Comments on Distinguishability in Electrical Impedance Imaging

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Abstract—**In Electrical Impedance Imaging (EII), the spatial distribution of electrical resistivity of a volume conductor is reconstructed from measurements of potential which result from an externally applied electric field. The ability of such a system to distinguish a target inhomogeneity and the discussion of several measures of distinguishability has been a subject of a recent study [1]. In this communication a few comments are added to those of [1].**

The medical device safety regulations limit the maximum total input current which can be applied to the human thorax [2]–[5]. Based on the currently existing safety regulations, if the input current to the thorax is limited and constant, patterns applied between opposite electrodes result in higher distinguishability of a centered target than the cosine current patterns as suggested by [1].

I. Introduction

The distinguishability in electrical impedance imaging has been defined by Isaacson [6] as the following: Two conductivities σ_1 and σ_2 are distinguishable with a measurement precision of ϵ, if there is a current $\|j\| = 1$ and $\|V(.;\sigma_1,j) - V(.;\sigma_2,j)\| > \epsilon$, where $\|f\|$ is defined as $\|f\| \equiv \left[\int S|f(p)|^2 dS\right]^{1/2}$. Isaacson [6] also showed that under these conditions, currents of the form $j(\theta) = \frac{\cos\theta}{\sqrt{\pi}}$ or $j(\theta) = \frac{\sin\theta}{\sqrt{\pi}}$ are the best current patterns to distinguish a central circular inhomogeneity inside an otherwise homogeneous circular conductor.

Gisser *et al.* [7] compared the distinguishabilities by using the adjacent electrode, the opposite electrode, and the cosine current patterns on a circular conductor model with and without a centered circular target. Setting the maximum (peak) cosine current equal to the current injected with the opposite or the adjacent electrodes, Gisser *et al.* [7] demonstrated that the maximum $\|V(.;\sigma_1,j) - V(;\sigma_2,j)\|$ can be determined when the cosine current pattern is used.

Newell *et al.* [8] compared the distinguishabilities $\delta = \frac{\|V^T - V^0\|}{\|I\|}$ obtainable by the three current injection methods. In this expression, $\|X\| = \sum_{n=1}^{L} X_n^2$ (subscript n represents the nth elect rode and L is the total number of electrodes), $V^T = V(.;\sigma_1,j)$ are surface potentials measured from a conductor with a target object to be detected, $V^0 = V(.;\sigma_2,j)$ are the surface potentials from the same conductor without the target, and I is the current injected into the conductor. Under these conditions, Newell *et al.* [8] demonstrated that the cosine current pattern can distinguish smaller inhomogeneities than the use of the opposite electrode and the adjacent electrode current patterns. Recently, Cheney and Isaacson [1] demonstrated that the use of different norms in the definition of distinguishability may result in different measures of a system's ability to differentiate between different conductivity distributions. Cheney and Isaacson [1] calculated the size of the smallest object which can be distinguished if the conductor is probed with different current patterns. The authors of [1] showed that, for constant total power input $P_{in} = \sum_{n=1}^{L} I_n V_n$, the difference between the size of the smallest detectable object by cosine and by the other two patterns is not so large as the differences calculated if the mean squared currents are kept constant. Medical device safety regulations require a limit of 5 mA (rms) on the maximum current at 50 kHz which can be applied to the human body for electroplethysmography measurements [2]-[5]. Therefore, when comparing different current patterns according to the existing regulations, the total input current applied to the thorax should be kept constant. In this communication, the size of the smallest detectable object for constant input current flow is calculated for different current patterns used to probe the conductor.

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II. Total Input as a Current Test Criterion

In the examples of this communication, a circular disk model described in [7] and used by the authors of [1], [7], [8] is employed. This disk has a height $k = 3.6$ cm, lxa radius $a = 15$ cm, and 32 electrodes of width $\frac{2\pi a}{32} = 2.94$ cm. The conductivity of the conductor is equal to $1(\mathrm{ohm-cm})^{-1}$, and the conductivity of the target is $1/350(\mathrm{ohm-cm})^{-1}$.

