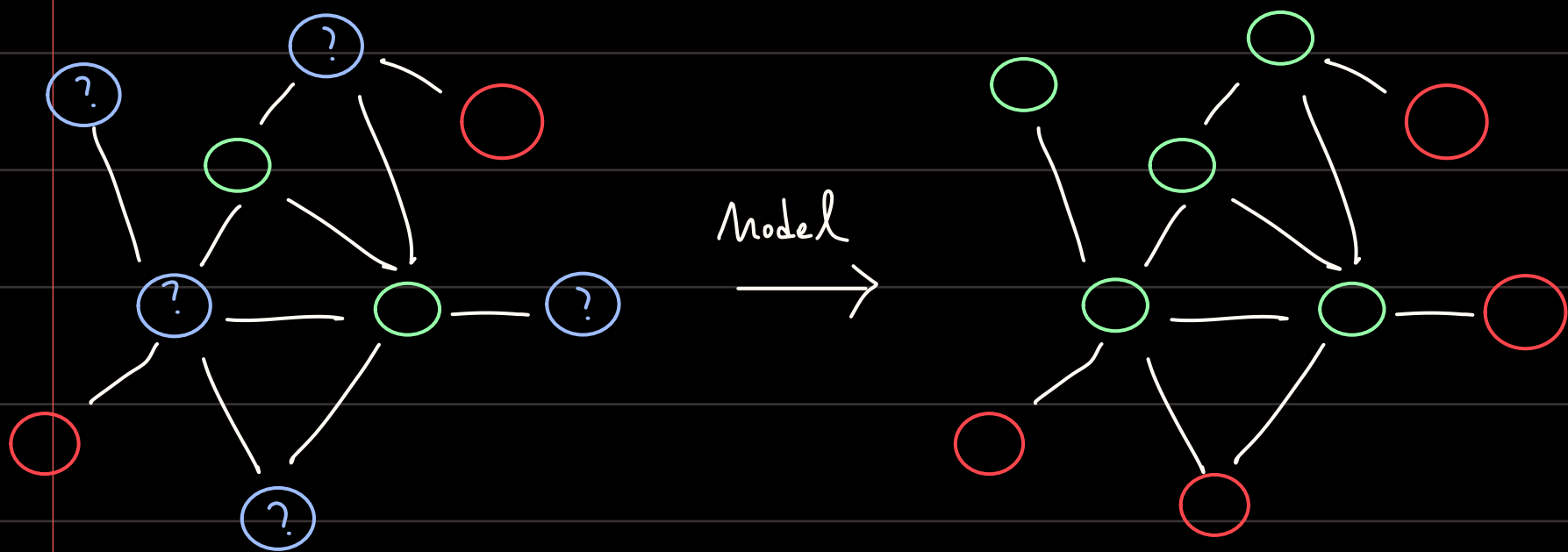


Node-Level Features

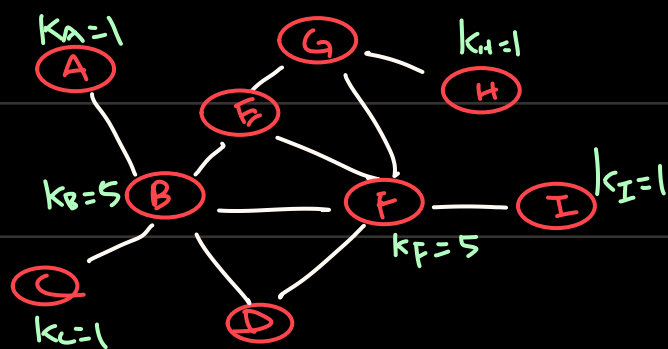
Node Classification \rightarrow Consider 2 labels: $\{\text{green circle}, \text{red circle}\}$



Characterize Node Features: Node Degree, Node Centrality

① Node Degree

Closing Coefficient, Graphlets



Weak: $k_A = k_C = k_H = k_I$
 $k_B = k_F \Rightarrow$ model can't distinguish

② Node Centrality \rightarrow How Important the Node is

(Def) Eigenvector Centrality

Node v is Important if surrounding by important nodes $u \in N(v)$

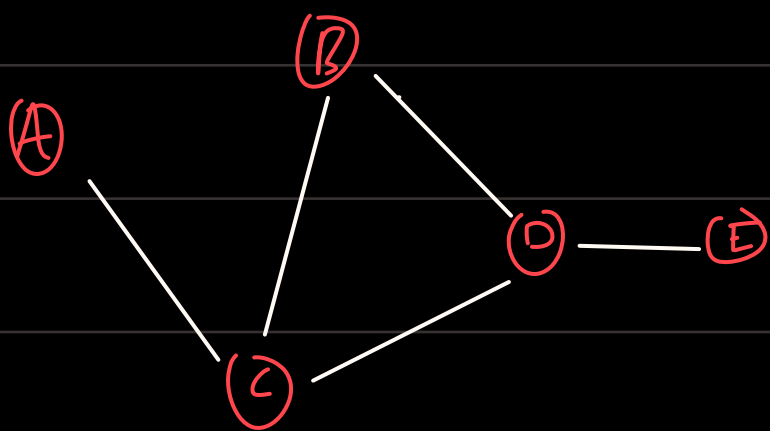
$$C_v = \frac{1}{\lambda} \sum_{u \in N(v)} C_u, \lambda > 0 \text{ iff } \lambda C = AC, \text{ where } A: \text{Adjacency Matrix}, C: \text{Centrality Vector}$$

