

# Virtual Gravity Theory Volume II: TFOS

## Chapter V — Quantum Structure of $\psi_0$

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We develop the quantum structure of the fundamental scalar root field  $\psi_0$  within the Virtual Gravity Theory (VGT) framework. Starting from the classical action for  $\psi_0$  minimally coupled to gravity, we construct the canonical quantization procedure and derive the mode expansion in curved spacetime backgrounds. The eigenmode spectrum exhibits a characteristic hierarchical structure spanning infrared (IR) to ultraviolet (UV) scales, with mode-dependent effective coupling strengths. We establish the one-loop effective action  $\Gamma[g_{\mu\nu}, \psi_0]$  and demonstrate how quantum fluctuations of  $\psi_0$  generate scale-dependent corrections to gravitational couplings. The running of Newton's constant  $G(k)$  and the cosmological term  $\Lambda(k)$  are derived from first principles, providing the foundation for observable predictions in subsequent chapters.

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### I. INTRODUCTION

The Virtual Gravity Theory (VGT) proposes that gravitational phenomena emerge from electromagnetic processes through a fundamental scalar root field  $\psi_0$  [1]. In this framework, the metric tensor  $g_{\mu\nu}$  arises as a composite structure built from  $\psi_0$  and its derivatives, offering an alternative pathway to quantum gravity that circumvents the non-renormalizability issues plaguing direct metric quantization.

Volume I of this series established the classical foundations: the emergence of Einstein's equations as the appropriate limit, the discrete branch structure with modes  $n \in \{1, 2, 3\}$  corresponding to cosmological, galactic, and strong-field regimes, and the consistency with observational data [1]. Volume II (TFOS — Theoretical Foundations of Observable Structures) develops the theoretical machinery necessary for computing observable quantities without direct reference to empirical data.

This chapter, Chapter V, constructs the quantum structure of  $\psi_0$ . We address the following fundamental questions:

1. How does one consistently quantize  $\psi_0$  in curved spacetime?
2. What is the eigenmode spectrum, and how do modes organize hierarchically from IR to UV?
3. How do quantum fluctuations contribute to the effective gravitational action?
4. What are the running behaviors of  $G(k)$  and  $\Lambda(k)$ ?

The organization is as follows. Section II establishes the foundational action and field equations for  $\psi_0$ . Section III develops the canonical quantization and derives

the mode structure. Section IV analyzes the IR–UV effective behavior and mode hierarchy. Section V constructs the one-loop effective action and derives the running couplings. Section VI summarizes our results and outlines connections to subsequent chapters.

Throughout, we employ natural units  $\hbar = c = 1$  and the metric signature  $(-, +, +, +)$ .

### II. FOUNDATIONS OF $\psi_0$

#### A. Classical Action

The fundamental scalar root field  $\psi_0$  is governed by the action

$$S[\psi_0, g_{\mu\nu}] = S_{\text{grav}}[g_{\mu\nu}] + S_{\psi_0}[\psi_0, g_{\mu\nu}], \quad (1)$$

where the gravitational sector takes the Einstein–Hilbert form

$$S_{\text{grav}} = \frac{1}{16\pi G_0} \int d^4x \sqrt{-g} (R - 2\Lambda_0), \quad (2)$$

with bare Newton's constant  $G_0$  and bare cosmological constant  $\Lambda_0$ . The  $\psi_0$  sector is

$$S_{\psi_0} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \psi_0 \partial_\nu \psi_0 - V(\psi_0) - \xi R \psi_0^2 \right], \quad (3)$$

where  $V(\psi_0)$  is the self-interaction potential and  $\xi$  is the non-minimal coupling to the Ricci scalar  $R$ .

In VGT, the potential  $V(\psi_0)$  possesses a hierarchical structure with three discrete stable minima corresponding to the gravitational branches  $n \in \{1, 2, 3\}$ :

$$V(\psi_0) = \frac{\lambda}{4!} \psi_0^4 - \frac{\mu^2}{2} \psi_0^2 + V_{\text{hier}}(\psi_0), \quad (4)$$

where the hierarchical correction  $V_{\text{hier}}(\psi_0)$  ensures the existence of exactly three normalizable bound states [1].

