Modeling Rare Events

Over/Undersampling, Priors, Decision Weights

Undersampling/ Oversampling and Prior Probabilities

Can be accounted for automatically in SAS EM

Undersampling and Prior Probabilities

- Say you have a rare event as target (<10% of data)
  - Fraud
  - Catastrophic failure
  - 10%+ single day change in value of stock market index

- May have trouble modelling because a model is accurate for classifying everything as nonevent!

- Potential Solution: Create a biased sample

Undersampling and Prior Probabilities

- Potential Solution: Create a biased sample
  - **Undersample:** under-represent common events in training data.
    - Keep all rare events and only a fraction of common events
    - Ratio of Common:Rare events is up for debate.
      - 70:30 ought to be fine.
      - 50:50 is sometimes encouraged.
  - **Oversample:**
    - replicate the rare events in training.
    - do this *after* the training/validation split so don't have the same observation in both training and validation set!
    - OR, use a hybrid technique like **SMOTE** (Chawla, 2002) that creates new data points *like* the rare events (not exact replicates)

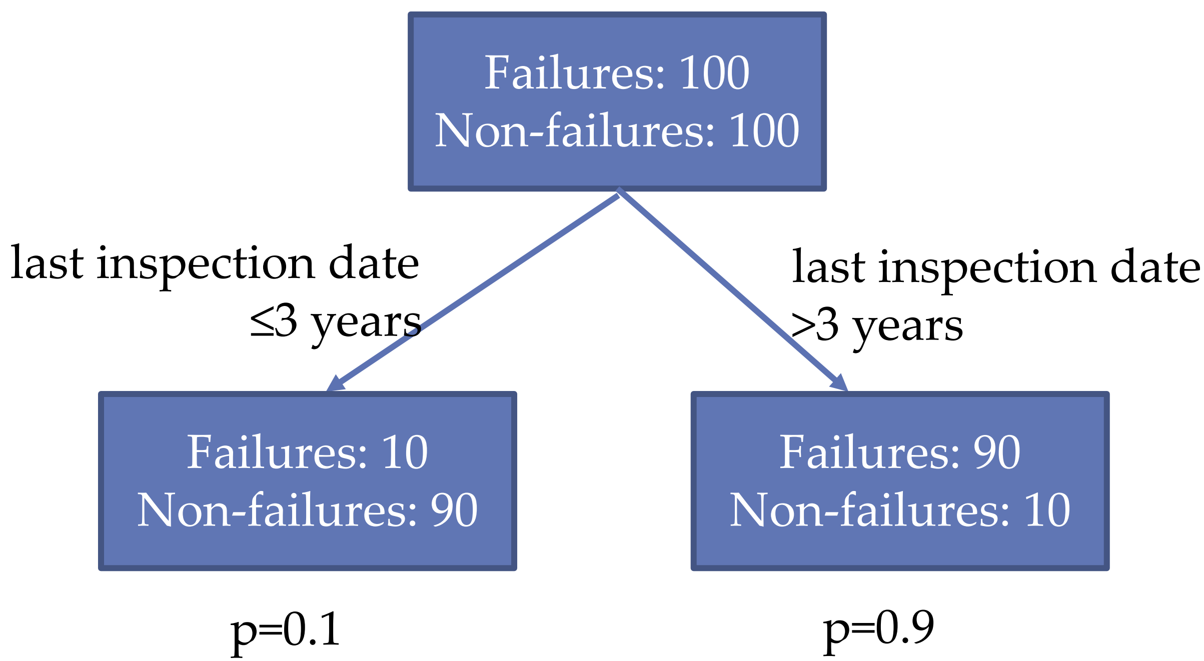
Undersampling and Prior Probabilities

- Models provide **posterior probabilities** for events.
- The accuracy of the posterior probabilities rely on a representative sample.
- If we bias our sample, must adjust the posterior probabilities to account for this.

