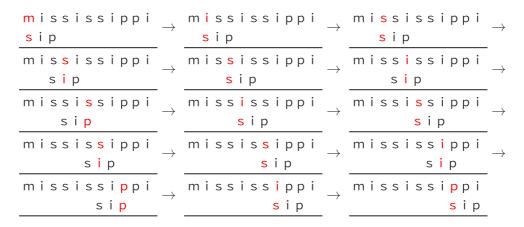
# Sprout 2020 Algorithm - Week 4

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# **Problem 1**

(a)



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# (b)

令 A 為  $aa\cdots ax\cdots xx$  ,其中 x 為任意字元,共  $10^3$  個,每個皆不相同,長度  $len(A)=10^6$  , B 為  $aaa\cdots aab$  ,長度  $len(B)=5*10^5$ 

因  $B \notin A$  且  $\forall i \in \mathbb{N}, i + len(B) - 2 < len(A) - 1000$ :  $B_{1,len(B)-1} = A_{i,i+len(B)-2}$ , 因此共有 499,001 次匹配都需要比較 len(B) 次

得至少比較  $len(B) * 499,001 = 250,000,500,000 > 10^9$  次

## (c)

令 A 為  $ab\cdots abx\cdots xx$  ,其中 x 為任意字元,共  $10^3$  個,每個皆不相同且不為 b 或 a ,長度  $len(A)=10^6$  , B 為  $ab\cdots abac$  長度  $len(B)=5*10^5$ 

因  $B \notin A$  且當 i 為奇數時  $\forall i \in \mathbb{N}, i + len(B) - 2 < len(A) - 1000: B_{1,len(B)-1} \in A_{i,i+len(B)-2}$ ,因此共有 249,501 次匹配需要比較 len(B) 次

得至少比較  $len(B) * 249,501 = 124,750,500,000 > 10^9$  次

### **Problem 2**

### (a)

有顏色者為目前處理區間,橘色為將左右子區間合併完成之狀態

```
 \begin{array}{l} [\mathbf{1}\,\mathbf{8}\,\mathbf{5}\,\mathbf{3}\,\mathbf{2}\,\mathbf{6}\,\mathbf{4}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{3}\,\mathbf{5}\,\mathbf{8}\,\mathbf{2}\,\mathbf{6}\,\mathbf{4}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{3}\,\mathbf{5
```

```
有顏色者為目前處理區間,藍色為 pivot ,pivotIndex = (n-1)/2 ,橘色為已分完堆之數字
```

```
 \begin{array}{l} [\mathbf{1}\,\mathbf{8}\,\mathbf{5}\,\mathbf{3}\,\mathbf{2}\,\mathbf{6}\,\mathbf{4}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{8}\,\mathbf{5}\,\mathbf{7}\,\mathbf{2}\,\mathbf{6}\,\mathbf{4}\,\mathbf{3}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{5}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{3}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to \\ [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{2}\,\mathbf{1}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{7}\,\mathbf{8}\,\mathbf{6}\,\mathbf{4}\,\mathbf{5}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{8}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{8}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{8}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{8}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{8}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{7}\,\mathbf{8}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf{4}\,\mathbf{5}\,\mathbf{6}\,\mathbf{7}] \to [\mathbf{1}\,\mathbf{2}\,\mathbf{3}\,\mathbf
```

(c)

