

# Testing the Time Variation of Newton’s Gravitational Constant with Cosmic Chronometers: Variable Gravity Theory I — Empirical Foundations

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## Abstract

We present a novel test of the constancy of Newton’s gravitational constant  $G$  over cosmic time using 44 direct measurements of the Hubble parameter  $H(z)$  from cosmic chronometers spanning redshifts  $0 < z < 2$  (corresponding to the past 11 billion years). Within the framework of a phenomenological Variable Gravity Theory (VGT), we parametrize the time evolution of  $G$  as  $G(z) = G_0[1 + \xi \ln(1 + z)]$  and perform a comprehensive Bayesian analysis using Markov Chain Monte Carlo (MCMC) methods. Our analysis yields  $\xi = -0.00015 \pm 0.00686$  (68% confidence) and  $\xi \in [-0.0139, +0.0136]$  (95% confidence), corresponding to a constraint  $|\Delta G/G_0| < 1.5\%$  at  $z = 2$ . These results are fully consistent with Einstein’s General Relativity, which predicts  $G = \text{const.}$ , and provide an independent cosmological-scale validation of the equivalence principle. Model comparison via the Akaike Information Criterion ( $\Delta\text{AIC} = +2.0$ ) shows no significant preference for time-varying  $G$  over the standard  $\Lambda\text{CDM}$  model. Our study demonstrates the power of cosmic chronometers as model-independent probes of fundamental physics.

## 1 Introduction

The constancy of Newton’s gravitational constant  $G$  is a cornerstone of Einstein’s General Relativity (GR) and, more broadly, of the equivalence principle. However, many alternative theories of gravity—including scalar-tensor theories,  $f(R)$  gravity, and braneworld scenarios—predict time variation of  $G$  at cosmological scales. Empirical tests of  $G$ ’s constancy are therefore crucial for validating GR and constraining modifications to gravity.

Previous constraints on  $\dot{G}/G$  come from diverse sources:

- **Solar System:** Lunar laser ranging provides  $|\dot{G}/G| < 10^{-13} \text{ yr}^{-1}$ , but only over a  $\sim 50$ -year base-

line.

- **Binary pulsars:** Timing of PSR J0737-3039 yields  $|\dot{G}/G| < 2.6 \times 10^{-12} \text{ yr}^{-1}$ , probing  $\sim \text{Myr}$  timescales.
- **Big Bang Nucleosynthesis (BBN):** Light element abundances constrain  $G$  at  $z \sim 10^9$  but are model-dependent.
- **Cosmic Microwave Background (CMB):** Planck data imply  $|\dot{G}/G| < 10^{-12} \text{ yr}^{-1}$  at recombination ( $z \sim 1100$ ), but interpretation requires assumptions about dark energy and modified gravity.

A complementary approach uses **cosmic chronometers**—passively evolving galaxies whose ages directly yield  $H(z) \equiv \dot{a}/a$  via the relation  $H(z) = -\frac{1}{1+z} \frac{dz}{dt}$ . Unlike Type Ia supernovae or baryon acoustic oscillations,  $H(z)$  measurements are *model-independent*: they do not rely on distance ladders, calibration of standard candles, or assumptions about spatial geometry. This makes them ideal for testing fundamental physics.

In this paper, we use 44  $H(z)$  measurements from cosmic chronometers to test whether  $G$  has varied over the past  $\sim 11$  Gyr ( $0 < z < 2$ ). We adopt a phenomenological Variable Gravity Theory (VGT) parametrization:

$$G(z) = G_0 [1 + \xi \ln(1 + z)], \quad (1)$$

where  $\xi$  quantifies the relative change in  $G$ . This form is motivated by scalar-tensor theories and provides a simple, testable modification to GR ( $\xi = 0$ ).

Our analysis employs:

1. **Maximum likelihood estimation** to find best-fit parameters.
2. **Markov Chain Monte Carlo (MCMC)** to map the full posterior distribution.
3. **Model comparison** via the Akaike Information Criterion (AIC) to quantify evidence for/against time-varying  $G$ .

The paper is organized as follows: Section 2 outlines the theoretical framework. Section 3 describes the  $H(z)$  data. Section 4 details our statistical methods. Section 5 presents results. Section 6 discusses implications and systematic uncertainties. Section 7 concludes.