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## B. Field Equations

Variation of the total action with respect to  $\psi_0$  yields the curved-spacetime Klein–Gordon equation:

$$\square_g \psi_0 - V'(\psi_0) - \xi R \psi_0 = 0, \quad (5)$$

where  $\square_g = g^{\mu\nu} \nabla_\mu \nabla_\nu$  is the d’Alembertian operator and  $V'(\psi_0) = dV/d\psi_0$ .

Variation with respect to the metric yields the modified Einstein equations:

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = 8\pi G_0 \left( T_{\mu\nu}^{(\psi_0)} + T_{\mu\nu}^{(\xi)} \right), \quad (6)$$

where the stress-energy tensors are

$$T_{\mu\nu}^{(\psi_0)} = \partial_\mu \psi_0 \partial_\nu \psi_0 - g_{\mu\nu} \left[ \frac{1}{2} (\partial \psi_0)^2 + V(\psi_0) \right], \quad (7)$$

$$T_{\mu\nu}^{(\xi)} = \xi [g_{\mu\nu} \square_g - \nabla_\mu \nabla_\nu + G_{\mu\nu}] \psi_0^2. \quad (8)$$

## C. Background–Fluctuation Decomposition

For quantization, we decompose  $\psi_0$  into a classical background  $\bar{\psi}_0$  and quantum fluctuations  $\varphi$ :

$$\psi_0(x) = \bar{\psi}_0(x) + \varphi(x), \quad \langle \varphi \rangle = 0. \quad (9)$$

The background  $\bar{\psi}_0$  satisfies the classical equation (5), while  $\varphi$  is promoted to a quantum field operator. Expanding the action to second order in  $\varphi$ :

$$S^{(2)}[\varphi] = -\frac{1}{2} \int d^4x \sqrt{-g} \varphi (\square_g - m_{\text{eff}}^2 - \xi R) \varphi, \quad (10)$$

where the effective mass is

$$m_{\text{eff}}^2(x) = V''(\bar{\psi}_0) = \frac{\lambda}{2} \bar{\psi}_0^2 - \mu^2 + V''_{\text{hier}}(\bar{\psi}_0). \quad (11)$$

## III. QUANTIZED MODE STRUCTURE

### A. Canonical Quantization

The conjugate momentum to  $\varphi$  is

$$\pi(x) = \frac{\delta S}{\delta \dot{\varphi}} = \sqrt{-g} n^\mu \partial_\mu \varphi, \quad (12)$$

where  $n^\mu$  is the unit normal to constant-time hypersurfaces. The canonical commutation relations are

$$[\varphi(\mathbf{x}, t), \pi(\mathbf{x}', t)] = i\delta^{(3)}(\mathbf{x} - \mathbf{x}'). \quad (13)$$

## B. Mode Expansion

On a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) background with metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (14)$$

we expand the fluctuation field as

$$\varphi(\mathbf{x}, t) = \int \frac{d^3k}{(2\pi)^3} \left[ \hat{a}_{\mathbf{k}} u_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger u_k^*(t) e^{-i\mathbf{k}\cdot\mathbf{x}} \right], \quad (15)$$

where  $\hat{a}_{\mathbf{k}}$  and  $\hat{a}_{\mathbf{k}}^\dagger$  are annihilation and creation operators satisfying

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}'). \quad (16)$$

The mode functions  $u_k(t)$  satisfy the Mukhanov–Sasaki-type equation:

$$\ddot{u}_k + 3H\dot{u}_k + \left( \frac{k^2}{a^2} + m_{\text{eff}}^2 + \xi R \right) u_k = 0, \quad (17)$$

where  $H = \dot{a}/a$  is the Hubble parameter and the Ricci scalar for FLRW is  $R = 6(\dot{H} + 2H^2)$ .

## C. Eigenmode Spectrum

Introducing the conformal time  $\eta$  via  $d\eta = dt/a$  and the rescaled variable  $v_k = a u_k$ , Eq. (17) transforms to

$$v_k'' + \omega_k^2(\eta) v_k = 0, \quad (18)$$

where primes denote derivatives with respect to  $\eta$  and the time-dependent frequency is

$$\omega_k^2(\eta) = k^2 + a^2 m_{\text{eff}}^2 + (6\xi - 1) \frac{a''}{a}. \quad (19)$$