藍色為目前處理區間,由左至右依序放入 bucket ,橘色為從 bucket 取出後之狀態

```
[15\ 27\ 35\ 17\ 36\ 28\ 16] 
ightarrow
0: [], 1: [], 2: [], 3: [], 4: [], 5: [], 6: [26], 7: [], 8: [], 9: []
[27\ 35\ 17\ 36\ 28\ 16] \rightarrow
0: [], \ 1: [], \ 2: [], \ 3: [], \ 4: [], \ 5: [15], \ 6: [26], \ 7: [], \ 8: [], \ 9: []
[35\ 17\ 36\ 28\ 16] 
ightarrow
0: [1, 1: [1, 2: [1, 3: [1, 4: [1, 5: [15], 6: [26], 7: [27], 8: [1, 9: [1, 4: [1, 5: [15], 6: [26], 7: [27], 8: [1, 9: [1, 5: [15], 6: [26], 7: [27]]]]
[17\ 36\ 28\ 16] \to
0: [1, 1: [1, 2: [1, 3: [1, 4: [1, 5: [15 35], 6: [26], 7: [27], 8: [1, 9: [1, 4: [1, 5: [15 35], 6: [26], 7: [27], 8: [1, 9: [1, 5: [1, 5: [1, 5: [2, 5], 6: [2, 5], 7: [2, 5]]]]]
[36\ 28\ 16] 
ightarrow
0: [], 1: [], 2: [], 3: [], 4: [], 5: [15 35], 6: [26], 7: [27 17], 8: [], 9: []
[28\ 16] 
ightarrow
0: [1, 1: [1, 2: [1, 3: [1, 4: [1, 5: [15 35], 6: [26 36], 7: [27 17], 8: [1, 9: [1, 4: [1, 5: [15 35], 6: [26 36], 7: [27 17]]]]
[16] \rightarrow
0: [1, 1: [1, 2: [1, 3: [1, 4: [1, 5: [15 35], 6: [26 36], 7: [27 17], 8: [28], 9: [1, 4: [1, 4: [1, 5: [26 36], 7: [27 17], 8: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [1, 4: [1, 5: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: [28], 9: 
0: [], 1: [], 2: [], 3: [], 4: [], 5: [15 35], 6: [26 36 16], 7: [27 17], 8: [28], 9: []
[15 35 26 36 16 27 17 28]
[35\ 26\ 36\ 16\ 27\ 17\ 28] 
ightarrow
0: [], 1: [15], 2: [], 3: [], 4: [], 5: [], 6: [], 7: [], 8: [], 9: []
[26\ 36\ 16\ 27\ 17\ 28] 
ightarrow
0: [], 1: [15], 2: [], 3: [35], 4: [], 5: [], 6: [], 7: [], 8: [], 9: []
[36\ 16\ 27\ 17\ 28] \rightarrow
0: [], \ 1: [15], \ 2: [26], \ 3: [35], \ 4: [], \ 5: [], \ 6: [], \ 7: [], \ 8: [], \ 9: []
[16\ 27\ 17\ 28] 
ightarrow
0: [], 1: [15], 2: [26], 3: [35 36], 4: [], 5: [], 6: [], 7: [], 8: [], 9: []
[27\ 17\ 28] 
ightarrow
0: [], 1: [15\ 16], 2: [26], 3: [35\ 36], 4: [], 5: [], 6: [], 7: [], 8: [], 9: []
[17\ 28] 
ightarrow
0: [], \ 1: [15\ 16], \ 2: [26\ 27], \ 3: [35\ 36], \ 4: [], \ 5: [], \ 6: [], \ 7: [], \ 8: [], \ 9: []
[28] \rightarrow
0: [], 1: [15\ 16\ 17], 2: [26\ 27], 3: [35\ 36], 4: [], 5: [], 6: [], 7: [], 8: [], 9: []
0: [], 1: [15\ 16\ 17], 2: [26\ 27\ 28], 3: [35\ 36], 4: [], 5: [], 6: [], 7: [], 8: [], 9: []
```

[15 16 17 26 27 28 35 36]

# **Problem 3**

(a)

### 1. Merge sort

首先考慮合併時,若一子區間中有相同元素,必為連續出現,又因為若左右子區間內有相同元素,會優先放入屬於左子區間的,且同一區間中之元素也是由左而右放入新序列,因此僅考慮合併前、後時,是 stable ;並且任意長度超過 1 之子區間皆為其子區間合併而成,且長度為 1 之子區間是 stable ,得任意子區間對於排序前之原序列皆為 stable

### 故 Merge sort 為 stable

### 2. Quick sort

考慮一序列[33'4],因 pivot 之選取為隨機,因此以下排序過程為可能的:

$$[3\ 3'\ 4] \rightarrow [4\ 3'\ 3] \rightarrow [3'\ 4\ 3] \rightarrow [3'\ 3\ 4]$$

得一反例證明 Quick sort 並非 stable

### 3. Radix sort

由於元素由左至右放入其 *bucket* 之末端,因此相同元素放入 *bucket* 之操作為 **stable** ;提出來時是由 *bucket* 之前端提出,放入序列末端,因此相同元素提出 *bucket* 之操作也是 **stable** 

### 得 Radix sort 為 stable

(b)

在 Data 中使用一變數 pos ,計為該元素在序列中的位置,比較時若值相同則比較 pos ,這樣相同值 之元素間排序前兩元素 pos 較小者會位於右側,排序後仍 pos 較小者仍會位於右側,故為 stable