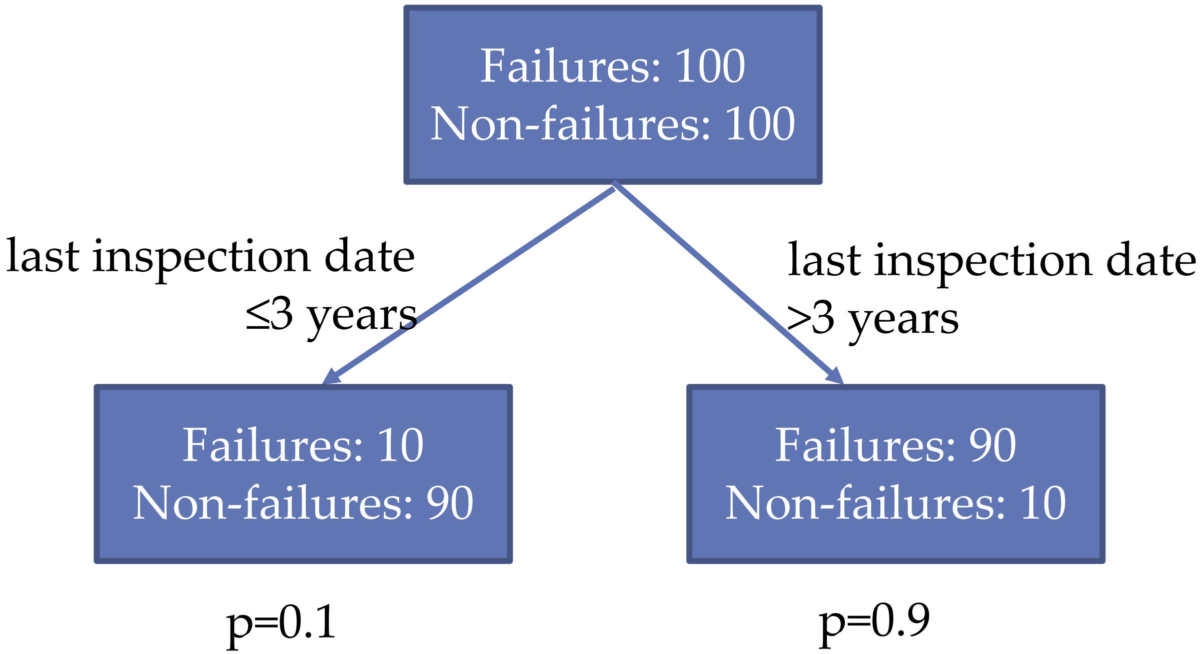
Why Adjustment is Necessary

Predict voting machine failure. Only 100 voting machines failed out of 10,000.

Undersample. Dataset has 100 failures and 100 non-failures.



Why Adjustment is Necessary



Does a new machine with last inspection date >3 years really have a 90% probability of failing?

Why Adjustment is Necessary

- We'd have to go back to the data to answer this question.
- Assuming the 100 non-failures chosen were random, representative sample, we expect inspection date to be ≤ 3 years 90% of the time.
- That is 8,910 non-failing machines with inspection date ≤ 3 years.
- Similarly, 10% of non-failures have expect inspection date >3 years ago. This is 990 machines.

	≤ 3 years	>3 years
Failures	10	90
Nonfailures	8910	990

P(Failure | last inspection date >3 years)
90/(90+990) = 8%
(Still failing at 8 times the rate of recently inspected machines)

Summary: Adjusting for Undersampling

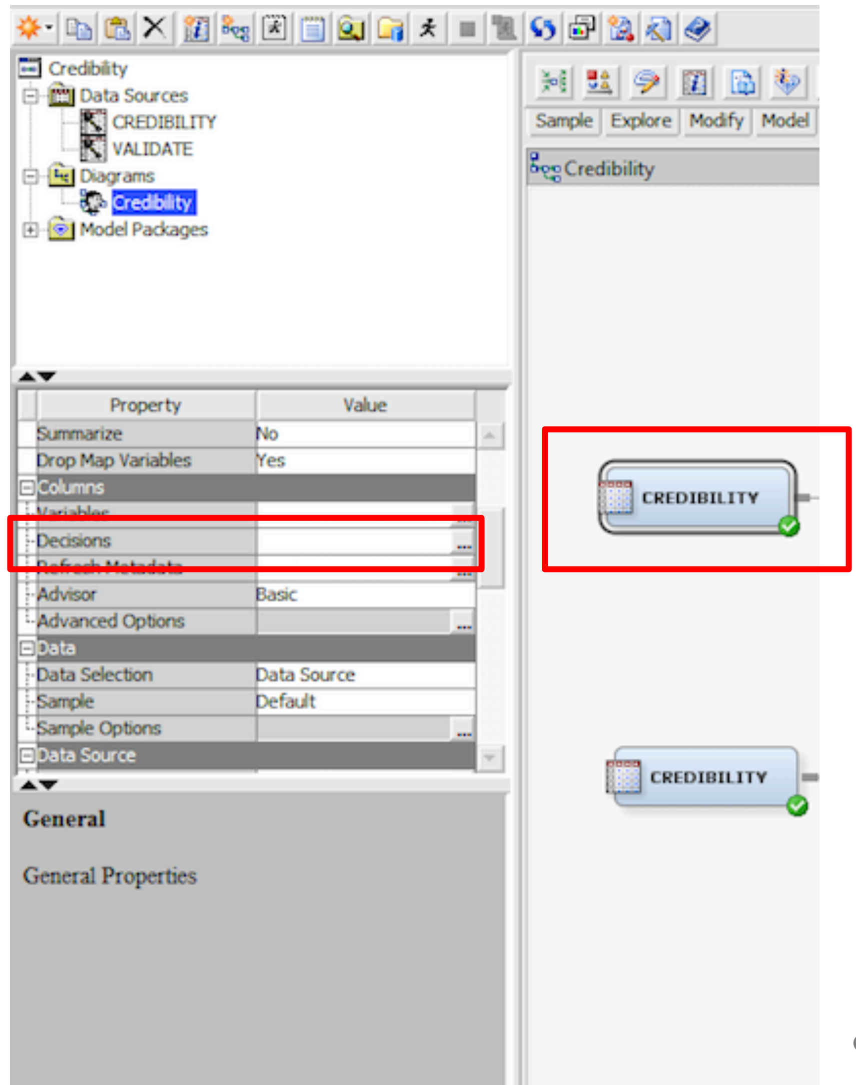
- Let $l = l_1, l_2, \ldots, l_L$ be the levels of the target variable
- Let $i = 1,2, \ldots, n$ index the observations in the data
- Let $OldPost(i,l)$ be the posterior probability from the model on oversampled data
- Let $OldPrior(l)$ be the proportion of target level in the oversampled data
- Let $Prior(l)$ be the correct proportion of target level in true population

$$NewPost(i,l) = \frac{OldPost(i,l)\dfrac{Prior(l)}{OldPrior(l)}}{\sum_{j=1}^{L} OldPost(i,l_j)\dfrac{Prior(l_j)}{OldPrior(l_j)}}$$

