22.1.6

Most graph algorithms that take an adjacency-matrix representation as input require time $\Omega(V^2)$, but there are some exceptions. Show how to determine whether a directed graph G contains a ***universal sink***—a vertex with in-degree $|V| - 1$ and out-degree 0—in time $O(V)$, given an adjacency matrix for G.

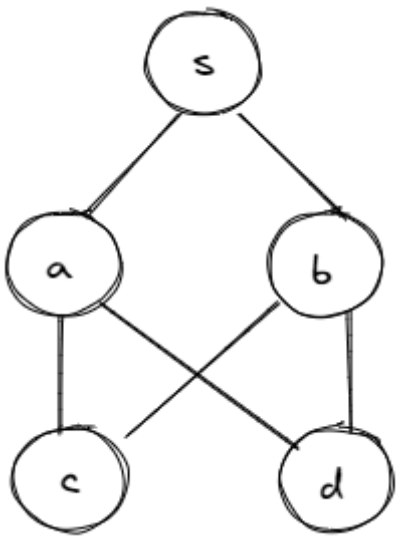
从邻接矩阵的左上角$M[1][1]$开始扫描，若$M[i][j]$为0，则扫描$M[i][j+1]$，若为1，则扫描$M[i+1][j]$。当i或j为V时，停止扫描，若该图存在通用汇点，则一定为i。

该算法的正确性可由通用汇点的性质得到。通用汇点所在的行的值一定是全为0的，所在列的值除了其本行的值为0，其他都为1。因此可以得到，在查找过程中，最后一定会落在通用汇点所在的行。总复杂度为$O(V)$。

22.2.6

Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$, and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.

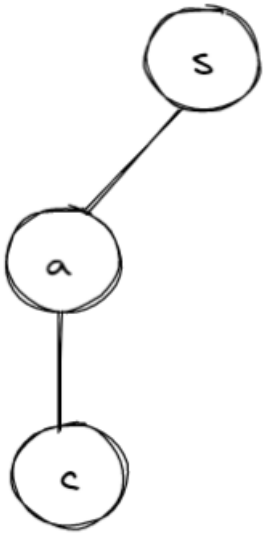
对于本题，可以通过首先构造一个广度优先树，再添加一些额外的边来完成。如下图：



22.3.9

Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, then any depth-first search must result in $v.d \leq u.f$.

考虑对下图从a开始按a->c->a->s的顺序运用dfs，则对路径c->s有
$s.d = 4 \geq c.f = 3$，构成了反例。

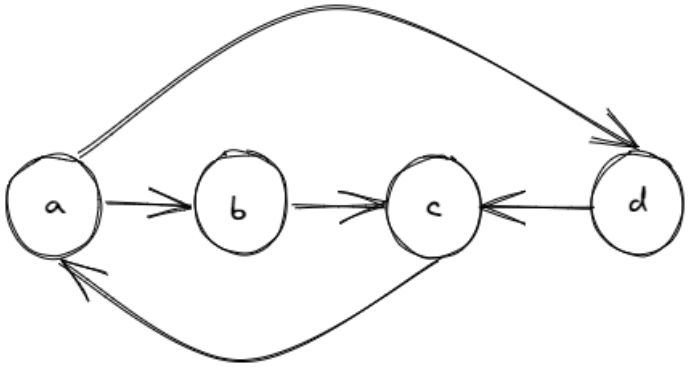


22.4.4

Prove or disprove: If a directed graph G contains cycles, then TOPOLOGICAL-SORT(G) produces a vertex ordering that minimizes the number of "bad" edges that are inconsistent with the ordering produced.

这种说法是不对的。考虑下图：

假设拓扑排序算法的dfs从节点c开始，可能得到的一种结果为c a d b，这样产生了两条坏边：b->c和d->c。若考虑序列a b d c，则只会产生一条坏边：c->a。因此拓扑排序算法不能总是得到最少的坏边。


a
b
c
d