A Mixed-Integer Programming Model for Sustainable Textile Repair Operations

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Abstract

The increasing emphasis on circular economy strategies in the textile and fashion sector has renewed industrial interest in large-scale repair operations as a means to extend product lifetime and reduce waste. This study develops a Mixed-Integer Programming (MIP) model to optimize the allocation and scheduling of repair activities across a network of specialized repair centres. The formulation captures key operational trade-offs by jointly considering defect–repairer assignment, batch shipment formation, and dispatch timing under capacity and service-level constraints. A composite objective function balances lead time, shipment cost repair cost, quality retention, and environmental impact, reflecting the multifaceted goals of sustainable operations management. Computational experiments inspired by real textile repair processes demonstrate the model's capability to identify efficient repair and shipment policies while supporting strategic decision-making for circular textile supply chains.

Keywords: Mixed-Integer Programming, Textile Repair, Sustainable Operations, Fashion Supply Chain, Optimization, Scheduling, Circular Economy.

1 Introduction

The increasing adoption of circular economy principles has placed reverse logistics and product recovery at the forefront of sustainable supply chain management. Reverse flows — including returns, repairs, remanufacturing and recycling — can recover substantial economic value while reducing environmental externalities; as such, the study of quantitative models for reverse logistics has been an active area of operations research for decades [1].

In particular, the textile and fashion industry is undergoing a significant transformation driven by the incorporation of circular-economy principles, with repair operations emerging as a critical enabler of resource efficiency and extended product life. For garments and accessories, the ability to repair rather than discard contributes directly to value retention and waste reduction. Recent empirical work underlines that repair forms an integral component of sustainable consumption behaviour and product-lifecycle extension [2].

From an operations-management viewpoint, executing repair operations at scale within fashion supply chains presents several intertwined challenges. Items arrive with heterogeneous defects, repair centres differ in capacity, turnaround time and environmental performance, and shipments must be batched and scheduled to meet service-level targets. Moreover, product quality after repair, shipping costs and carbon emissions form conflicting objectives that must be balanced in practice. Policies and industry reports emphasise that infrastructure for repair and redistribution in fashion remains under-developed, despite the potential value of reverse logistics networks tailored to repair services [3].

On the methodological side, decision-support models and mathematical formulations for repair operations are comparatively scarce. While reverse-logistics and closed-loop supply-chain design have been studied widely, the specific process of defect-level assignment, batch shipment dispatching and service-level timing in textile/fashion repair has received limited attention [4]. This motivates the current study, which develops a deterministic MIP model tailored to fashion-product repair operations: assigning defective items to external repair centres, forming batch shipments, and scheduling dispatches under capacity, lead-time and quality constraints.

2 Methodology

The proposed MIP formulation integrates assignment, batching, and shipment scheduling decisions required to operate a distributed textile repair network. Products enter the system over a discrete planning horizon and must be assigned to a repairer, accumulated into batches, and shipped while respecting capacity limits and lead-time requirements. In the following, we discuss the logic of the formulation and comment on each constraint in detail.

We are given the following sets:

- R — set of repairers.
- D — set of defect types.
- P — set of products.
- T — set of time periods in the planning horizon.
- D_p — defects associated with product p.
- P_t — products arriving at time t.

and the following parameters:

- χ^r_{dpr} — unit repair cost of defect d on product p when repaired by r.
- τ — maximum lead-time admitted for all products.
- τ_p — arrival time of product p.
- β_r — maximum batch capacity for repairer r.
- λ_r — lead time of repairer r.
- σ_{dpr} — percentage of quality reduction caused by repair of defect d in p by r.
- χ^s_r — shipping cost per batch (round trip) for repairer r.
- π_r — carbon emissions (CO_2 [g]) per batch shipment to repairer r.