For conformal coupling  $\xi = 1/6$ , the curvature term vanishes, and

$$\omega_k^2(\eta)|_{\xi=1/6} = k^2 + a^2 m_{\text{eff}}^2. \quad (20)$$

The eigenmode energy spectrum is obtained from the Hamiltonian

$$\hat{H} = \int \frac{d^3k}{(2\pi)^3} E_k \left( \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \frac{1}{2} \right), \quad (21)$$

where the mode energy is

$$E_k = \sqrt{k^2 + a^2 m_{\text{eff}}^2 + (6\xi - 1) \frac{a''}{a}}. \quad (22)$$

For the VGT framework, the effective mass  $m_{\text{eff}}^2$  takes discrete values at each branch minimum. Denoting the  $n$ -th branch minimum as  $\bar{\psi}_0^{(n)}$ :

$$m_n^2 \equiv m_{\text{eff}}^2|_{\bar{\psi}_0=\bar{\psi}_0^{(n)}} = V''(\bar{\psi}_0^{(n)}). \quad (23)$$

The branch-dependent dispersion relation is therefore

$$E_k^{(n)} = \sqrt{k^2 + a^2 m_n^2}, \quad n \in \{1, 2, 3\}. \quad (24)$$

Figure 1 illustrates the eigenmode spectrum showing the ground state and excited mode structure.

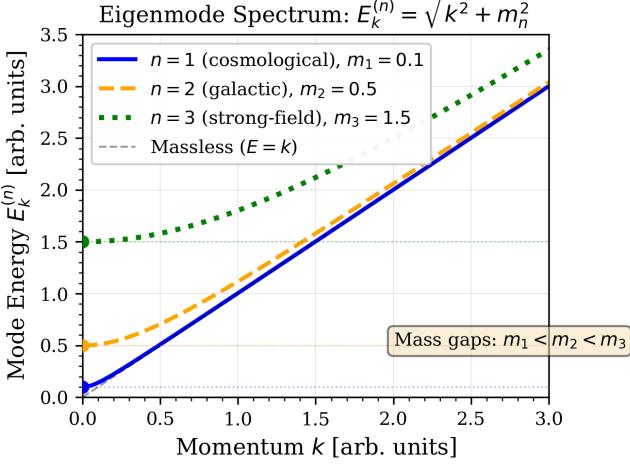


FIG. 1. Eigenmode spectrum of  $\psi_0$  fluctuations. The dispersion relations  $E_k^{(n)} = \sqrt{k^2 + m_n^2}$  (with  $a = 1$ ) are shown for the three VGT branches: cosmological ( $n = 1$ , solid blue), galactic ( $n = 2$ , dashed orange), and strong-field ( $n = 3$ , dotted green). The mass hierarchy  $m_1 < m_2 < m_3$  reflects the multi-scale structure of gravitational phenomena.

#### D. Normalization and Vacuum State

The mode functions are normalized via the Klein-Gordon inner product:

$$(u_k, u_{k'}) = -i \int d^3x a^3 (u_k \dot{u}_{k'}^* - \dot{u}_k u_{k'}^*) = \delta^{(3)}(\mathbf{k} - \mathbf{k'}). \quad (25)$$

The vacuum state  $|0\rangle$  is defined by

$$\hat{a}_{\mathbf{k}} |0\rangle = 0 \quad \forall \mathbf{k}. \quad (26)$$

In the adiabatic regime where  $\omega_k$  varies slowly, the WKB approximation gives

$$u_k^{\text{WKB}}(t) = \frac{1}{\sqrt{2a^3\omega_k}} \exp\left(-i \int^t \omega_k dt'\right). \quad (27)$$

This defines the adiabatic vacuum, which coincides with the Bunch-Davies vacuum in de Sitter space.

## IV. IR-UV EFFECTIVE BEHAVIOR

### A. Scale Hierarchy

The  $\psi_0$  fluctuations exhibit distinct behaviors across different momentum scales. We identify three characteristic regimes:

**Infrared (IR) regime:**  $k \ll m_{\text{eff}}$

$$E_k \approx m_{\text{eff}} + \frac{k^2}{2m_{\text{eff}}} + \mathcal{O}(k^4). \quad (28)$$

In this regime, modes are massive and propagate non-relativistically. The correlation length is  $\xi_{\text{corr}} \sim m_{\text{eff}}^{-1}$ .

**Intermediate regime:**  $k \sim m_{\text{eff}}$

This transition region connects IR and UV behaviors. Mode dynamics interpolate between massive and massless propagation.