As an example, assume that j_D is the current density applied to the opposite electrodes and that the current density applied to the adjacent electrodes j_F is equal to j_D. When a cosine current pattern of the form $j_B+ = j_m \cos\theta$ is applied to the surface of the circular disk, the total input current flow into the conductor is

$$I_B+ = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} j_m \cos\theta \mathrm{had}\theta = 108 j_m \tag{1}$$

for the continuum problem (when the electrodes are covering the whole surface). Gisser *et al.* [7] have used equal input currents for the opposite electrode and for the adjacent electrode current pattern $I_D = I_F = 10.6 j_D$. However, the total input current injected by the cosine pattern is about 10.2 times larger than the total input current injected with the opposite and the adjacent electrodes. The total input current required for each current pattern is shown in Table I, a table corresponding to "Currents with Maximum Magnitude 1 mA" in [1]. The authors of [1] used the "gap" model in the calculations. In this paper, a similar approach to the "gap" model is taken, by assuming that the edges of the electrodes are separated by approximately 0.005 cm. When the cosine pattern is applied through discrete electrodes, the total input current $I_B+ = 9.07$ mA, the total input current applied between the opposite and the adjacent pairs of electrodes is $I_B+ = 0.996$ mA (Table I). The total input current injected with the adjacent or the opposite electrode current patterns is 9.1 times smaller than the input current resulting from the cosine pattern. The radius of the smallest object which can be detected with a cosine current pattern is 2.8 times and 9.05 times smaller than the radius of the smallest object which can be detected by the opposite electrode current pattern and the adjacent electrode current pattern, respectively. Current patterns in the form of Walsh functions [1] result in the highest input current flow, almost 16 times more than I_D and I_F, while having the smallest detectable object size.

For different current patterns, the approach of Newell *et al.* [8] sets the sum squared current $\|I\| = \sum_{n=1}^{L} I_n^2$ to be a constant using

TABLE I
CURRENTS WITH MAXIMUM CURRENT DENSITY 1 mA

current pattern	max. current density in mA/cm^2	power input in μW	target radius in cm	total current input in mA
B+	0.094	7922	0.95	9.07
W	0.094	13540	0.84	15.94
D	0.094	238	2.7	0.996
F	0.094	85	8.6	0.996

discrete electrodes. In the continuum problem, if a cosine pattern of the form $j_B+ = j_m\cos\theta$ is applied to the circular conductor, the mean squared current is

$$||j_B|| = \int_0^{2\pi} j_m^2\cos^2\theta \text{had}\theta = 169.65 j_m^2. \quad (2)$$

For the opposite or the adjacent electrode current patterns $||j_D|| = 21.2 j_D^2$. For the circular conductor, to keep $||j||$ constant for all the current patterns, the maximum of the cosine current j_m must be equal to 0.354 times the current applied to the adjacent or to the opposite electrodes. The total current input to the conductor as a result of a pattern type $||j_{B+}|| = 0.354 j_D\cos\theta$ is $I_B+ = 3.6 I_D$. For a cosine current pattern of the form $j_{B+} = j_m\cos\theta$ injected to a circular homogeneous conductor, the total power input to the homogeneous circular conductor is

$$P_B+ = \int_{\frac{-pi}{2}}^{\frac{\pi}{2}} j_B+V_B+\text{had}\theta = \int_{\frac{-pi}{2}}^{\frac{\pi}{2}} j_m^2\cos^2\theta ha^2 d\theta = 1272.3 j_m^2. \quad (3)$$

For the opposite electrode current patterns, the potential at the electrodes can be calculated analytically as (see the Appendix)

$$V_D = 3.76 j_D. \quad (4)$$

The power input is

$$P_D = \int_{\frac{-pi}{32}}^{\frac{\pi}{32}} j_D V_D \text{had}\theta = 3.76 j_D^2 10.6 = 39.86 j_D^2. \quad (5)$$

To keep the input power constant for all the current patterns, the peak of the cosine current pattern should be $j_m = 0.177 j_D$. The total input current flow into the thorax becomes $I_B+ = 1.8 I_D$.

Table II, a table corresponding to "Currents with Power 7.9 mW" in [1], shows the total input current required for each current pattern. In this case, the input current in the opposite patterns is 1.6 times smaller than the input current of the cosine pattern. Total input current Bow with Walsh function current patterns reduces, therefore, as the size of the smallest detectable object increases. The total input current flow with the adjacent electrode current patterns increases almost 10 times from its value in Table I as the size of the smallest detectable object decreases.

Table III shows the object sizes for a constant input current of 9.07 mA. When the input current is constant, current injected between pairs of opposite electrodes can detect the smallest object. Opposite electrode current patterns are followed by the cosine patterns, Walsh function current patterns, and the adjacent electrode current patterns. Opposite electrode current patterns has the highest power input to the conductor. Distinguishability with the adjacent electrode current patterns remains the poorest although the difference in the size of the smallest detectable object is not so marked as in Table I.