To address the mathematical formulation of the optimization model we define the decision variables:

- $u_{dpr} = \mathbb{1}\{\text{product } p \text{ assigned to repairer } r \text{ for defect } d\}$;
- $u_{pr} = \mathbb{1}\{\text{product } p \text{ is assigned to repairer } r\}$;
- $x_{prt} = \mathbb{1}\{\text{product } p \text{ shipped to repairer } r \text{ at time } t\}$;
- $z_{rt} = \mathbb{1}\{\text{batch for repairer } r \text{ shipped at time } t\}$;
- $b_{rt} \in \mathbb{Z}^+$ — products accumulated in repairer r's batch at time t;
- $a_{rt} \in \mathbb{Z}^+$ — products newly assigned to repairer r's batch at time t;
- $l \in \mathbb{R}^+$ — total lead time.

2.1 Mathematical Formulation

Since each repairer can process at most one batch of limited size per time slot, batch capacity is enforced through

$$b_{rt} \leq \beta_r \qquad \forall r \in R,\ \forall t \in T, \tag{1}$$

which guarantees that no batch assigned to repairer r exceeds its maximum admissible size. Whenever a batch is full this event must trigger the shipment. This logic is encoded through

$$b_{rt} - \beta_r + 1 \leq z_{rt} \qquad \forall r \in R,\ \forall t \in T, \tag{2}$$

ensuring that each time $b_{rt} = \beta_r$ we have also $z_{rt} = 1$.

Shipment decisions are represented by the binary variable x_{prt}, which indicates whether product p assigned to repairer r is shipped at time t. The shipment of a product $p \in P$ to repairer $r \in R$ at time $t \in T$ can occur only if the whole batch of repairer r is sent at time t. This can be enforced by

$$x_{prt} \leq z_{rt} \qquad \forall p \in P,\ \forall r \in R,\ \forall t \in T \tag{3}$$

so that no product departs independently of its batch. Conversely, whenever a batch is activated, it must contain at least one shipped product; this prevents "empty" batches and is expressed by

$$z_{rt} \leq \sum_{p \in P} x_{prt} \qquad \forall r \in R,\ \forall t \in T \tag{4}$$

The dynamics of batch accumulation follow a flow-balance logic. At the beginning of the horizon $t = 0$, we suppose that each basket is empty i.e. $b_{r-1} = 0$ for each r. Then the number of items accumulated equals the previous stock plus newly assigned products a_{rt}, minus the products shipped in that period. This is formalised as

$$b_{rt} = b_{r,t-1} + a_{rt} - \sum_{p \in P} x_{prt} \qquad \forall r \in R,\ \forall t \in T \tag{5}$$

and with the constraint:

$$b_{r-1} = 0 \qquad \forall r \in R \tag{6}$$

As a consistency requirement, in any period the number of shipped products cannot exceed the number already accumulated, giving rise to

$$\sum_{p \in P} x_{prt} \leq b_{r,t-1} + a_{rt} \qquad \forall r \in R,\ \forall t \in T \tag{7}$$

Even when enough products are available, shipments cannot surpass the batch capacity β_r, and thus

$$\sum_{p \in P} x_{prt} \leq \beta_r \qquad \forall r \in R, \ \forall t \in T \tag{8}$$

Because products arrive exogenously at specific periods, the assignment variables must reflect this temporal structure. All the products arriving in each period t must be sorted into repairer batches

$$\sum_{r \in R} a_{rt} = |P_t| \qquad \forall t \in T \tag{9}$$

and each product must be uniquely assigned to exactly one repairer,

$$\sum_{r \in R} u_{pr} = 1 \qquad \forall p \in P \tag{10}$$

The previous flows are able to connect the variables by noting that the total number of products assigned to repairer r across the entire horizon must coincide with the cumulative inflow to the associated basket

$$\sum_{t \in T} a_{rt} = \sum_{p \in P} u_{pr} \qquad \forall r \in R \tag{11}$$

Once assigned, each product must be shipped exactly once. This condition binds the assignment and shipment variables:

$$\sum_{t \in T} x_{prt} = u_{pr} \qquad \forall p \in P, \ \forall r \in R \tag{12}$$

Since each product may exhibit multiple defects, the defect-level assignment variable u_{dpr} must be consistent with the product-level assignment u_{pr}. The following constraint enforces that a product is assigned to repairer r if and only if all its defects are:

$$|D_p| \, u_{pr} = \sum_{d \in D_p} u_{dpr} \qquad \forall p \in P, \ \forall r \in R \tag{13}$$