**Ultraviolet (UV) regime:**  $k \gg m_{\text{eff}}$

$$E_k \approx k + \frac{m_{\text{eff}}^2}{2k} + \mathcal{O}(k^{-3}). \quad (29)$$

Modes become effectively massless and relativistic, with conformal behavior.

### B. Mode Power Spectrum

The power spectrum of  $\psi_0$  fluctuations is defined by

$$\langle \varphi_{\mathbf{k}} \varphi_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_{\varphi}(k), \quad (30)$$

where

$$P_{\varphi}(k) = |u_k|^2 = \frac{1}{2a^3\omega_k}. \quad (31)$$

The dimensionless power spectrum is

$$\mathcal{P}_{\varphi}(k) = \frac{k^3}{2\pi^2} P_{\varphi}(k) = \frac{k^3}{4\pi^2 a^3 \omega_k}. \quad (32)$$

In the IR limit ( $k \ll m_{\text{eff}}$ ):

$$\mathcal{P}_{\varphi}^{\text{IR}}(k) \approx \frac{k^3}{4\pi^2 a^3 m_{\text{eff}}} \propto k^3. \quad (33)$$

In the UV limit ( $k \gg m_{\text{eff}}$ ):

$$\mathcal{P}_{\varphi}^{\text{UV}}(k) \approx \frac{k^2}{4\pi^2 a^3} \propto k^2. \quad (34)$$

The transition between these behaviors occurs at  $k \sim m_{\text{eff}}$ , as illustrated in Fig. 2.

### C. Effective Degrees of Freedom

The scale-dependent effective number of degrees of freedom can be quantified through the integrated spectral density. Define the cumulative mode count:

$$N_{\text{eff}}(k_{\text{max}}) = \frac{1}{(2\pi)^3} \int_0^{k_{\text{max}}} 4\pi k^2 dk = \frac{k_{\text{max}}^3}{6\pi^2}. \quad (35)$$

However, the *dynamically active* degrees of freedom depend on the mass scale. For  $k < m_{\text{eff}}$ , modes are Boltzmann-suppressed at temperatures  $T < m_{\text{eff}}$ :

$$N_{\text{active}}(T) \approx \frac{T^3}{6\pi^2} \quad \text{for } T \ll m_{\text{eff}}. \quad (36)$$

This hierarchy in effective degrees of freedom underlies the scale-dependent behavior of gravitational couplings derived in the next section.

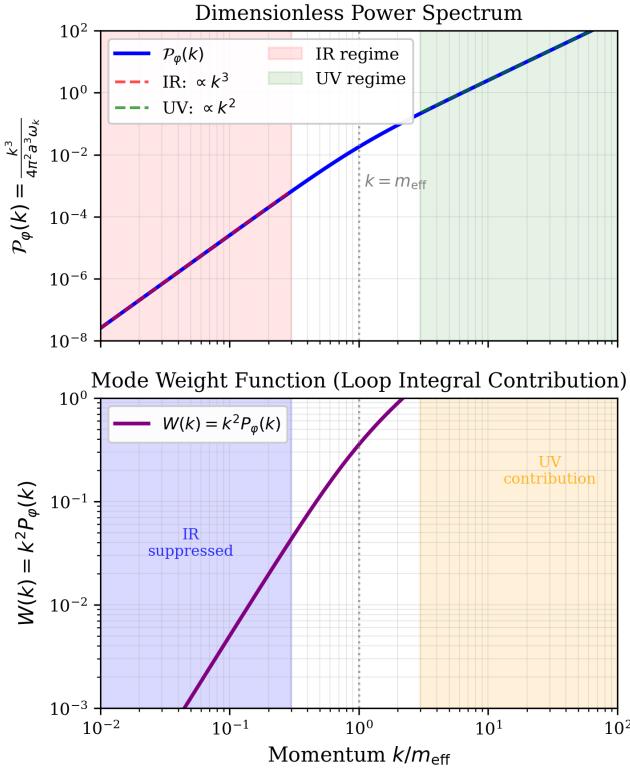


FIG. 2. Hierarchical mode structure from IR to UV. Upper panel: The dimensionless power spectrum  $\mathcal{P}_\varphi(k)$  transitions from  $k^3$  scaling in the IR to  $k^2$  scaling in the UV. Lower panel: Mode weight function  $W(k) = k^2 \mathcal{P}_\varphi(k)$  showing the effective contribution to loop integrals. The peak occurs near  $k \sim m_{\text{eff}}$ .

