Rationale for definitions for 2C negative numbers!

For n-bit #, there's 2^n values.

For every positive # p, define

$-p = \text{flip\_bits\_add\_one}(p)$

Values are: 0, $2^{n-1}-1$ positive #s, $2^{n-1}-1$ negative numbers, totalling $1+2(2^{n-1}-1) = 1+2^n-2 = 2^n-1$ values. We note 10...0 is unused. It begins w/ 1, so define $10\ldots0 = -(2^{n-1})$.

Define:

dec	bin
$-(2^{n-1})$	10...0
$-(2^{n-1}-1)$	$1\underbrace{0\ldots0}_{n-1\ 0s}+1 = 1\underbrace{0\ldots0}_{n-2\ 0s}1$
⋮	
-2	1...101 + 1 = 1...110
-1	1...110 + 1 = 1...111
0	$\underbrace{0\ldots000}_{n\ \text{digits}}$
1	0...001
2	0...010
⋮	⋮
$2^{n-1}-1$	01...1
2^{n-1}	$\underbrace{10\ldots0}_{n\ \text{digits}}$

counting down by 1

*

counting up by 1

$2^{n-1}-1$ negative #s (signed)

Count up from $\boxed{1}0\ldots01$ to $\boxed{1}1\ldots11$. * Remove trailing 1, you get count up from $0\ldots01 = 1$ to $1\ldots11 = 2^{n-1}-1$ (unsigned) which is distance from $-(2^{n-1})$. This proves any negative # can be expressed as $-(2^{n-1}) + cnt$, where $cnt > 0$

$2^{n-1}-1$ positive #s compatible w/ both signed & unsigned

// unsigned binary, reference only (to help convert $2^{n-1}-1$ to binary)

2C signed binary range = -2^{n-1} to $2^{n-1}-1$

* Count up from 0...01 to 01...1, flipped will be count down from 11...11 to 10...01

Proof on next page!

Proof by cases: Count up from 0...01 to 01...1, flipped will be count down from 11...11 to 10...01

Note: X... means keep repeating X

Case 0: 从一开始

Original (count up) 0...01 → 0...10
Flipped: 1...10 → 1...01
Flip-add-one (indeed count down) 1...11 → 1...10

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inductively count down

Case 1: 不进位 (一半的情况)

Original (count up) 0...1X...0 → 0...1X...1
Flipped (indeed count down) 1...0Y...1 → 1...0Y...0

Flip-add-one: if LHS − 1 = RHS (from above), then (LHS+1) − 1 = (RHS+1), indeed count down

Case 2: 从零进位 (另一半的情况)

Original (count up) 0X...01...1 → 0X...10...0
Flipped (indeed count down) 1Y...10...0 → 1Y...01...1

Flip-add-one: if LHS − 1 = RHS (from above), then (LHS+1) − 1 = (RHS+1), indeed count down

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Case 2: 最后的进位

Original (count up) 01...10 → $\underbrace{\text{01...11}}_{\text{End}}$

Flip 10...01 → 10...00
Flip-add-one (indeed count down) 10...10 → 10...01