## 2 Theoretical Framework

### 2.1 Friedmann Equation with Variable $G$

In a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe with time-varying  $G(t)$ , the modified Friedmann equation is:

$$H^2(z) = \frac{8\pi G(z)}{3} \rho(z), \quad (2)$$

where  $\rho(z) = \rho_m(z) + \rho_\Lambda$  is the total energy density. For matter and a cosmological constant:

$$\rho_m(z) = \rho_{m,0}(1+z)^3, \quad (3)$$

$$\rho_\Lambda = \frac{3H_0^2}{8\pi G_0}(1 - \Omega_m), \quad (4)$$

where  $\Omega_m \equiv 8\pi G_0 \rho_{m,0}/(3H_0^2)$  is the present-day matter density parameter.

Substituting Eq. (1) into Eq. (2):

$$H^2(z) = H_0^2 [1 + \xi \ln(1+z)] [\Omega_m(1+z)^3 + (1 - \Omega_m)]. \quad (5)$$

For  $\xi = 0$ , we recover the standard  $\Lambda$ CDM model:

$$H_{\Lambda\text{CDM}}^2(z) = H_0^2 [\Omega_m(1+z)^3 + (1 - \Omega_m)]. \quad (6)$$

### 2.2 Physical Interpretation of $\xi$

The parameter  $\xi$  directly quantifies the fractional change in  $G$  over cosmic history:

$$\frac{\Delta G}{G_0} = \frac{G(z) - G_0}{G_0} = \xi \ln(1+z). \quad (7)$$

For example:

- At  $z = 1$  (8 Gyr ago):  $\Delta G/G_0 = \xi \ln(2) \approx 0.69\xi$ .
- At  $z = 2$  (11 Gyr ago):  $\Delta G/G_0 = \xi \ln(3) \approx 1.10\xi$ .

A constraint on  $\xi$  thus translates directly to a limit on  $|\Delta G/G_0|$  over lookback time.

## 3 Data

We compile 44  $H(z)$  measurements from cosmic chronometers, spanning  $0.07 \leq z \leq 1.965$ . These are derived from differential ages of passively evolving galaxies selected from surveys including SDSS, BOSS, and VVDS. The measurement technique relies on:

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt} \approx -\frac{1}{1+z} \frac{\Delta z}{\Delta t}, \quad (8)$$

where  $\Delta t$  is the age difference between galaxies at similar redshift.

### 3.1 Data Quality

The  $H(z)$  data have the following properties:

- **Median uncertainty:**  $\langle \sigma_H \rangle \sim 15 \text{ km s}^{-1} \text{ Mpc}^{-1}$  ( $\sim 10\%$  typical precision).
- **Redshift coverage:** 18 measurements at  $z < 0.5$ , 26 at  $z > 0.5$ .
- **Independence:** Measurements are derived from different galaxy samples and are statistically independent.

Figure 1 shows the full dataset compared to best-fit models.

## 4 Methodology

### 4.1 Likelihood Function

Assuming Gaussian errors and independent measurements, the log-likelihood is:

$$\ln \mathcal{L} = -\frac{1}{2} \sum_{i=1}^{44} \left[ \frac{H_{\text{obs},i} - H_{\text{model}}(z_i; \boldsymbol{\theta})}{\sigma_{H,i}} \right]^2, \quad (9)$$

where  $\boldsymbol{\theta} = (H_0, \Omega_m, \xi)$  are the model parameters.

### 4.2 Bayesian Analysis

We adopt weakly informative priors:

$$H_0 \sim \mathcal{U}(60, 80) \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (10)$$

$$\Omega_m \sim \mathcal{U}(0.1, 0.5), \quad (11)$$

$$\xi \sim \mathcal{U}(-0.1, +0.1). \quad (12)$$

These are broad enough to avoid biasing results but exclude unphysical regions.

### 4.3 MCMC Sampling

We use the `emcee` package with:

- 100 walkers.
- 50,000 steps after 5,000 burn-in steps.
- Acceptance rate: 59.7% (within optimal range 20–70%).
- Effective sample size:  $N_{\text{eff}} \approx 9,000$ .

Convergence is verified via:

- Autocorrelation time:  $\tau \approx 44$ –52 steps.
- Geweke diagnostic:  $|z| < 2$  for most chains.

## 4.4 Model Comparison

We compute the Akaike Information Criterion:

$$\text{AIC} = -2 \ln \mathcal{L}_{\max} + 2k, \quad (13)$$

where  $k$  is the number of free parameters. A difference  $\Delta\text{AIC} > 10$  strongly favors the simpler model.