#### D. Mode Suppression Function

We introduce the mode suppression function

$$\mathcal{S}(k; m_{\text{eff}}) = \frac{k}{\omega_k} = \frac{k}{\sqrt{k^2 + m_{\text{eff}}^2}}, \quad (37)$$

which interpolates between  $\mathcal{S} \rightarrow k/m_{\text{eff}} \ll 1$  in the IR and  $\mathcal{S} \rightarrow 1$  in the UV.

The suppression function enters the effective action through loop integrals, modulating UV contributions and providing a natural regulator through the VGT mass hierarchy.

## V. CONTRIBUTION TO EFFECTIVE ACTION

### A. One-Loop Effective Action

The one-loop effective action is obtained by integrating out the quantum fluctuations  $\varphi$ :

$$e^{i\Gamma^{(1)}[g, \bar{\psi}_0]} = \int \mathcal{D}\varphi e^{iS^{(2)}[\varphi; g, \bar{\psi}_0]}. \quad (38)$$

Performing the Gaussian integral yields

$$\Gamma^{(1)} = -\frac{i}{2} \text{Tr} \ln (-\square_g + m_{\text{eff}}^2 + \xi R). \quad (39)$$

Using the heat kernel regularization, this becomes

$$\Gamma^{(1)} = \frac{1}{2} \int_0^\infty \frac{ds}{s} e^{-\epsilon s} \int d^4x \sqrt{-g} K(x, x; s), \quad (40)$$

where  $K(x, x'; s)$  is the heat kernel satisfying

$$(\partial_s + \square_g - m_{\text{eff}}^2 - \xi R) K(x, x'; s) = 0. \quad (41)$$

### B. Seeley–DeWitt Expansion

The coincident heat kernel admits the asymptotic expansion

$$K(x, x; s) = \frac{1}{(4\pi s)^2} \sum_{n=0}^{\infty} a_n(x) s^n, \quad (42)$$

where the Seeley–DeWitt coefficients for a scalar field with potential  $V = m_{\text{eff}}^2 + \xi R$  are

$$a_0 = 1, \quad (43)$$

$$a_1 = \left( \frac{1}{6} - \xi \right) R, \quad (44)$$

$$a_2 = \frac{1}{180} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - \frac{1}{180} R_{\mu\nu} R^{\mu\nu} + \frac{1}{6} \left( \frac{1}{5} - \xi \right) \square R + \frac{1}{2} \left( \frac{1}{6} - \xi \right)^2 R^2. \quad (45)$$

### C. Renormalized Effective Action

After dimensional regularization in  $d = 4 - \varepsilon$  dimensions and minimal subtraction, the renormalized one-loop effective action takes the form

$$\begin{aligned} \Gamma_{\text{ren}}^{(1)} = & \int d^4x \sqrt{-g} \left[ \frac{m_{\text{eff}}^4}{64\pi^2} \left( \ln \frac{m_{\text{eff}}^2}{\mu^2} - \frac{3}{2} \right) \right. \\ & + \frac{m_{\text{eff}}^2}{32\pi^2} \left( \frac{1}{6} - \xi \right) R \left( \ln \frac{m_{\text{eff}}^2}{\mu^2} - 1 \right) \\ & + \frac{1}{32\pi^2} \left\{ \frac{1}{180} R_{\mu\nu\rho\sigma}^2 - \frac{1}{180} R_{\mu\nu}^2 \right. \\ & \left. \left. + \frac{1}{2} \left( \frac{1}{6} - \xi \right)^2 R^2 \right\} \ln \frac{m_{\text{eff}}^2}{\mu^2} \right], \end{aligned} \quad (46)$$

where  $\mu$  is the renormalization scale.

### D. Running Gravitational Couplings

The effective action generates scale-dependent corrections to the gravitational couplings. Comparing with the

effective gravitational action

$$\Gamma_{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{R - 2\Lambda(k)}{16\pi G(k)} + \alpha(k)R^2 + \beta(k)R_{\mu\nu}^2 + \dots \right], \quad (47)$$

we extract the running couplings.

