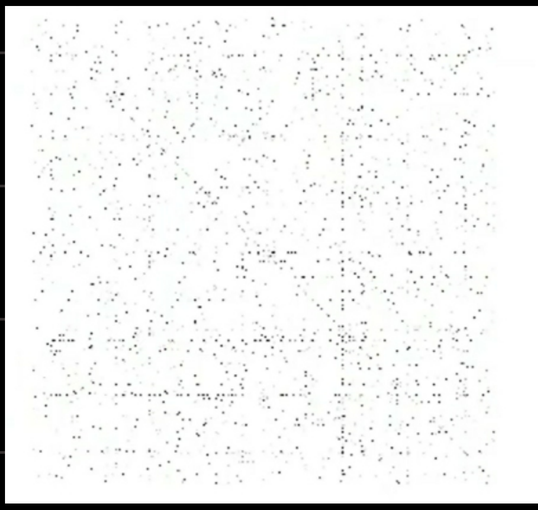
Graph Representation

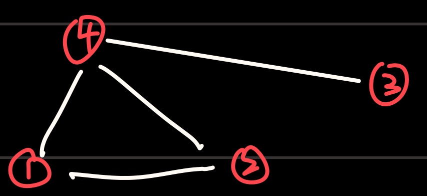
Def Adjacency Matrix

$A \in M_{N \times N}$ s.t $\begin{cases} A_{ij} = 1 & \text{undirect } i \leftrightarrow j \text{ ; direct } i \to j \\ A_{ij} = 0 & \text{O.W} \end{cases}$

Property → A very sparse matrix →



① Undirected Graph



①: 12, 14
②: 21, 24
③: 34
④: 41.42.43

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & \boxed{0} & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix},$$

Node②

$A_{ij} = A_{ji}$

$A_{ii} = 0$

Node Degree

$$\boxed{k_i = \sum_j^N A_{ij} = \sum_j^4 A_{ij}}$$

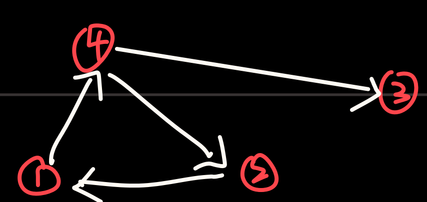
$$\boxed{k_j = \sum_i^N A_{ij} = \sum_i^4 A_{ij}}$$

$$L = \frac{1}{2}\sum_i^N k_i = \frac{1}{2}\sum_{i,j}^N A_{ij}$$

Avg nd

$$\bar{k} = \frac{2E}{N} = \frac{2 \cdot 4}{4}$$

② directed Graph



①: 14
②: 21
③: no
④: 42.43

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & \boxed{0} \end{pmatrix},$$

out

In

Node ④

$A_{ij} \neq A_{ji}$

$A_{ii} = 0$

Node Degree

$$\boxed{k_i^{out} = \sum_j^N A_{ij} = \sum_j^4 A_{ij}}$$

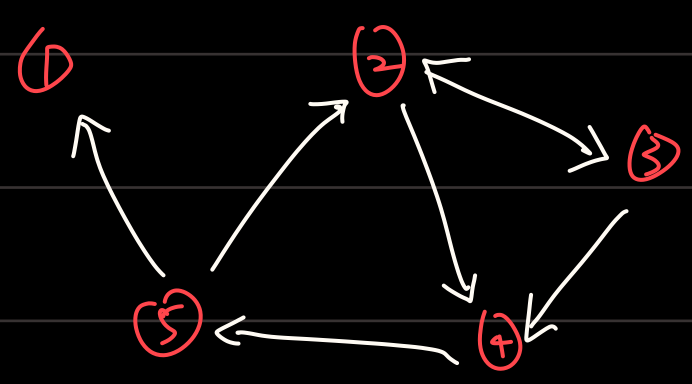
$$\boxed{k_j^{in} = \sum_i^N A_{ij} = \sum_i^4 A_{ij}}$$