TABLE II
CURRENTS WITH CONSTANT POWER 7.9 mW

current pattern	max. current density in mA/cm^2	power input in mW	target radius in cm	total current input in mA
B+	0.094	7.9	0.953	9.07
W	0.072	7.9	0.958	12.21
D	0.545	7.9	1.123	5.78
F	0.909	7.9	2.776	9.64

TABLE III
CURRENTS WITH TOTAL INPUT CURRENT EQUAL TO 9.074 Ma

current pattern	max. current density in mA/cm^2	power input in mW	target radius in cm	total current input in mA
B+	0.094	7.9	0.95	9.07
W	0.054	4.4	1.11	9.07
D	0.856	19.7	0.89	9.07
F	0.856	7.0	2.85	9.07

III. CONCLUSION

In this article, the importance of the selection criteria in comparing different injection current patterns used in Eli is demonstrated. Present medical device safety regulations limit the maximum total current which can be applied to the human thorax. Therefore, when comparing different current patterns, the constant total current input should be the first consideration. When the input current to the thorax is limited, current injected between diametrically opposite electrodes result in better distinguishability of a centered target than the cosine current patterns as suggested by [1] and [7], [8]. For applications other than medical, the total input power criterion remains a factor because of the instrumentation design. The current medical device safety regulations for impedance measuring devices are written based on impedance plethysmography equipment. Health care facilities and the medical instrument industry still follow these regulations. Regulations based on the safety considerations for EII measurement schemes need to be clearly defined. These regulations should state safe levels of frequency of the applied current and the current density in the myocardium in order not to damage the cardiac tissue and not to interfere with its physiological functioning. In addition, care must be taken not to cause respiratory paralysis by involuntary stimulation of respiratory muscles. Then safe levels of current, which can be applied to the electrodes, can be calculated based on the allowed current densities in the cardiac and other tissues. Maximum current density at the electrodes must be defined as well as the total current through the electrode to avoid tissue damage.

APPENDIX

A. Formulation of the Potential Function Inside a Circular Conductor

The solution to the potential function, developed as a result of an externally applied current flow between two surface electrodes and adopted from [9], is given in this Appendix. A circular disk of conducting region with homogeneous conductivity $\sigma(r, \theta)$, radius a, height h with two surface electrodes of width δ and of height h at angular positions θ_1 and θ_2 is considered.

When a current I is applied between the two electrodes, the resultant voltage distribution within the region can be expressed as

$$\Phi(r,\theta) = \sum_{m=0}^{\infty} a_m r^m \cos(m\theta) + \sum_{m=0}^{\infty} B_m r^m \sin(m\theta) \qquad \text{(A.1)}$$

Boundary conditions are

$$j(a,\theta) = \sigma \frac{\partial \Phi(r,\theta)}{\partial r}\Big|_{r=a} \qquad \text{(A.2)}$$

$$j(a,\theta) = \begin{cases} \frac{I}{h\delta} & \text{for} \quad \theta_1 - \frac{B}{2} \le \theta \le \theta_1 + \frac{B}{2} \\ \frac{-I}{h\delta} & \text{for} \quad \theta_2 - \frac{B}{2} \le \theta \le \theta_2 + \frac{B}{2} \\ 0 & \text{elsewhere} \end{cases} \qquad \text{(A.3)}$$

where $j(\theta)$ is the normal component of the electric current density at the surface, a the normal derivative to the surface, and $B = \frac{\delta}{a}$ is angular width of a current injecting electrode. Substituting (A.1) into (A.2)

$$\frac{\partial \Phi(r,\theta)}{\partial r}\Big|_{r=a} = \sum_{m=0}^{\infty} m a^{(m-1)}[A_m \cos(m\theta) + B_m \sin(m\theta)] = \frac{j(a,\theta)}{\sigma}. \qquad \text{(A.4)}$$

Multiplying both sides with $\sin(n\theta)d\theta$ and integrating from $\theta = 0$ to $\theta = 2\Pi$ and using the previously specified boundary conditions and orthogonality principles, one shows that

$$B_n = \frac{2j}{\sigma \pi n^2 a^{(n-1)}} \sin\left(\frac{n\delta}{2a}\right)[\sin(n\theta_1) - \sin(n\theta_2)]. \qquad \text{(A.5)}$$