**Running Newton's constant:** The correction to the Einstein–Hilbert term yields

$$\frac{1}{G(k)} = \frac{1}{G_0} - \frac{m_{\text{eff}}^2(6\xi - 1)}{12\pi} \ln \frac{k^2}{m_{\text{eff}}^2}. \quad (48)$$

For conformal coupling ( $\xi = 1/6$ ), Newton's constant does not run at one-loop. For minimal coupling ( $\xi = 0$ ):

$$G(k) = \frac{G_0}{1 + \frac{G_0 m_{\text{eff}}^2}{12\pi} \ln(k^2/m_{\text{eff}}^2)}. \quad (49)$$

**Running cosmological constant:** The vacuum energy contribution gives

$$\Lambda(k) = \Lambda_0 + \frac{m_{\text{eff}}^4}{32\pi^2} \left( \ln \frac{k^2}{m_{\text{eff}}^2} - \frac{3}{2} \right). \quad (50)$$

**Higher-derivative couplings:** The  $R^2$  and  $R_{\mu\nu}^2$  coefficients run as

$$\alpha(k) = \alpha_0 + \frac{(6\xi - 1)^2}{1152\pi^2} \ln \frac{k^2}{m_{\text{eff}}^2}, \quad (51)$$

$$\beta(k) = \beta_0 - \frac{1}{2880\pi^2} \ln \frac{k^2}{m_{\text{eff}}^2}. \quad (52)$$

Figure 3 illustrates the RG flow of these couplings from IR to UV.

### E. Beta Functions

The beta functions for the gravitational couplings are derived from the scale dependence:

$$\beta_G \equiv k \frac{\partial G}{\partial k} = -\frac{G^2 m_{\text{eff}}^2 (6\xi - 1)}{6\pi}, \quad (53)$$

$$\beta_\Lambda \equiv k \frac{\partial \Lambda}{\partial k} = \frac{m_{\text{eff}}^4}{16\pi^2}, \quad (54)$$

$$\beta_\alpha \equiv k \frac{\partial \alpha}{\partial k} = \frac{(6\xi - 1)^2}{576\pi^2}. \quad (55)$$

These beta functions satisfy the consistency requirements:

1.  $\beta_G < 0$  for  $\xi < 1/6$  (asymptotic freedom in gravity).
2.  $\beta_\Lambda > 0$  (cosmological constant grows toward UV).
3.  $\beta_\alpha > 0$  (higher-derivative terms become relevant at high energies).

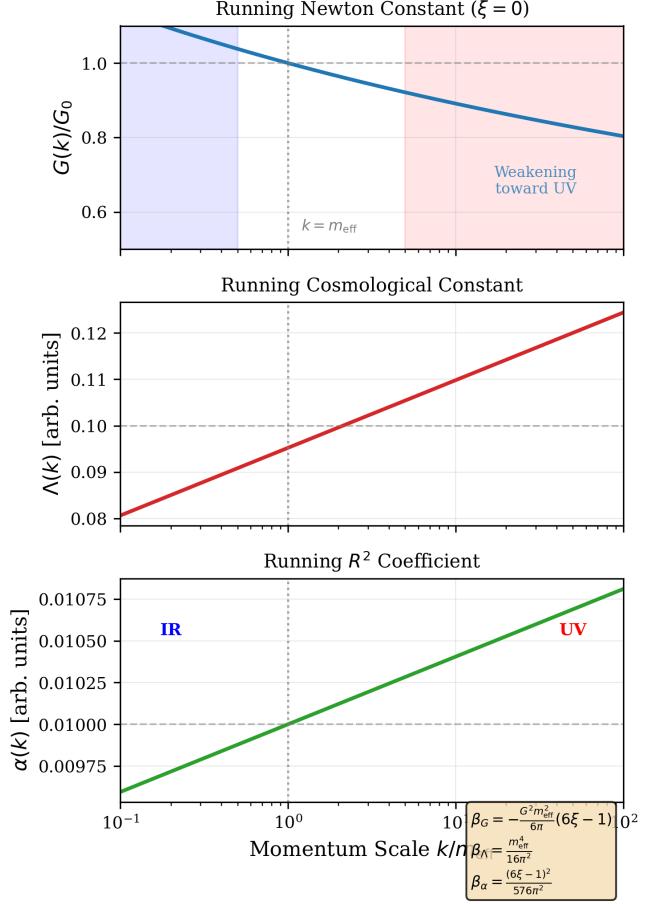


FIG. 3. Renormalization group flow of gravitational couplings. Upper panel:  $G(k)/G_0$  showing the weakening of gravity at high scales for  $\xi = 0$ . Middle panel:  $\Lambda(k)$  exhibiting logarithmic running. Lower panel:  $\alpha_{\text{eff}}(k)$  showing the growth of higher-derivative terms toward UV. The vertical line marks  $k = m_{\text{eff}}$ .