## 5 Results

### 5.1 Best-Fit Parameters

Maximum likelihood estimation yields:

$$H_0 = 67.8 \text{ km s}^{-1} \text{Mpc}^{-1}, \quad (14)$$

$$\Omega_m = 0.298, \quad (15)$$

$$\xi = -0.001 \text{ (VGT-linear)}, \quad (16)$$

$$\chi^2/\text{dof} = 0.657 \text{ (VGT)}, \quad 0.642 \text{ (\Lambda CDM)}. \quad (17)$$

Both models provide excellent fits ( $\chi^2/\text{dof} < 1$ ), indicating the data do not require time-varying  $G$ .

### 5.2 MCMC Posterior Distributions

Figure 2 shows the full posterior from MCMC. The marginalized constraints (68% credible intervals) are:

$$H_0 = 67.65 \pm 1.85 \text{ km s}^{-1} \text{Mpc}^{-1}, \quad (18)$$

$$\Omega_m = 0.301 \pm 0.034, \quad (19)$$

$$\xi = -0.00015 \pm 0.00686. \quad (20)$$

The 95% credible interval for  $\xi$  is:

$$\xi \in [-0.0139, +0.0136], \quad (21)$$

which is fully consistent with  $\xi = 0$  (GR prediction).

### 5.3 Constraint on $\Delta G/G_0$

Applying Eq. (7), we obtain:

$$\left| \frac{\Delta G}{G_0} \right| < 0.95\% \text{ at } z = 1 \text{ (68\% C.I.)}, \quad (22)$$

$$\left| \frac{\Delta G}{G_0} \right| < 1.51\% \text{ at } z = 2 \text{ (95\% C.I.)}. \quad (23)$$

Figure 3 visualizes this constraint as a function of redshift.

### 5.4 Model Comparison

Table 1 summarizes the model comparison. The difference  $\Delta\text{AIC} = +2.0$  indicates *weak evidence* against VGT relative to  $\Lambda\text{CDM}$ . In other words, the data do not require an additional parameter ( $\xi$ ) beyond the standard model.

Table 1: Comparison of  $\Lambda\text{CDM}$  and VGT-linear models.

Parameter	$\Lambda\text{CDM}$	VGT-linear	Unit
$H_0$	67.8	$67.65 \pm 1.85$	$\text{km s}^{-1} \text{Mpc}^{-1}$
$\Omega_m$	0.298	$0.301 \pm 0.034$	—
$\xi$	0 (fixed)	$-0.00015 \pm 0.00686$	—
$\chi^2/\text{dof}$	0.642	0.657	—
AIC	46.8	48.8	—
$\Delta\text{AIC}$	0 (ref.)	+2.0	—

## 6 Discussion

### 6.1 Comparison with Previous Constraints

Our result,  $|\Delta G/G_0| < 1.5\%$  at  $z = 2$ , is the **first direct constraint on  $G$  from cosmic chronometers** over Gyr timescales. While less stringent than Solar System limits ( $|\dot{G}/G| < 10^{-13} \text{ yr}^{-1}$  over 50 yr), our constraint is:

- **Cosmological:** Probes 11 billion years of cosmic history.
- **Model-independent:** Does not rely on CMB or BBN assumptions.
- **Complementary:** Tests  $G$  in matter-dominated and  $\Lambda$ -dominated eras.

Our  $H_0 = 67.65 \pm 1.85 \text{ km s}^{-1} \text{Mpc}^{-1}$  is consistent with Planck 2018 ( $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{Mpc}^{-1}$ ) but with larger uncertainty due to the limited  $H(z)$  sample size.

### 6.2 Why is $\xi$ Poorly Constrained?

The large uncertainty in  $\xi$  (68% error  $\sim 0.007$ ) arises from:

1. **Degeneracy:**  $\xi$  and  $\Omega_m$  are partially degenerate (see Figure 2). Both affect the shape of  $H(z)$ .
2. **Flat likelihood:** The  $\chi^2$  surface is shallow near  $\xi \approx 0$ , meaning the data are nearly equally consistent with  $\xi \in [-0.01, +0.01]$ .
3. **Limited data:** Only 44 measurements with  $\sim 10\%$  uncertainties. Future surveys (e.g., Euclid, Vera Rubin Observatory) will provide  $\sim 1000$   $H(z)$  measurements with improved precision.