Service-level agreements require that repaired items return within a maximum admissible lead time τ. Since for a product p arrived at time τ_p and shipped at time t the completion time is $t + \lambda_r - \tau_p$, we must impose that:

$$\sum_{t \in T} x_{prt}(t + \lambda_r) - \tau_p \leq \tau \qquad \forall p \in P, \ \forall r \in R \tag{14}$$

and that shipments cannot occur before the product arrives i.e.

$$\sum_{\substack{t \in T \\ t < \tau_p}} x_{prt} = 0 \qquad \forall p \in P, \ \forall r \in R \tag{15}$$

Finally, the variable l represents the maximum lead time across all products and is defined through:

$$l \geq \sum_{t \in T} x_{prt}(t + \lambda_r - \tau_p) \qquad \forall p \in P, \ \forall r \in R \tag{16}$$

The final aim is to minimize an objective function that is a weighted combination of total lead time, shipment cost, quality deterioration, repair costs and environmental impact.

$$\alpha_1 l + \alpha_2 \sum_{t \in T} \sum_{r \in R} \chi^s_r z_{rt} + \alpha_3 \sum_{d \in D_p} \sum_{p \in P} \sum_{r \in R} \sigma_{dpr} u_{dpr} + \alpha_4 \sum_{d \in D_p} \sum_{p \in P} \sum_{r \in R} \chi^r_{dpr} u_{dpr} + \alpha_5 \sum_{t \in T} \sum_{r \in R} \pi_r z_{rt} \tag{17}$$

where coefficients $\alpha_1, \ldots, \alpha_5$ represent the relative importance of each component.

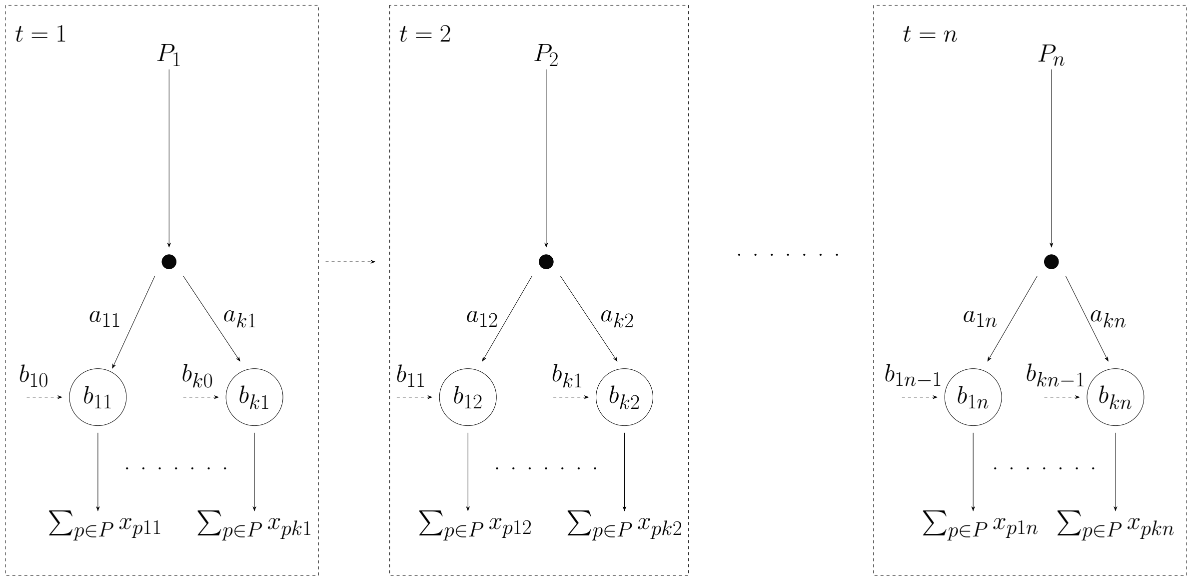

Figure 1: Schematic representation of the product repair and shipment process across multiple repairers and time periods.