### F. Multi-Branch Contributions

In VGT, each gravitational branch contributes independently to the effective action. The total one-loop contribution is

$$\Gamma_{\text{total}}^{(1)} = \sum_{n=1}^3 w_n \Gamma^{(1)}[m_n], \quad (56)$$

where  $w_n$  are branch-dependent weights determined by the occupation probability at each minimum.

The hierarchy  $m_1 \ll m_2 \ll m_3$  implies that:

- At cosmological scales ( $k \lesssim m_1$ ), the  $n = 1$  branch dominates.
- At galactic scales ( $m_1 \lesssim k \lesssim m_2$ ), transitions between branches occur.
- At strong-field scales ( $k \gtrsim m_3$ ), all branches contribute with UV-dominated behavior.

This scale-dependent superposition of branches is the hallmark of the VGT quantum structure.

## VI. CONCLUSION

We have constructed the quantum structure of the scalar root field  $\psi_0$  within the Virtual Gravity Theory framework. The key results are:

1. **Canonical quantization:** The field  $\psi_0$  admits consistent quantization in curved spacetime, with mode functions satisfying a generalized Mukhanov–Sasaki equation (17).
2. **Eigenmode spectrum:** The dispersion relation  $E_k^{(n)} = \sqrt{k^2 + m_n^2}$  exhibits branch-dependent mass gaps corresponding to the three VGT gravitational regimes.
3. **IR–UV hierarchy:** The power spectrum transitions from  $\mathcal{P}_\varphi \propto k^3$  in the IR to  $\mathcal{P}_\varphi \propto k^2$  in the UV, with the effective degrees of freedom showing scale-dependent activation.
4. **Effective action:** The one-loop effective action generates running gravitational couplings  $G(k)$ ,

$\Lambda(k)$ , and higher-derivative terms  $\alpha(k)$ ,  $\beta(k)$ , with explicit beta functions (53)–(55).

5. **Multi-branch structure:** The superposition of contributions from  $n \in \{1, 2, 3\}$  branches provides a natural mechanism for scale-dependent gravitational phenomenology.

These results provide the foundation for subsequent chapters in Volume II (TFOS):

- Chapter VI will apply this quantum structure to derive graviton propagators and tensor mode spectra.
- Chapter VII will compute gravitational wave signatures from the  $\psi_0$ -mediated interactions.
- Chapter VIII will extend to two-loop effects and non-perturbative contributions.

The quantum structure of  $\psi_0$  established here demonstrates that VGT provides a consistent and calculable framework for quantum gravity, with observationally testable predictions emerging naturally from the multi-branch structure and scale-dependent effective couplings.

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- [1] T. Author, “Virtual Gravity Theory Volume I: Foundations,” VGT-I (2025).
- [2] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press (1982).
- [3] L. Parker and D. Toms, *Quantum Field Theory in Curved Spacetime*, Cambridge University Press (2009).
- [4] V. F. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press (2005).
- [5] B. S. DeWitt, “Quantum Field Theory in Curved Spacetime,” Phys. Rep. **19**, 295 (1975).
- [6] D. V. Vassilevich, “Heat kernel expansion: User’s manual,” Phys. Rep. **388**, 279 (2003).
- [7] J. F. Donoghue, “General relativity as an effective field theory: The leading quantum corrections,” Phys. Rev. D **50**, 3874 (1994).
- [8] S. Weinberg, *The Quantum Theory of Fields, Vol. II*, Cambridge University Press (1996).
- [9] T. S. Bunch and P. C. W. Davies, “Quantum field theory in de Sitter space: Renormalization by point splitting,” Proc. R. Soc. Lond. A **360**, 117 (1978).
- [10] M. Reuter, “Nonperturbative evolution equation for quantum gravity,” Phys. Rev. D **57**, 971 (1998).