$$\hat{H} = \hat{M}_{ll} + \sum_{i=1}^{2} -\frac{1}{2}\frac{\partial^2}{\partial r_i^2} - \frac{z}{r_i} + \frac{l(l+1)}{r_i^2}$$

$$\hat{M}_{ll'} = \sum_{k=|l-l'|}^{l+l'} C^k(l,l')\frac{r_<^k}{r_>^{k+1}}$$

$$C^k(l,l') = \frac{1}{2}[(2l+1)(2l'+1)]^{\frac{1}{2}} \times \int P_l(\cos(\theta))P_k(\cos(\theta))P_{l'}(\cos(\theta))$$

$$l = 0 \rightarrow H = \frac{1}{r_>} + \sum_{i=1}^{2} -\frac{1}{2}\nabla_i^2 - \frac{z}{r_i}$$

$$
A = \begin{array}{c}
\quad \quad |1\rangle \quad |2\rangle \quad |3\rangle \\
\begin{array}{c}
\langle 1| \\
\langle 2| \\
\langle 3|
\end{array}
\begin{array}{ccc}
\ddots & & \\
& \ddots & \\
& & \ddots
\end{array}
\end{array}
$$

$$A_1 = A \otimes I$$
$$A_2 = I \otimes A$$
$$A_{1,2} = A \otimes A$$

$$D = \frac{\partial^2 \psi(r_1, r_2)}{\partial r_1^2} = \psi(r_1 + h, r_2) + \psi(r_1 - h, r_2) - 2\psi(r_1, r_2)$$

$$= \begin{bmatrix} -2 & 1 & 0 & \dots \\ 1 & -2 & 1 & \dots \\ 0 & 1 & -2 & \dots \\ \dots & \dots & \dots & \ddots \end{bmatrix}$$

$$\frac{\partial^2 \psi(r_1, r_2)}{\partial r_1^2} + \frac{\partial^2 \psi(r_1, r_2)}{\partial r_1^2} = D \otimes I + I \otimes D$$

$$V = \frac{z}{r_i}$$

$$= \begin{bmatrix} \frac{z}{h} & 0 & 0 & \dots \\ 0 & \frac{z}{2h} & 0 & \dots \\ 0 & 0 & \frac{z}{3h} & \dots \\ \dots & \dots & \dots & \ddots \end{bmatrix}$$

$$V(r_1) + V(r_2) = V \otimes I + I \otimes V$$

$$O = \frac{1}{r_>}$$

$$
= \begin{array}{c}
\quad \quad |11\rangle \quad |12\rangle \quad |\dots\rangle \quad |31\rangle \quad |\dots\rangle \\
\begin{array}{c}
\langle 11| \\
\langle 12| \\
\langle \dots| \\
\langle 31| \\
\langle \dots|
\end{array}
\begin{array}{ccccc}
1 & & & & \\
& \frac{1}{2} & & & \\
& & \ddots & & \\
& & & \frac{1}{3} & \\
& & & & \ddots
\end{array}
\end{array}
$$

$$\hat{H} = -\frac{1}{2h^2}(D \otimes I + I \otimes D) - V \otimes I - I \otimes V + O$$

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$\hat{H}|\psi_l\rangle = E_{l-1}|\psi_l\rangle - \sum_{l'\neq l}^{l}\hat{M}_{ll'}|\psi_{l'}\rangle$$

$$\hat{H} = \sum_\chi \lambda_\chi |\chi\rangle\langle\chi|$$

$$\sum_\chi \lambda_\chi |\chi\rangle\langle\chi|\psi_l\rangle = \sum_\chi E_{l-1}|\chi\rangle\langle\chi|\psi_l\rangle - \sum_\chi \sum_{l'\neq l}^{l}|\chi\rangle\langle\chi|\hat{M}_{ll'}|\psi_{l'}\rangle$$

$$\langle\chi|\psi_l\rangle = \gamma_\chi, \ \langle\chi|\hat{M}_{ll'}|\psi_{l'}\rangle = c_{\chi,l'}$$

$$\sum_\chi \lambda_\chi \gamma_\chi |\chi\rangle = \sum_\chi \left(E_{l-1}\gamma_\chi - \sum_{l'\neq l}^{l} c_{\chi,l'}\right)|\chi\rangle$$

$$|\psi_l\rangle = \sum_\chi \frac{\sum_{l'\neq l}^{l} c_{\chi,l'}}{E_{l-1}-\lambda_\chi}|\chi\rangle$$

$$\hat{H}|\psi_k\rangle = E|\psi_k\rangle - \sum_{l'\neq k}^{l}\hat{M}_{ll'}|\psi_{l'}\rangle$$

$$\hat{H}|\psi_k\rangle + \sum_{l'\neq k}^{l}\hat{M}_{ll'}|\psi_{l'}\rangle = E_l|\psi_k\rangle$$

$$\langle\psi_k|\hat{H}|\psi_k\rangle + \sum_{l'\neq k}^{l}\langle\psi_k|\hat{M}_{ll'}|\psi_{l'}\rangle = \langle\psi_k|E_l|\psi_k\rangle$$

$$E_l = \langle\hat{H}\rangle_k + \sum_{l'\neq l}^{l}\langle\psi_k|\hat{M}_{ll'}|\psi_{l'}\rangle$$

$$E_l = \langle\hat{H}\rangle_0 + \sum_{l'=1}^{l}\langle 0|\hat{M}_{0l'}|\psi_{l'}\rangle$$

$$E_l = E_{l-1} + \langle 0|M_{0l}|l\rangle$$

Grid Size	S Limit								
9/20	-2.417778098								
/25	-2.549148048	-2.782694627							
/30	-2.633740690	-2.825996694	-2.860638347						
/35	-2.690595776	-2.848040629	-2.871003062	-2.876028379					
/40	-2.730287122	-2.859945519	-2.875251805	-2.877975359	-2.878624352				
/45	-2.758923859	-2.866732750	-2.877125698	-2.878624811	-2.878914745	-2.878986227			
/50	-2.780178951	-2.870792764	-2.878010568	-2.878860737	-2.878993445	-2.879019678	-2.879026050		
/55	-2.796344911	-2.873325674	-2.878454815	-2.878953618	-2.879016829	-2.879026733	-2.879028571	-2.879028955	
/60	-2.808902262	-2.874964849	-2.878690247	-2.878992945	-2.879024406	-2.879028314	-2.879028841	-2.879028898	-2.879028891