3 Results and Discussion

To assess the quality of the proposed model, we tested our method on real data from a company in the province of Prato, Tuscany (Italy). This company's business model is based on the design and production of clothing made exclusively from recycled raw materials. A few months ago, it also launched a clothing repair service, which involves customers opening an online ticket and sending their garments to the company. The dataset contains only 118 repairing requests and two possible repairers (A and B), since it refers to a very short period of time, as this service has only recently been launched. By analyzing the data, it was possible to identify the main categories of defects, which were then used to define the sets on which the model operates.

ID	Description
0	Hole Less Than 1cm
1	Hole 1 To 2cm
2	Hole Greater Than 2cm
3	Ladder
4	Multiple Holes
5	Seam Unstitched
6	Armpit Damage
7	Collar Wear
8	Pulled Thread
9	Pilling
10	Cuff Hem Repair

Table 1: Categorization of possible defects in the data

For the purpose of an initial experimentation, we simulated the arrival of all items within an horizon of one week, by randomly associating a week day (natural number in $\{0, ..., 6\}$) to each product and we also estimated each parameter randomly using expected values that we deduced from the data provided.

Parameter	A	B	Distribution Type
Shipping cost per batch (χ_r^s)	8.0	8.0	Deterministic
Batch capacity (β_r)	15	9	Deterministic
Repairer lead time (λ_r)	12	6	Deterministic
Quality degradation (σ_{dpr})	0.08 (8%)	0.05 (5%)	Deterministic
Unit repair cost (χ_{dpr}^r)	$\mathcal{N}(10,2)+\mathcal{U}(2,10)$	$\mathcal{N}(10,2)$	Normal and Uniform
Carbon emissions per batch (π_r)	8 kg	1.360 kg	Deterministic

Table 2: Distribution of the parameters for each repairer

3.1 Discussion

The model was implemented in Python using the `Gurobi` solver, and solved on an Intel i5 3.2GHz CPU with 32GB RAM. Given the scarcity of data, in this first experimentation we focused to solve the problem with all the coefficients α_1, α_2, α_3, α_4, α_5 equal to 1 and we set the maximum feasible lead time for all the product, τ, equal to 15 days.

Results reported in Table 3 indicate that the MIP formulation effectively balances operational and sustainability goals. In this first analysis we are able to observe that the model is able to provide an high-quality solution with respect to all the components of the objective. The maximum lead time for each product has been minimized to 13 days, thus improving by 2 days the maximum lead time constraint. Moreover the total shipping cost has been necessarily forced to 88€ (11 shipments) saying that, with a less number of shipments, the model can't be feasible on the given data. Finally, note that the emission component is clearly correlated to the number of shipments but in this case, the repairer with the most convenient repair cost is also the one nearest to the company i.e. the one who can minimize the carbon emission every time it is possible thus, this two objective components are not adversaries in this case.

Component	Value
Maximum lead time (days)	13
Total shipping cost (€)	88
Avg quality degradation per product (%)	6.4
Total repair cost (€)	1397.90
Total carbon emissions (kg)	41.52

Table 3: Objective Function Components with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1.0$

Figure 2 summarises the system behaviour generated by the proposed optimisation model over a representative week. The upper panel reports the daily arrivals of defective fashion products, revealing substantial variability across the horizon. This exogenous pattern constitutes the demand stream that the model must allocate to repairers while jointly determining batching and shipment decisions.

The central panel illustrates the evolution of baskets for each repairer. Since repairers operate under fixed batch-capacity limits, the model accumulates incoming products until shipment becomes necessary (full batch or maximum lead time reached for a product in the batch) or convenient. The trajectory shows how the optimizer exploits cross-repairer flexibility: on days with higher inflows, the model tends to saturate the repairer with the minimum repairing cost while additional volumes are directed to the repairer with greater residual capacity. The vertical marker on Sunday highlights the end-of-week inventory position.

The bottom panel reports the batch shipments scheduled by the model. Here the timing decisions arise from the trade-off between efficiency and service-level constraints. The optimizer dispatches full batches to the most convenient repairer each time that it is possible (as observed on each day), but also releases smaller batches to the the other repairer when further accumulation would violate the allowed lead time.