Entering **Priors** and Decision **Weights** into SAS EM

Entering Priors into SAS EM

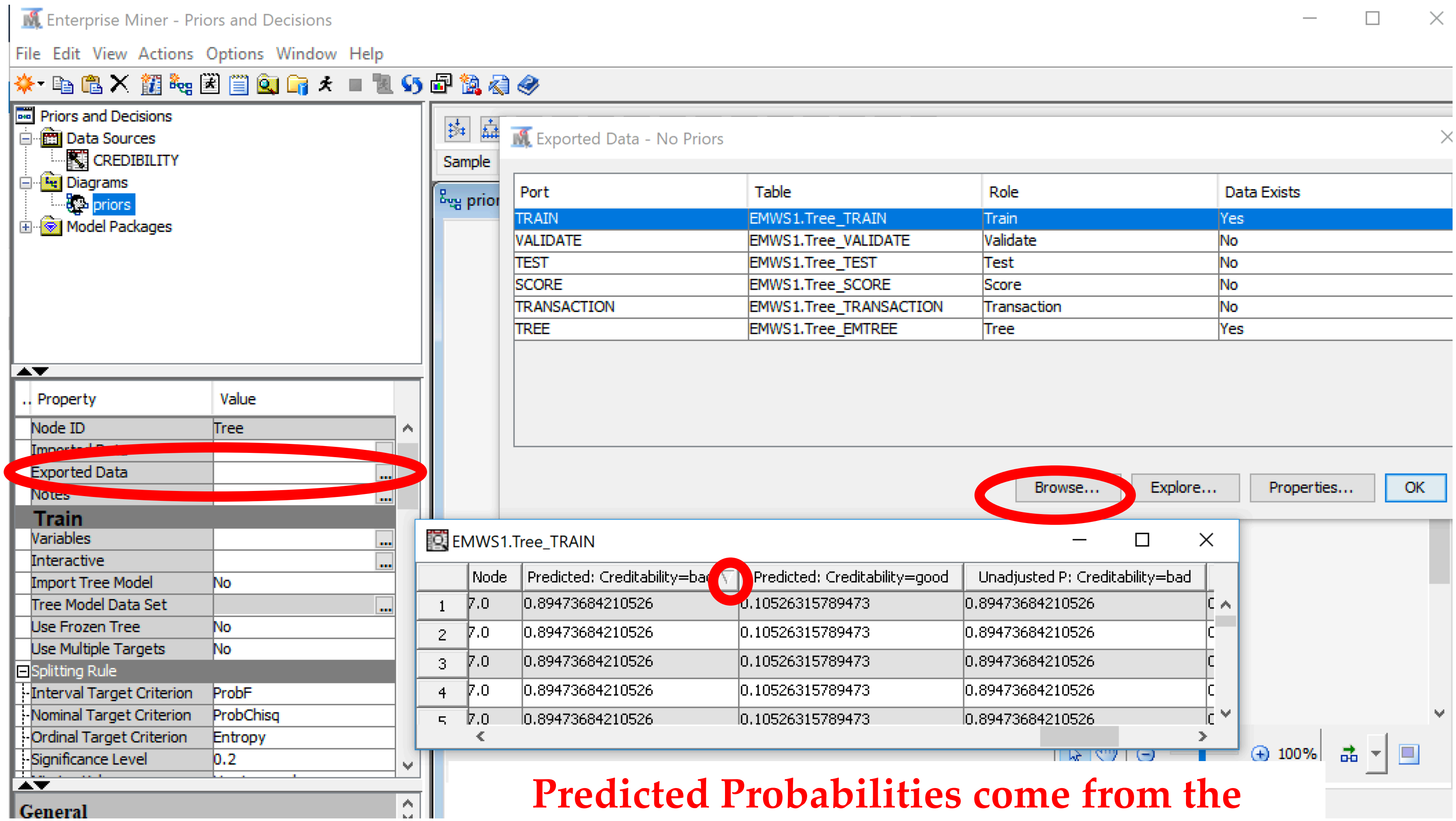
- Priors are also adjusted in the "decisions" on a dataset panel.
- Click "Build" when first opening the prompt, then click priors tab.



