Reactor Engineering Week 7 Problem Set

Question 1 - A PFR Recycle Stream
The following first-order reaction,

$$\mathrm{A} \xrightarrow{\mathrm{k_1}} \mathrm{B} \tag{1}$$

occurs in the aqueous phase in an isothermal ideal plug flow reactor of volume V. The volumetric flow rate through the reactor is υ. Tim is thinking of splitting a fraction $\xi = R/(R+1)$ of the total liquid flow coming out of the reactor and recycling it back to the reactor entrance (here R is the *recycle ratio*; ξ and R are just two ways of quantifying the same thing.) Is this a good idea? Find an expression for the overall conversion of A as a function of the fraction recycled, ξ. Is there an optimal recycle fraction, or is this just a bad idea?

Question 2 - Alice's PFR
As we are now well aware, Alice desperately wants to be rich. She's starting to wonder if she chose the wrong profession. Determined as ever, she's now reconsidering an old system: a reaction occuring in a plug flow reactor with reaction rate constant given by

$$r_A = -\frac{k}{((X_A - 0.5)^2 + 0.5)}. \tag{2}$$

The inlet concentration is C_{A0}, the reactor volume is V, and the volumetric flow rate is υ. She is wondering about splitting the outlet stream and recycling some fraction of it back to the start of the reactor. Is this a good idea? Use either an analytical or a graphical method to find the optimal recycle ratio.

Question 3 - A Recycle on an MFR
Why don't we consider recycle streams for MFR reactors? Provide either an intuitive or mathematical answer.

Question 4 - A Separator rather than a Splitter
Tim is reconsidering the system we discussed in question 1. He thinks he can get better performance by *separating* out the unused reactant, A, from the product B, and recycling only A back to the reactor inlet. He has purchased a membrane filtration rig, which he will place at the end of the reactor to separate out all the unreacted A and pump it back to the beginning of the PFR reactor. His filter works perfectly: it separates A from B with no trace amounts of either in the incorrect stream. The A being pumped back must be returned with some water: assume that, of all the water entering the filtration rig, the fraction $\xi = R/(R+1)$ is split into the recycle stream, and the remaining fraction $\xi = 1/(R+1)$ goes with the product stream containing pure B. Find an expression for the conversion of A as a function of ξ. Is it larger than in Question 1? Is this reasonable?

Super-Challenging-Fredo-Frog-Challenge-Question

Note: This is a challenging question. Any student who can solve this question within a week is eligible for a Fredo Frog.

*Jeremy has been playing with favourite plug flow reactor, the PFR-*3000®*, and has been conducting the following two-step, dilute, isothermal, aqueous-phase reaction inside it:*

$$\mathrm{A} \xrightarrow{\mathrm{k_1}} \mathrm{B} \tag{3}$$

$$\mathrm{B} \xrightarrow{\mathrm{k_2}} \mathrm{C} \tag{4}$$

The first reaction is first order in A with reaction rate constant k_1, while the second reaction is second order in B with reaction rate constant k_2. The reactor has volume V. At present, his feed stream contains A with concentration C_{A0}, no B or C (so $C_{B0} = C_{C0} = 0$), and has volumetric flow rate v. His goal is to produce as much C as possible, and he's happy with how his rig is doing. However, his friend Wendy has come around, and she's suggested he might be able to get better performance by separating *out the B product and recycling it back into the reactant stream. He has built a little membrane filtration rig that separates B from A and C. The rig works perfectly - it creates one stream with only B and no A or C, and a second stream containing only A and C and no B. The separation rig always splits the stream so that some fraction $\xi = R/(R+1)$ of the water goes with the B to create a recycle stream, while the remaining fraction $1-\xi = 1/(R+1)$ of the water goes with the products. Because the reactants and products are dilute, it may be assumed that the water in the stream makes up effectively all of its volume.*

Preliminary Question: *If all the B in the outlet stream were separated out and recycled back into the reactor entrance, find an expression for the yield of C, C_C^{out}/C_{A0}, where C_C^{out} is the concentration of C in the product stream. Compare this with the plug flow reactor without recycle. Does the recycle improve the yield of C?*

Secondary Question: *Jeremy has realised that it may not be ideal to recycle the B stream all the way back to the start of the reactor: this may dilute the A and slow down the reaction $A \to B$. Instead, Jeremy plans to mix in the recycle stream some distance down the reactor, after the first input stream containing only A has travelled through a volume of $V' < V$. How far down the reactor should Jeremy place his recycle stream in order to maximise the yield of C?*

Solutions

Question 1

We know from the notes that, for a PFR recycle reactor,

$$\frac{\tau}{C_{A0}} = (R+1)\int_{RX_f/(R+1)}^{X_f} \frac{dX_A}{-r_A} \tag{5}$$

or, for $-r_A = k_1 C_{A0}(1 - X_A)$,

$$\frac{\tau}{C_{A0}} = (R+1)\int_{RX_f/(R+1)}^{X_f} \frac{dX_A}{k_1 C_{A0}(1 - X_A)} \tag{6}$$

If we take $R > 0$ as a variable, it is clear from the graphical interpretation discussed in your notes that R is maximised when $R = 0$. Hence, for simple reactions like this, a splitting-based recycle is a bad idea.