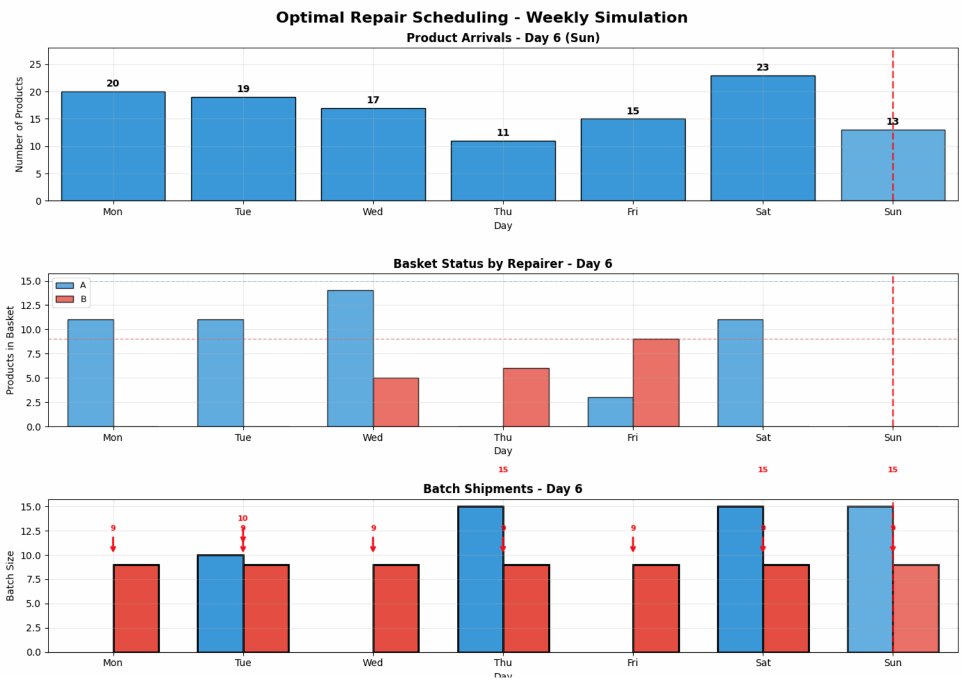

Figure 2: Screenshot of an animated simulation of the optimal repair planning on the given data

4 Conclusions and Future Work

This study introduces a comprehensive MIP model for optimizing repair operations in a multi-repairer reverse logistics network. The model integrates economic, quality, and environmental considerations into a unified optimization framework. Preliminary experiments show promising results in balancing multiple performance metrics while maintaining feasibility under realistic capacity constraints.

Future research will extend this work in two directions:

1. Obtaining or simulating a large-scale dataset to develop a detailed sensitivity analysis to compare different optimal solutions varying α parameters;

2. Developing a *Rolling Horizon* approach for dynamic rescheduling under uncertain product arrivals;

3. Integrating stochastic repair durations and data-driven estimation of defect probabilities.

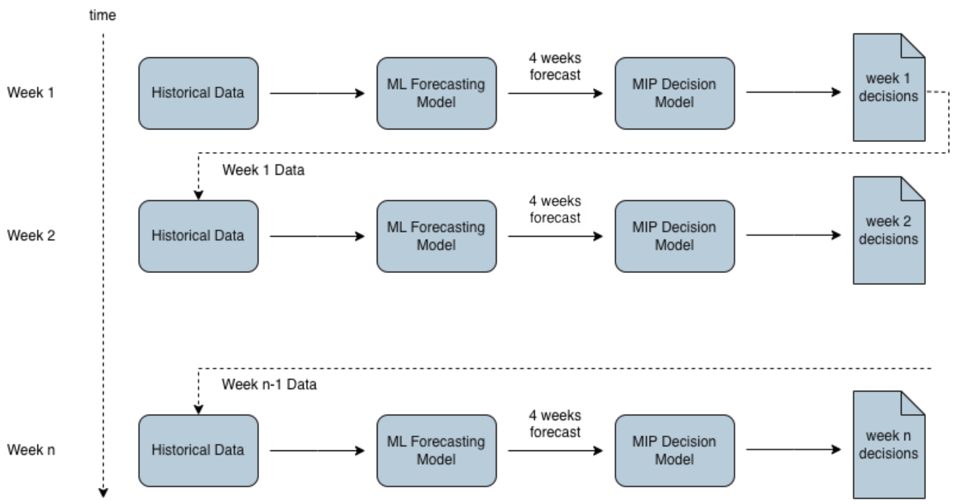

Figure 3: Conceptual representation of the Rolling Horizon decision-making process for dynamic repair planning.