Only Some Output Uses the Prior Information

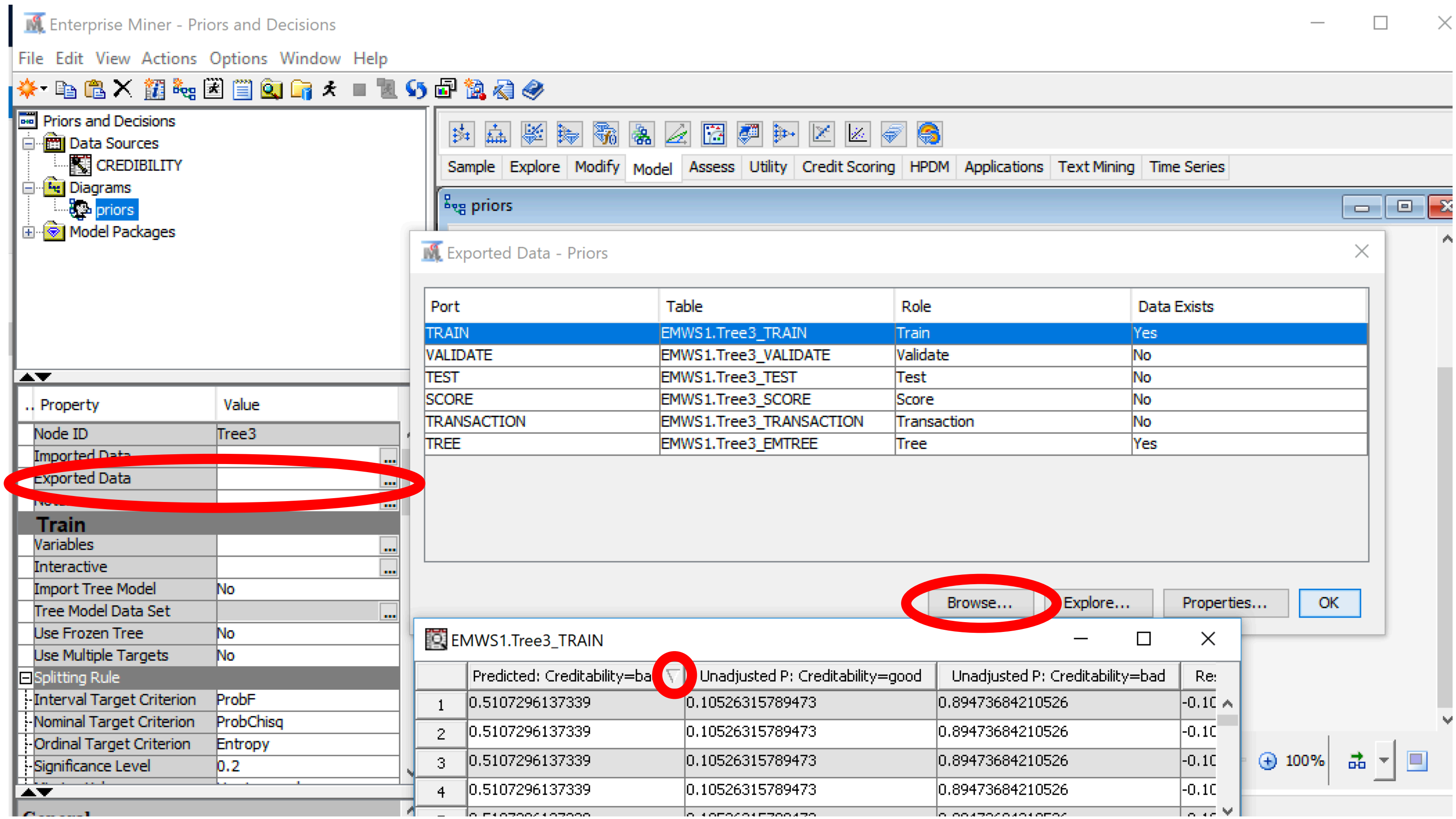
- In SAS EM, accounting for priors *has no effect on*:
  - Growing decision trees
  - Misclassification Rate (The cutoff probability is still 0.5 by default)
- Priors *do affect:*
  - *Pruning* decision trees
  - Once we account for a prior, a given split may not have a reasonable gain
- Net Effects:
  - Increasing a prior probability increases the posterior probability
  - Decreasing a prior decreases the posterior probability
  - Changing prior will have more noticeable effect if the original posterior is near 0.5 than if it is near 0 or 1.

Oversampled Data with No Priors



Predicted Probabilities come from the decision tree as expected

Oversampled Data *with* Priors



Predicted Probabilities are adjusted according to the priors. The tree is pruned according to those adjustments too. Default cutoff probability is still 0.5!

