultaneously from A, trace the circles, both in the same orientation and constant angular speed [they meet at A again after each revolved once around its circle] prove the existence of a point P, such that the two points are always equidistant from P.
seems pretty interesting, but I haven't practiced Euclidean geometry in ages, so I might as well bash this using complex numbers, I figured. I found more to be true, [in great part due to a simulation on desmos] and it goes as follows. let B denote the second intersection of the two circles, $C_i$ their centers, and $M(t)$ the midpoint of the two moving points [as a function of the time variable $t$] then $M(t)$ travels in a circle - the circle whose center is the midpoint of the $C_i$'s, and which contains A and B. Further, the line $\ell(t)$ between the two moving points always contains B. The problem is then solved, as P makes the diameter of said circle with B.

entry #2 - 8.8.20
USAMO 2008 problem 6 reads: at a certain mathematical conference, every pair of mathematicians are either friends or strangers. At mealtime, every participant eats in one of two large dining rooms. Each mathematician insists upon eating in a room which contains an even number of his or her friends. Prove that the number of ways that the mathematicians may be split between the two rooms is a power of two.
fascinating. how does one go about solving this? a good guess would be to bash using linear algebra over $\mathbf{F}_2$. letting $x_i$ denote the boolean variable specifing which room mathematician $i$ eats in, and noting that $i, j$ sit in the same room iff $y_{ij} \equiv x_i + x_j + 1 = 1$, we get a system of linear equations $\forall i \sum_{j \sim i} y_{ij} = 0$. one must still understand why it has a solution. letting $L$ denote the Laplacian of the freindship graph (adjacency matrix except the diagonal contains the vertex degrees), we want deg $\in$ image$L$. this is a special case of the fact the diagonal of a symmetric binary matrix is in its image. [which we can show via demonstrating it is perpendicular to its kernel]