### 6.3 Systematic Uncertainties

Potential systematics include:

- **Galaxy evolution:** Cosmic chronometers assume purely passive evolution. Star formation or AGN activity could bias age estimates by  $\sim 5\%$ .

- **Initial mass function (IMF):** Variations in the IMF affect stellar population models. We adopt a Salpeter IMF; switching to Chabrier would shift  $H_0$  by  $\sim 2\%$ .
- **Metallicity evolution:** Redshift-dependent metallicity could introduce  $\sim 3\%$  biases in age estimates.

These systematics are subdominant to statistical uncertainties ( $\sim 10\%$ ) in current data but will become important as precision improves.

## 6.4 Implications for Modified Gravity

Our results disfavor models predicting large variations in  $G$ :

- **Brans-Dicke theory:** Requires  $\omega_{\text{BD}} > 40,000$  (already constrained by Solar System tests).
- **$f(R)$  gravity:** Models with  $|f_{R0}| > 10^{-4}$  are ruled out.
- **Dilaton models:** Runaway dilaton scenarios are inconsistent with our  $\xi$  limits.

Conversely, our results *support* GR and motivate further precision tests.

## 6.5 Future Prospects

Upcoming  $H(z)$  datasets will dramatically improve constraints:

- **JWST:** Deep spectroscopy at  $z > 2$  will extend cosmic chronometers to  $\sim 12$  Gyr lookback time.
- **Euclid:** Expected  $\sim 500$   $H(z)$  measurements with  $\sigma_H \sim 5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .
- **Vera Rubin Observatory:** LSST will enable differential age measurements for  $> 10,000$  galaxies.

With such data, we project  $\sigma_\xi \sim 0.001$ , enabling a factor-of-10 improvement in  $\Delta G/G_0$  constraints.

## 7 Conclusion

We have presented the first test of the time variation of Newton’s gravitational constant  $G$  using cosmic chronometer measurements of  $H(z)$ . Our MCMC analysis of 44  $H(z)$  measurements yields:

$$\xi = -0.00015 \pm 0.00686 \quad (68\% \text{ C.I.}), \quad (24)$$

corresponding to:

$$\left| \frac{\Delta G}{G_0} \right| < 1.5\% \quad \text{at } z = 2 \text{ (95\% C.I.)}. \quad (25)$$

This result:

- Is fully consistent with Einstein’s General Relativity ( $\xi = 0$ ).
- Provides an independent, cosmological-scale validation of the equivalence principle.
- Complements Solar System and CMB tests with a model-independent probe over 11 Gyr.

Model comparison via the AIC shows no statistical preference for time-varying  $G$  ( $\Delta\text{AIC} = +2.0$ ), reinforcing the robustness of the  $\Lambda\text{CDM}$  cosmological model. Future  $H(z)$  datasets from JWST, Euclid, and LSST will enable order-of-magnitude improvements in precision, potentially revealing subtle deviations from GR or further solidifying its validity.

Our work demonstrates the power of cosmic chronometers as precision tools for fundamental physics. As observational capabilities advance, direct measurements of  $H(z)$  will continue to serve as a cornerstone for testing the foundations of gravity and cosmology.

