

机器学习理论作业4

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P213 习题10.2:

10.2 考虑盒子和球组成的隐马尔可夫模型 $\lambda = (A, B, \pi)$ ，其中，

$$A = \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}, \quad \pi = (0.2, 0.3, 0.5)^T$$

设 $T = 8$ ， $O = (\text{红}, \text{白}, \text{红}, \text{红}, \text{白}, \text{红}, \text{白}, \text{白})$ ，用前向后向概率计算 $P(i_4 = q_3 | O, \lambda)$ 。

1.4/3: 前向算法计算前向概率:

$$\alpha_1(1) = 0.5 \times 0.2 = 0.1$$

$$\alpha_1(2) = 0.4 \times 0.3 = 0.12$$

$$\alpha_1(3) = 0.7 \times 0.5 = 0.35$$

$$\alpha_2(1) = (0.1 \times 0.5 + 0.12 \times 0.3 + 0.35 \times 0.2) \times 0.5 = 0.078$$

$$\alpha_2(2) = (0.1 \times 0.1 + 0.12 \times 0.5 + 0.35 \times 0.2) \times 0.6 = 0.084$$

$$\alpha_2(3) = (0.1 \times 0.4 + 0.12 \times 0.2 + 0.35 \times 0.6) \times 0.5 = 0.0822$$

$$\alpha_3(1) = (0.078 \times 0.5 + 0.084 \times 0.3 + 0.0822 \times 0.2) \times 0.5 = 0.04032$$

$$\alpha_3(2) = (0.078 \times 0.1 + 0.084 \times 0.5 + 0.0822 \times 0.2) \times 0.4 = 0.026496$$

$$\alpha_3(3) = (0.078 \times 0.4 + 0.084 \times 0.2 + 0.0822 \times 0.6) \times 0.7 = 0.068124$$

$$\alpha_4(1) = (0.04032 \times 0.5 + 0.026496 \times 0.3 + 0.068124 \times 0.2) \times 0.5 = 0.0208668$$

$$\alpha_4(2) = (0.04032 \times 0.1 + 0.026496 \times 0.5 + 0.068124 \times 0.2) \times 0.4 = 0.01236192$$

$$\alpha_4(3) = (0.04032 \times 0.4 + 0.026496 \times 0.2 + 0.068124 \times 0.6) \times 0.7 = 0.0436112$$

~~$\beta_8(1)$~~ = 后向算法计算后向概率:

$$\beta_8(1) = \beta_8(2) = \beta_8(3) = 1$$

$$\beta_7(1) = 0.5 \times 0.5 + 0.1 \times 0.6 + 0.4 \times 0.3 = 0.43$$

$$\beta_7(2) = 0.3 \times 0.5 + 0.5 \times 0.6 + 0.2 \times 0.3 = 0.51$$

$$\beta_7(3) = 0.2 \times 0.5 + 0.2 \times 0.6 + 0.6 \times 0.3 = 0.4$$

$$\beta_6(1) = 0.5 \times 0.5 \times 0.43 + 0.1 \times 0.6 \times 0.51 + 0.4 \times 0.3 \times 0.4 \\ = 0.1861$$

$$\beta_6(2) = 0.3 \times 0.5 \times 0.43 + 0.5 \times 0.6 \times 0.51 + 0.2 \times 0.3 \times 0.4 \\ = 0.2415$$

$$\beta_6(3) = 0.2 \times 0.5 \times 0.43 + 0.2 \times 0.6 \times 0.51 + 0.6 \times 0.3 \times 0.4 = 0.1762$$

$$\beta_5(1) = 0.5 \times 0.5 \times 0.1861 + 0.1 \times 0.4 \times 0.2415 + 0.4 \times 0.7 \times 0.1762 \\ = 0.105521$$

$$\beta_5(2) = 0.3 \times 0.5 \times 0.1861 + 0.5 \times 0.4 \times 0.2415 + 0.2 \times 0.7 \times 0.1762 \\ = 0.100883$$

$$\beta_5(3) = 0.2 \times 0.5 \times 0.1861 + 0.2 \times 0.4 \times 0.2415 + 0.6 \times 0.7 \times 0.1762 \\ = 0.111934$$

$$\beta_4(1) = 0.5 \times 0.5 \times 0.105521 + 0.1 \times 0.6 \times 0.100883 + 0.4 \times 0.3 \times 0.111934 \\ = 0.04586531$$

$$\beta_4(2) = 0.3 \times 0.5 \times 0.105521 + 0.5 \times 0.6 \times 0.100883 + 0.2 \times 0.3 \times 0.111934 \\ = 0.05276409$$

$$\beta_4(3) = 0.2 \times 0.5 \times 0.105521 + 0.2 \times 0.6 \times 0.100883 + 0.6 \times 0.3 \times 0.111934 = 0.04277618$$

故所求前向/后向根概率为:

$$P(i_4 = q_3 | 0, 2) = \frac{\alpha_4(3) \beta_4(3)}{\alpha_4(1) \beta_4(1) + \alpha_4(2) \beta_4(2) + \alpha_4(3) \beta_4(3)}$$

$$= \frac{0.0436112 \times 0.04277618}{0.0208668 \times 0.04586531 + 0.01236192 \times 0.05276409 + 0.0436112 \times 0.04277618} \\ \approx 0.5368638876$$