$$L = \sum_i^N k_i = \sum_j^N k_j^{out} = \sum_{ij}^N A_{ij}$$

Avg nd

$$\bar{k} = \frac{E}{N} = \frac{4}{4}$$

Def Adjacency List



Out list

1 : no

2 : 3,4

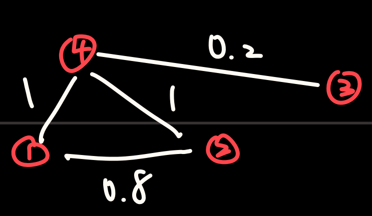
3 : 2,4

4 : 5

5 : 1. 2

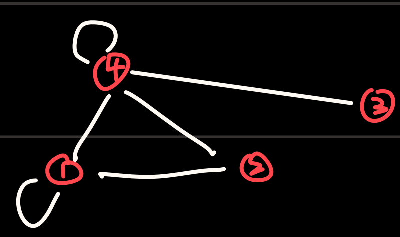
More Types

Weighted



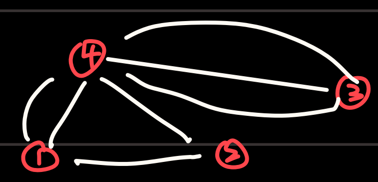
$$A = \begin{pmatrix} 0 & 0.8 & 0 & 1 \\ 0.8 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.2 \\ 1 & 1 & 0.2 & 0 \end{pmatrix}$$

self-edges



$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

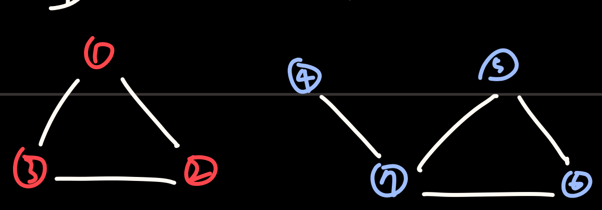
Multigraph



$$A = \begin{pmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{pmatrix}$$

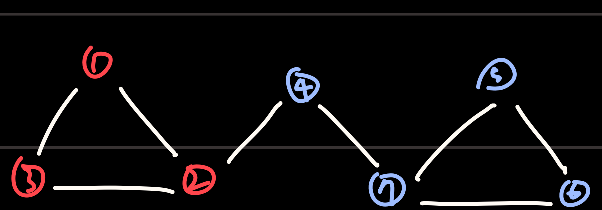
Connectivity

Disconnected



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Connected



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & \boxed{1} & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Types of Connectivity

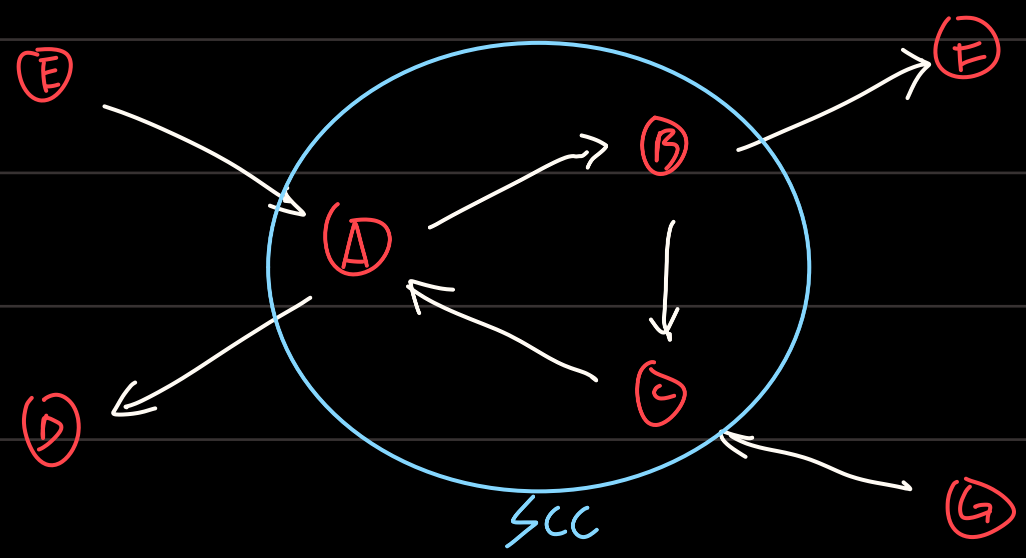
① Strongly connected directed graph

$A \longleftrightarrow B$

② Weakly connected directed graph

$A \longrightarrow B$

Strongly Connected Components (SCCs)



In - components : A, B, C

Out - components : E, D, F, G