Princeton Quantitative Traders Questions:

1. You roll a fair 6-sided die until you get a 6. Find the variance of the number of rolls.

2. A biased coin has probability p of heads. You flip until you get HT (a head followed immediately by a tail). Derive the expected number of flips.

3. Two players play the following game: Starting with 100, players alternate subtracting any prime number ≤ 19. The player who makes the total exactly 0 wins. Does the first or second player have a winning strategy?

4. A stock's daily log-returns are i.i.d. N(0, 0.01). What is the probability the stock price drops below half its original value after 200 trading days?

5. You have two envelopes. One has twice as much money as the other. You pick one and see $100. Describe a non-trivial switching strategy that improves your expected value.

6. There are 3 boxes: (GG), (SS), (GS). You pick a box at random and draw one coin: it's gold. What's the probability the other coin is also gold?

7. There are 100 prisoners and 100 boxes, each box containing one unique prisoner's number. Each prisoner may open up to 50 boxes. If all succeed, they live; otherwise all die. What strategy maximizes survival, and what's the approximate probability of success?

8. You play a game: Roll two dice. If the sum is 7, you win $20. Otherwise, you lose $3. What is the expected value?

9. You have 12 coins, one counterfeit, known to be either heavier or lighter. You have a balance scale. What is the minimum number of weighings to always identify it and its weight difference?

10. There are 100 light bulbs, all off. The 1st person toggles all, the 2nd toggles every 2nd, the 3rd every 3rd, … the 100th toggles every 100th. Which bulbs remain on?

11. Two players each choose a number between 0 and 100. The lower number wins that many dollars, unless it is more than 10 lower than the other's choice, in which case it pays 0. Find the equilibrium strategy (in broad terms).

12. You have a biased coin with unknown probability p of landing heads. You flip it 200 times and observe 120 heads. Construct a 95% confidence interval for p.