Oversampled Data *with* Priors

EMWS2.Tree2_TRAIN

Predicted: Creditability=good	Predicted: Creditability=bad	Unadjusted P: Creditability=good	Unadjusted P: Creditability=bad	Residual: Creditability=good	Residual: Creditability=bad	Decisio
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	0.89473684210527	-0.89473684210526	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	0.89473684210527	-0.89473684210526	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
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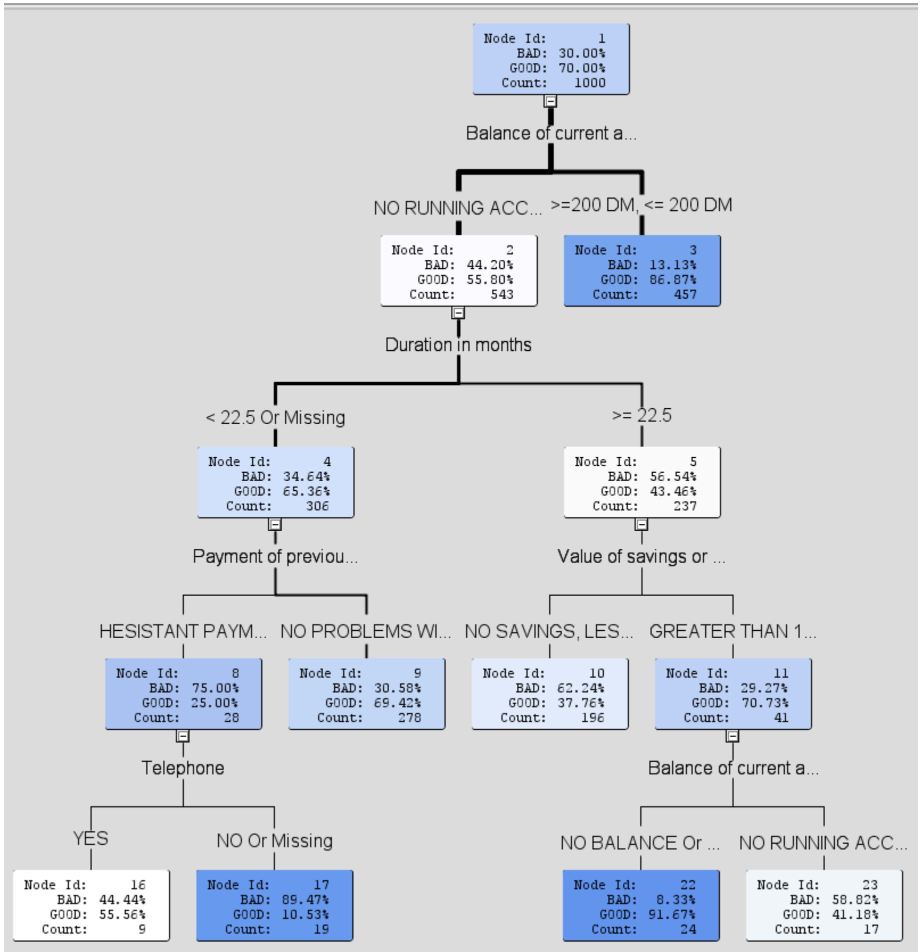
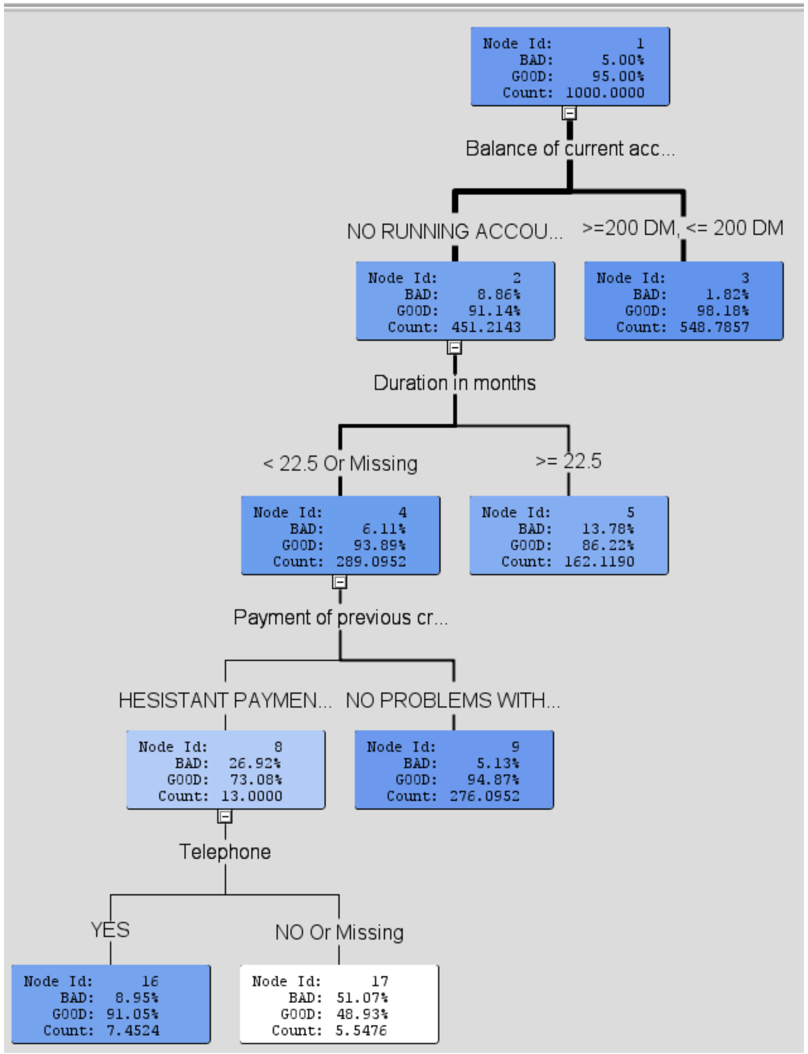
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.48927038626609	0.5107296137339	0.10526315789473	0.89473684210526	-0.10526315789473	0.10526315789473994	BAD
0.86224115141724	0.13775884858275	0.43459915611814	0.56540084388185	0.5654008438818601	-0.56540084388185	GOOD
0.86224115141724	0.13775884858275	0.43459915611814	0.56540084388185	0.5654008438818601	-0.56540084388185	GOOD
0.86224115141724	0.13775884858275	0.43459915611814	0.56540084388185	-0.43459915611814	0.43459915611815003	GOOD
0.86224115141724	0.13775884858275	0.43459915611814	0.56540084388185	0.5654008438818601	-0.56540084388185	GOOD

Default cutoff probability is still 0.5!