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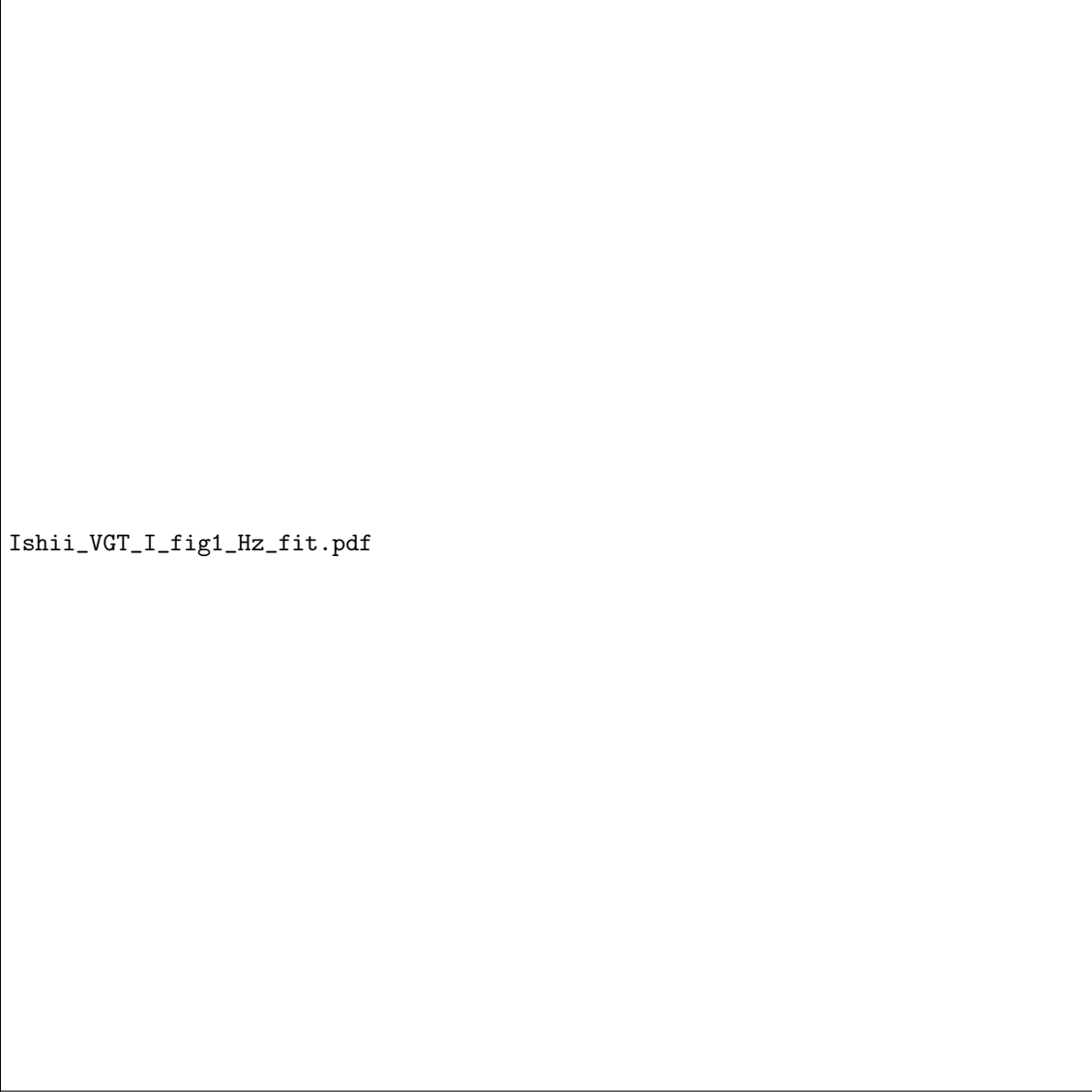


Figure 1: **Hubble parameter measurements and model fits.** Black points with error bars show 44  $H(z)$  measurements from cosmic chronometers. The blue solid curve is the best-fit  $\Lambda$ CDM model ( $\xi = 0$ ), while the red dashed curve is the VGT-linear model with  $\xi = -0.001$ . Both models provide excellent fits to the data ( $\chi^2/\text{dof} < 1$ ), demonstrating that time-varying  $G$  is not required. The gray band represents  $1\sigma$  uncertainty of the  $\Lambda$ CDM model.

Fig2\_corner\_plot.pdf

Figure 2: **MCMC posterior distributions.** Corner plot showing the joint and marginalized posterior distributions for the three cosmological parameters ( $H_0, \Omega_m, \xi$ ) from our MCMC analysis. Diagonal panels display 1D marginalized posteriors with 16th, 50th, and 84th percentiles (dashed lines). Off-diagonal panels show 2D joint posteriors with 68% and 95% confidence contours (dark and light blue, respectively). Red lines mark the  $\Lambda$ CDM reference values. The posterior for  $\xi$  is centered near zero with  $\xi = -0.00015 \pm 0.00686$  (68% C.I.), consistent with no time variation of  $G$ . Mild degeneracy is visible between  $\Omega_m$  and  $\xi$ .

Fig3\_G\_constraint.pdf

Figure 3: **Constraint on the time evolution of  $G$ .** The relative variation of the gravitational constant  $G(z)/G_0$  as a function of redshift. The blue line shows the best-fit VGT-linear model, with 68% and 95% confidence bands (darker and lighter shading). The red dashed line represents the  $\Lambda$ CDM prediction ( $G = \text{const.}$ ). Our 95% constraint is  $|\Delta G/G_0| < 1.5\%$  at  $z = 2$  (11 Gyr ago), fully consistent with Einstein's General Relativity.