5 MIP model without demand forecasting

In this case the proposed MIP formulation integrates assignment, batching, and shipment scheduling decisions required to operate a distributed textile repair network but data contains just the set of products already arrived at the warehouse. Thus, we are not aware of the demand in the future days in the planning horizon and we must take decision with only these given information.

The following sets and parameters are given:

- R — set of repairers.
- D — set of defect types.
- P — set of products arrived in stock.
- T — set of time periods in the planning horizon.
- D_p — defects associated with product p.

- χ^r_{dpr} — unit repair cost of defect d on product p when repaired by r.
- τ — maximum lead-time admitted for all products.
- τ_p — number of days product p has been in stock
- β_r — maximum batch capacity for repairer r.
- λ_r — lead time of repairer r.
- $\bar{\lambda}$ — minimum lead time among all repairers.
- σ_{dpr} — percentage of quality reduction caused by repair of defect d in p by r.
- χ^s_r — shipping cost per batch (round trip) for repairer r.
- π_r — carbon emissions (CO_2 [g]) per batch shipment to repairer r.

Then the decision variables:

- $u_{dpr} = \mathbb{1}\{\text{product } p \text{ assigned to repairer } r \text{ for defect } d\}$;
- $u_{pr} = \mathbb{1}\{\text{product } p \text{ is assigned to repairer } r\}$;
- $x_{prt} = \mathbb{1}\{\text{product } p \text{ shipped to repairer } r \text{ at time } t\}$;
- $z_{rt} = \mathbb{1}\{\text{batch for repairer } r \text{ shipped at time } t\}$;
- $b_{rt} \in \mathbb{Z}^+$ — products accumulated in repairer r's batch at time t;
- $a_{rt} \in \mathbb{Z}^+$ — products newly assigned to repairer r's batch at time t;
- $l \in \mathbb{R}^+$ — total lead time.

5.1 Mathematical Formulation

Since each repairer can process at most one batch of limited size per time slot, batch capacity is enforced through

$$b_{rt} \leq \beta_r \qquad \forall r \in R,\ \forall t \in T, \tag{1}$$

which guarantees that no batch assigned to repairer r exceeds its maximum admissible size. Whenever a batch is full this event must trigger the shipment. This logic is encoded through

$$b_{rt} - \beta_r + 1 \leq z_{rt} \qquad \forall r \in R,\ \forall t \in T, \tag{2}$$

ensuring that each time $b_{rt} = \beta_r$ we have also $z_{rt} = 1$.

Shipment decisions are represented by the binary variable x_{prt}, which indicates whether product p assigned to repairer r is shipped at time t. The shipment of a product $p \in P$ to repairer $r \in R$ at time $t \in T$ can occur only if the whole batch of repairer r is sent at time t. This can be enforced by

$$x_{prt} \leq z_{rt} \qquad \forall p \in P,\ \forall r \in R,\ \forall t \in T \tag{3}$$

so that no product departs independently of its batch. Conversely, whenever a batch is activated, it must contain at least one shipped product; this prevents "empty" batches and is expressed by

$$z_{rt} \leq \sum_{p \in P} x_{prt} \qquad \forall r \in R,\ \forall t \in T \tag{4}$$