Oversampled Data with No Priors



Oversampled Data *with* Priors

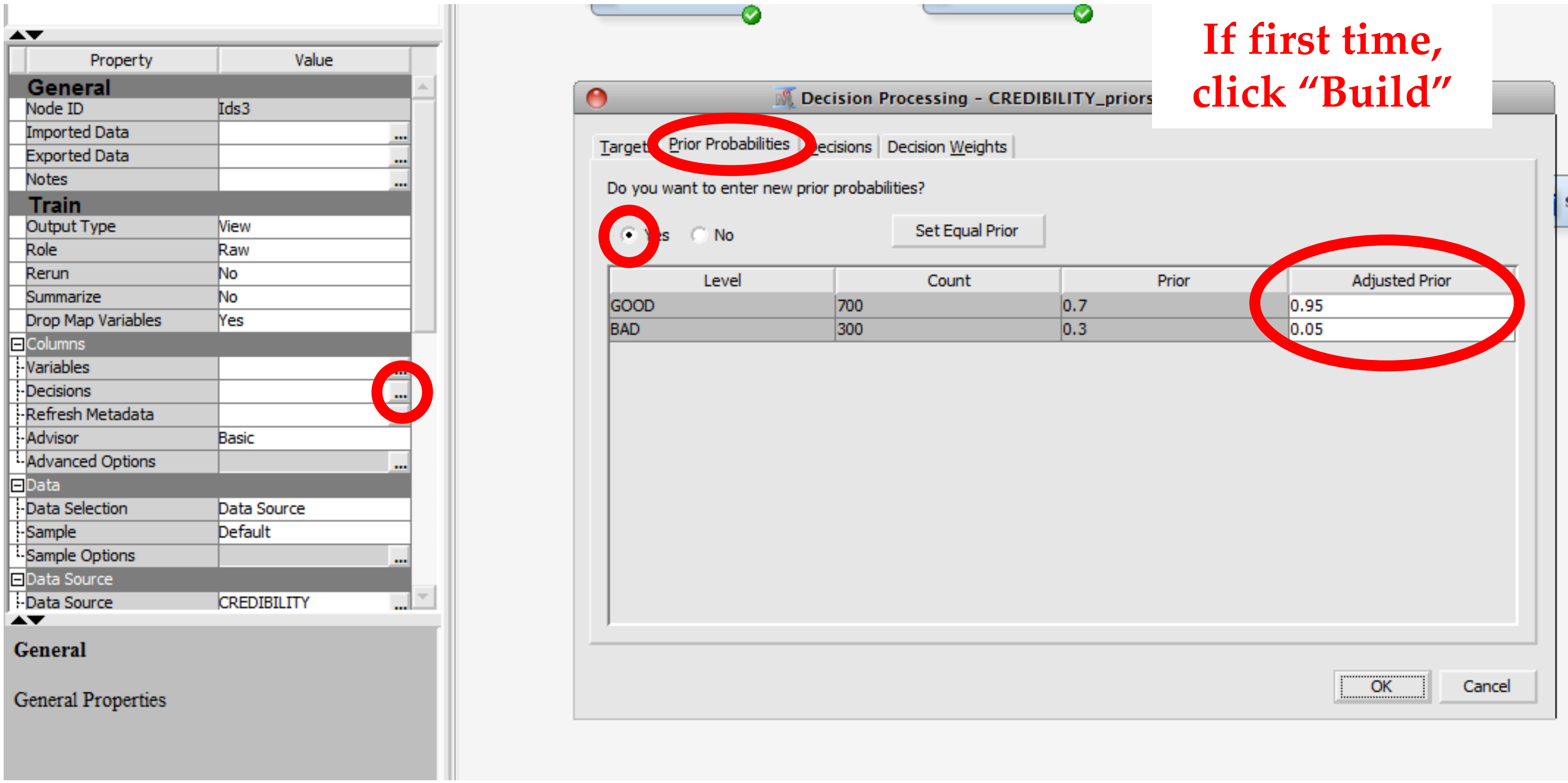


Using Inv. Priors as Decision Weights

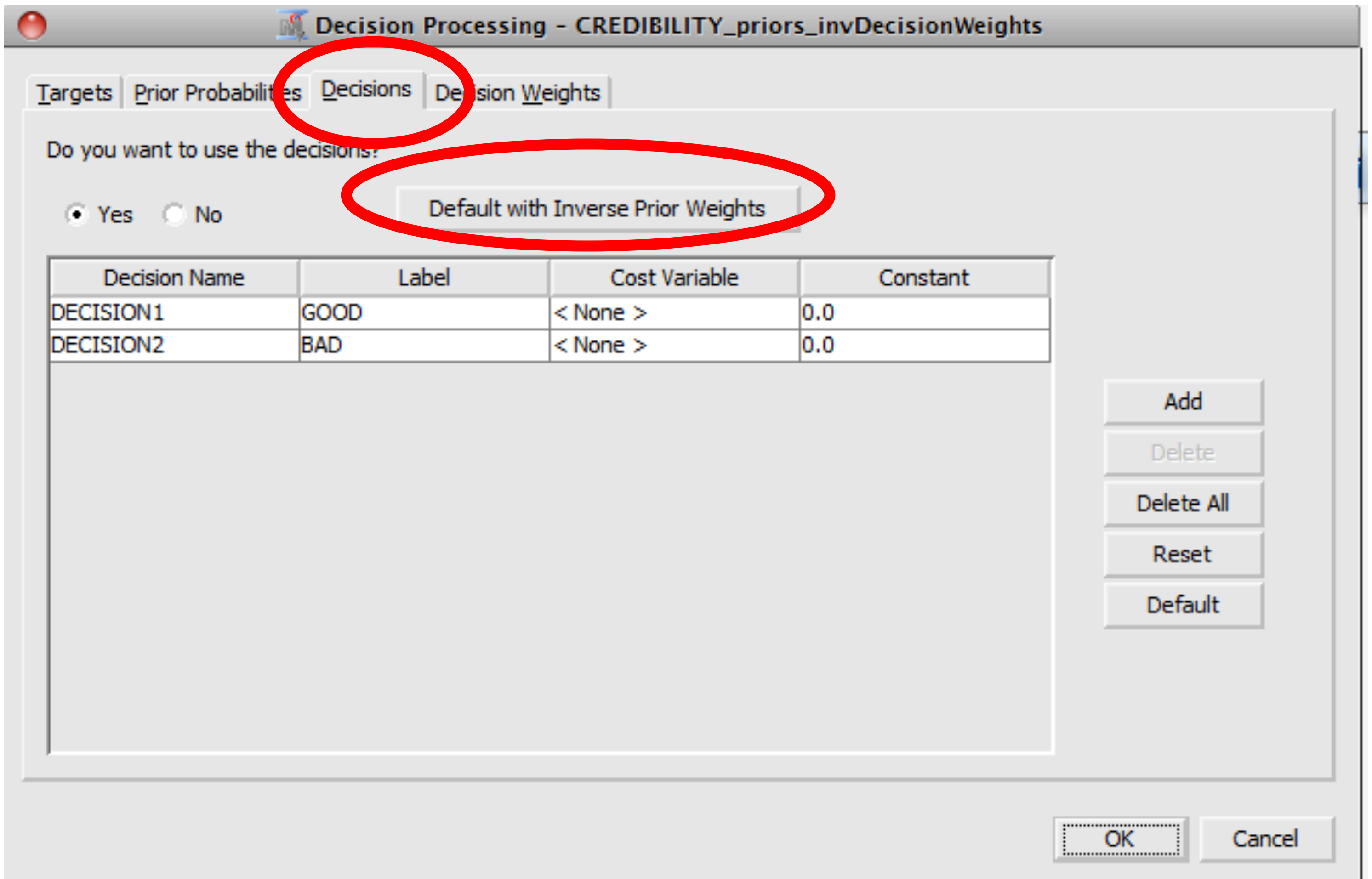
- To emphasize rare events in a modelling context, we may want to increase the "profit" of making a correct prediction of the rare event.
- The easiest way to do this is to weight the decisions with a profit (or cost – make errors negative) matrix:
- Priors: RareEvent = 0.02, CommonEvent = 0.98

