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CS 252

TA 8

5.1) Prove EQ_{CFG} is undecidable. Reduce ALL_{CFG}, which is undecidable, to EQ_{CFG}. Prove by contradiction: Assume EQ_{CFG} is decidable and M is the TM that decides it. Make a CFG G' where $L(G') = \Sigma^*$ and use it with M to compare the given grammar G with G'. If they are equal, ALL_{CFG} should accept. Construct the decider for ALL_{CFG}: submit $\langle G, G' \rangle$ to M. If it accepts, accept. If it rejects, reject. Because we have a decider for ALL_{CFG}, that is a contradiction, so EQ_{CFG} is undecidable.

5.4) A does not have to be a regular language. Suppose $A = \{1^n 0^n \mid n \geq 0\}$, which is not regular. Suppose we have a function f such that $f(s) = 0$ if $s \in A$ and $f(s) = 1$ if $s \notin A$. $f(A) = 0 = B$, which is a regular language, so we see that A does not also have to be regular when $A \leq_m B$ and B is a regular language.

5.12) Reduce A_{TM}, which is undecidable, to the stated problem. Proof by contradiction: Assume the language stated in the problem L is decidable by the decider D. Make a new TM M' which is a copy of the TM M described in the problem except it has an additional transition that draws a blank symbol before it transitions to the accept state. Run M' on D. If it accepts, accept. Else, reject. Because we made a decider for A_{TM}, this is a contradiction, so L is undecidable.