Similarly, multiplying both sides of (A.4) with $\cos(n\theta)d\theta$ and integrating from 0 to 2π, and using the previously specified boundary conditions and orthogonality principles yields

$$A_n = \frac{2j}{\sigma \pi n^2 a^{(n-1)}} \sin\left(\frac{n\delta}{2a}\right)[\cos(n\theta_1) - \cos(n\theta_2)]. \qquad \text{(A.6)}$$

Substituting (A.5) and (A.6) into (A.4) after appropriate simplifications results in the equation for the potential function inside a uniform circular conductor

$$\Phi(r,\theta) = \frac{2ja}{\sigma\pi} \sum_{n=1}^{\infty} \frac{1}{n^2}\left(\frac{r}{a}\right)^n \sin\left(\frac{n\delta}{2a}\right)[\cos n(\theta - \theta_1) - \cos n(\theta - \theta_2)]. \qquad \text{(A.7)}$$

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References

[1] M. Cheney and M. D. Isaacson, "Distinguishability in impedance imaging," *IEEE Trans. Biomed. Eng.*, vol. 39, pp. 852-860, Aug. 1992.

[2] A. Ghahary, "Electrical safety," in *Electrical Impedance Tomography*, J. G. Webster, Ed. Bristol, New York: Adam Hilger, 1990, pp. 58–69.

[3] Association for the Advancement of Medical Instrumentation AAMI, *Safe Current Limits for Electromedical Apparatus* (ANSI/AAMI esl-1985J, Arlington, VA, 1985.

[4] British Standard Inst., *Specifications for Safety of Medical Electrical Equipment—Part 1, General Requirements*, Sub-clause 19.3 (BS 5724, 1979), London, 1979.

[5] Int. Electrotechnical Commission, *Safety of Medical Electrical Equipment—Part I: General Requirements*, (601-1), Geneva, IEC, 1977.

[6] D. Isaacson, "Distinguishability of conductivities by electric current computed tomography," *IEEE Trans. Medical Imaging*, vol. MI-5, pp. 91–95, June 1986.

[7] D. C. Gisser, D. Isaacson, and J. C. Newell, "Theory and performance of adaptive current tomography system," *Clinical Physics and Physiological Meas.*, vol. 9, supp. A, pp. 35-42, 1988.

[8] J. C. Newell, D. C. Gisser, and D. Isaacson, "An electric current tomograph," *IEEE Trans. Biomed. Eng.*, vol. 35, pp. 823–33, Oct. 1988.

[9] B. M. Eyüboglu, "Development and application of electrical impedance tomography for thoracic imaging," Ph.D. dissertation, The University of Sheffield, Sheffield, UK, pp. 227-231, 297-303, Nov. 1988.

An Acoustical Guidance and Position Monitoring System for Endotracheal Tubes

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***Abstract*—A prototype instrument to guide the placement and continuously monitor the position of an endotracheal tube (ETT) was developed. An incident audible sound pulse is introduced into the proximal ETT and detected as it travels down the ETT via a miniature microphone located in the wall. This pulse is then emitted from the tube tip into the airways and the reflected signal from the airways is detected by the microphone. A well defined reflection arises from the point where the total cross sectional area of the airways increases rapidly, and the difference in timing between detection of the incident pulse and this reflection is used to determine ETT position or movement. This reflection is not observed if the ETT is erroneously placed in the esophagus. The amplitude and polarity of an additional reflection that occurs at the ETT tip is used to estimate the cross-sectional area of the airway in which the ETT is placed. This combined information allows discrimination between tracheal and bronchial intubation and can be used to insure an adequate fit between the ETT and trachea. The instrument has proven extremely reliable in multiple intubations in eight canines and offers the potential to noninvasively and inexpensively monitor ETT position in a continuous manner.**

I. Introduction

An endotracheal tube (ETT) is inserted into the trachea: 1) to establish an open airway, 2) to permit the use of mechanical ventilation, and 3) as part of an anesthesia delivery system. The intubation may be complicated by inadvertent insertion of the ETT into the esophagus or past the carina into one of the bronchi, as depicted in Fig. 1. Also, the ETT tip may move above the vocal folds after placement due to patient or ventilator tube movement. In all of these scenarios, the patient is ineffectively ventilated which may result in severe medical complications.

A number of techniques have been developed to aid clinicians in the determination of the location of an ETT[1]. Due to various

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