The dynamics of batch accumulation follow a flow-balance logic. At the beginning of the horizon $t = 0$, we suppose that each basket is empty i.e. $b_{r-1} = 0$ for each r. Then the number of items accumulated equals the previous stock plus newly assigned products a_{rt}, minus the products shipped in that period. This is formalized as

$$b_{rt} = b_{r,t-1} + a_{rt} - \sum_{p \in P} x_{prt} \qquad \forall r \in R,\ \forall t \in T \tag{5}$$

and with the constraint:

$$b_{r-1} = 0 \qquad \forall r \in R \tag{6}$$

As a consistency requirement, in any period the number of shipped products cannot exceed the number already accumulated, giving rise to

$$\sum_{p \in P} x_{prt} \leq b_{r,t-1} + a_{rt} \qquad \forall r \in R,\ \forall t \in T \tag{7}$$

Even when enough products are available, shipments cannot surpass the batch capacity β_r, and thus

$$\sum_{p \in P} x_{prt} \leq \beta_r \qquad \forall r \in R,\ \forall t \in T \tag{8}$$

During the period, at all the products in stock must be sorted into repairer batches:

$$\sum_{t \in T} \sum_{r \in R} a_{rt} = |P| \tag{9}$$

and each product must be uniquely assigned to one repairer,

$$\sum_{r \in R} u_{pr} = 1 \qquad \forall p \in P \tag{10}$$

The previous flows are able to connect the variables by noting that the total number of products assigned to repairer r across the entire horizon must coincide with the cumulative inflow to the associated basket

$$\sum_{t \in T} a_{rt} = \sum_{p \in P} u_{pr} \qquad \forall r \in R \tag{11}$$

Once assigned, each product can be shipped at most once. This condition binds the assignment and shipment variables:

$$\sum_{t \in T} x_{prt} \leq u_{pr} \qquad \forall p \in P, \ \forall r \in R \tag{12}$$

Since each product may exhibit multiple defects, the defect-level assignment variable u_{dpr} must be consistent with the product-level assignment u_{pr}. The following constraint enforces that a product is assigned to repairer r if and only if all its defects are:

$$|D_p| \, u_{pr} = \sum_{d \in D_p} u_{dpr} \qquad \forall p \in P, \ \forall r \in R \tag{13}$$

Service-level agreements require that repaired items return within a maximum admissible lead time τ. For this reason, if a product is not shipped during the horizon T it can't violate the constraint:

$$\tau_p + \bar{\lambda} + \sum_{r \in R} u_{pr}(\lambda_r - \bar{\lambda}) + (1 - \sum_{r \in R} \sum_{t \in T} x_{prt})|T| + \sum_{r \in R} \sum_{t \in T} t x_{prt} \leq \tau \qquad \forall p \in P \tag{14}$$

i.e., for a product p that has already been in stock for τ_p days and shipped at time t to the repairer r the completion time is constrained to $t + \lambda_r + \tau_p \leq \tau$ and, if the model decides not to send the product, we have to be sure that even with the minimum lead time $\bar{\lambda}$ the total time passed for the product is not greater than the maximum admitted. Thus, in case $u_{pr} = 0 \ \forall r$ and $x_{prt} = 0 \ \forall r, \ \forall t$ we have to impose: $\tau_p + \bar{\lambda} + |T| \leq \tau$, saying that even if the product remains in stock it does not violate the maximum lead time admitted.

Finally, the variable l represents the maximum lead time across all products and is defined through:

$$l \geq \sum_{t \in T} x_{prt}(t + \lambda_r + \tau_p) \qquad \forall p \in P, \ \forall r \in R \tag{15}$$

The final aim is to minimize an objective function that is a weighted combination of total lead time, shipment cost, quality deterioration, repair costs and environmental impact.

$$\alpha_1 l + \alpha_2 \sum_{t \in T} \sum_{r \in R} \chi^s_r z_{rt} + \alpha_3 \sum_{d \in D_p} \sum_{p \in P} \sum_{r \in R} \sigma_{dpr} u_{dpr} + \alpha_4 \sum_{d \in D_p} \sum_{p \in P} \sum_{r \in R} \chi^r_{dpr} u_{dpr} + \alpha_5 \sum_{t \in T} \sum_{r \in R} \pi_r z_{rt} \tag{16}$$

where coefficients $\alpha_1, \ldots, \alpha_5$ represent the relative importance of each component.

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