Decision Weights to emphasize correct classification of rare events		Predicted	
		RareEvent	CommonEvent
Actual	RareEvent	1/0.02 = **50**	0
	CommonEvent	0	1/0.98 = **1.02**

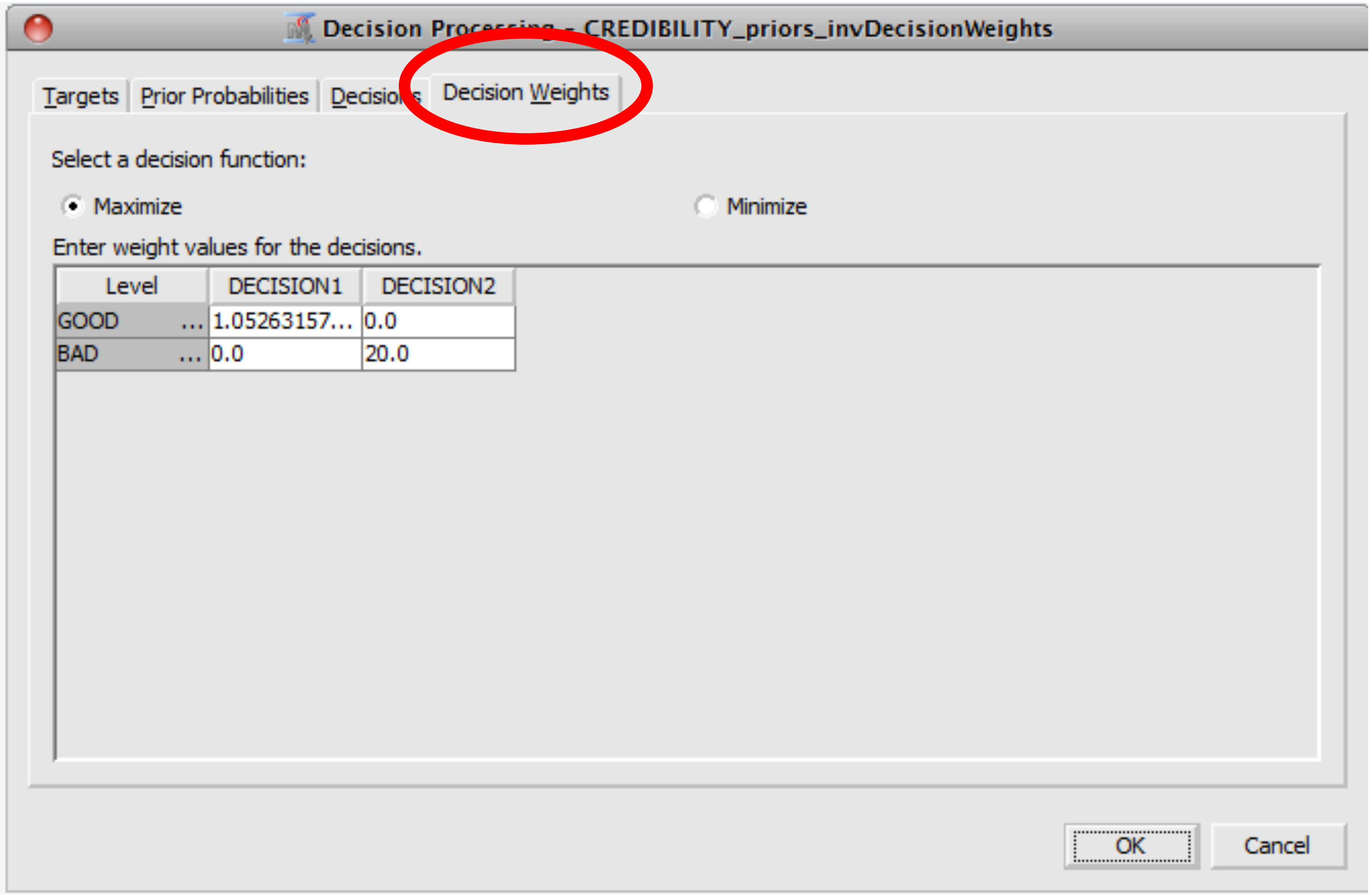
Creating Inv. Prior Decision Weights



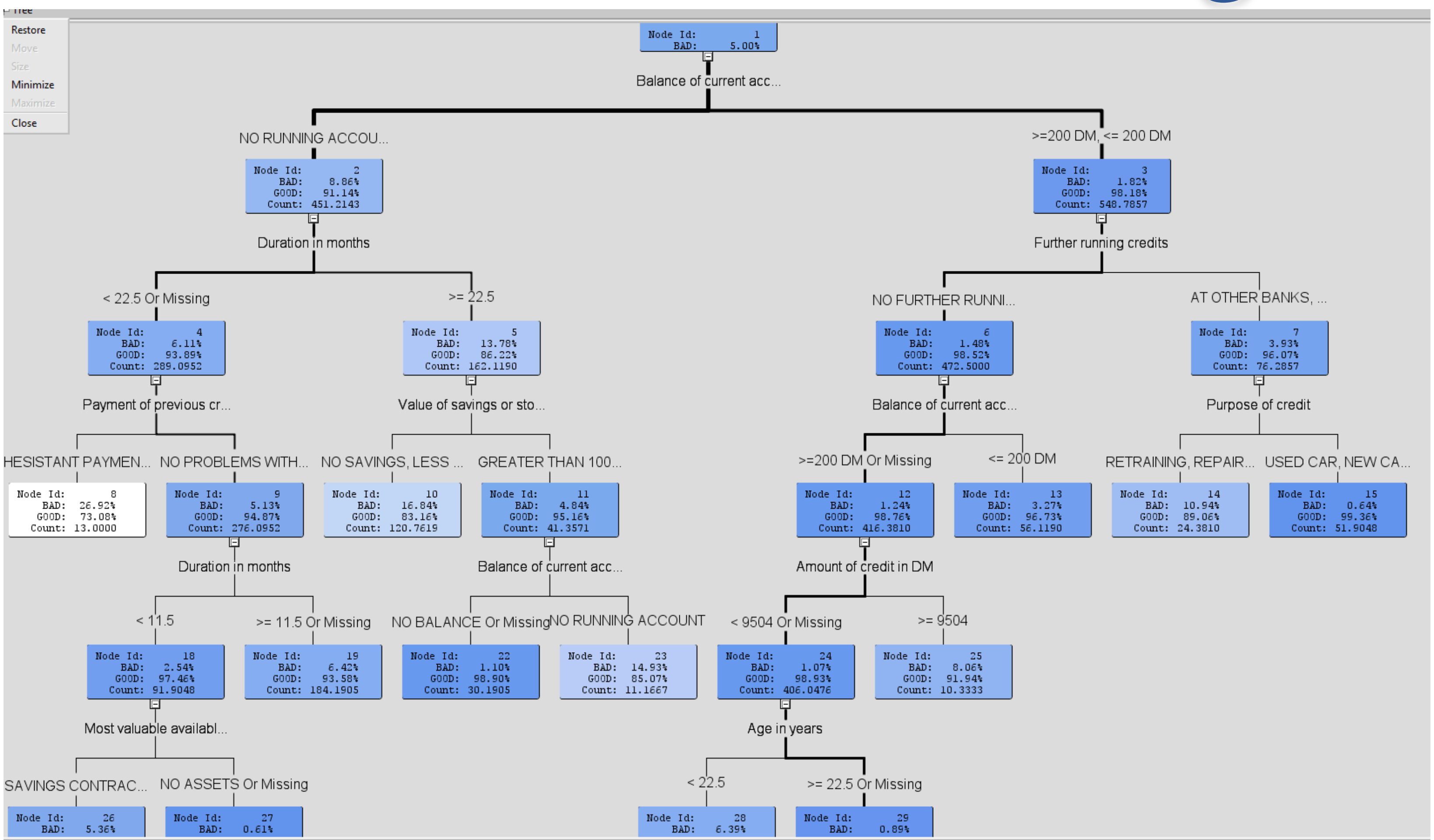
Creating Inv. Prior Decision Weights



Creating Inv. Prior Decision Weights



Oversampled Data with Priors *and* Decision Weights



Oversampled Data with Priors

Predicted: Creditability=bad	Unadjusted P: Creditability=good	Unadjusted P: Creditability=bad	Residual: Creditability=good	Residual: Creditability=bad	Decision: Creditability
0.26923076923076	0.25	0.75	0.75	-0.75	BAD
0.26923076923076	0.25	0.75	-0.25	0.25	BAD
0.26923076923076	0.25	0.75	-0.25	0.25	BAD
0.26923076923076	0.25	0.75	-0.25	0.25	BAD
0.26923076923076	0.25	0.75	-0.25	0.25	BAD
0.26923076923076	0.25	0.75	0.75	-0.75	BAD

Cutoff probability is now the population probability of the rare event, here p=0.05!