By Perron - Frobenius Thm $\rightarrow \lambda_{\max} > 0$ and unique
 $\rightarrow C_{\max}$ is used for Centrality

Def Betweenness Centrality

Node v is important if lies on many shortest path between other nodes

$$C_v = \sum_{s \neq v \neq t} \frac{\text{Count: shortest paths between } s \text{ and } t \text{ that contains } v}{\text{Count: shortest paths between } s \text{ and } t}$$



$$C_A = C_B = C_E = 0$$

$$C_C = 3 \rightarrow \begin{matrix} A \rightarrow B \\ A \rightarrow D \\ A \rightarrow E \end{matrix} \rightarrow ACB, ACD, ACDE$$

$$C_D = 3 \rightarrow \begin{matrix} B \rightarrow E \\ C \rightarrow E \\ A \rightarrow E \end{matrix} \rightarrow BDE, CDE, ACDE$$

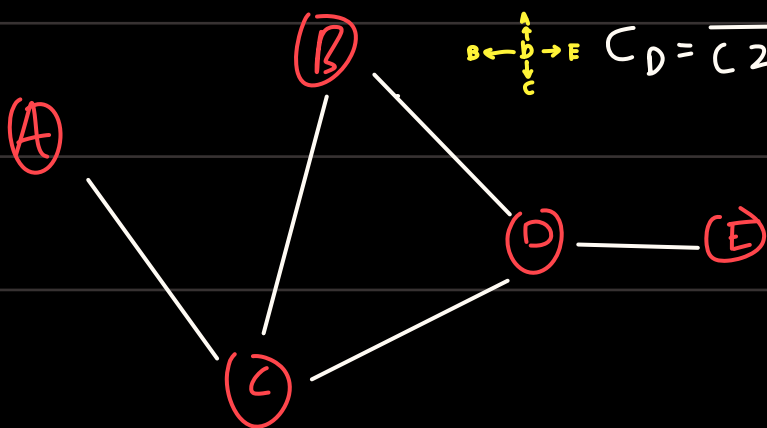
Def Closeness Centrality

Node v is important if it has small shortest path length to all other nodes

$$C_v = \frac{1}{\sum_{m \neq v} \text{shortest path length between } m \text{ and } v}$$

$$C_A = \frac{1}{(2+1+2+3)} = \frac{1}{8} \rightarrow (ACB, AC, ACD, ACDE)$$

$$C_D = \frac{1}{(2+1+1+1)} = \frac{1}{5} \rightarrow (DCA, DB, DC, DE)$$

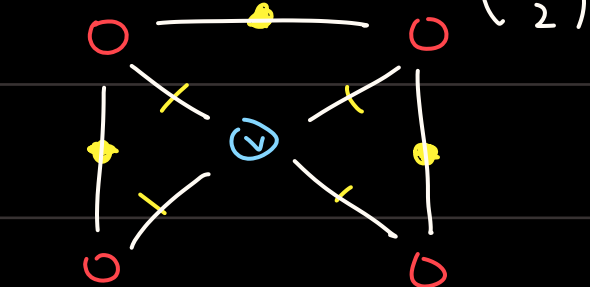


Importance: $C_D > C_A$

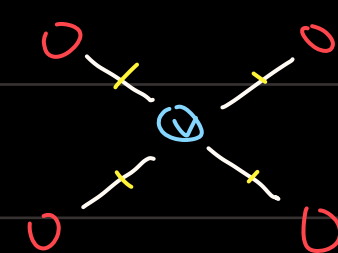
Def Clustering Coefficient

Measures how connected v 's neighboring nodes

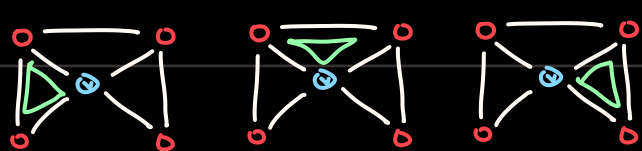
$$e_v = \frac{\text{Count: edges among neighboring nodes}}{\binom{k_v}{2}} \in [0, 1]$$



$$e_v = \frac{1}{\binom{4}{2}} \cdot 3 = \frac{1}{2}$$



$$e_v = \frac{1}{\binom{4}{2}} \cdot 0 = 0$$

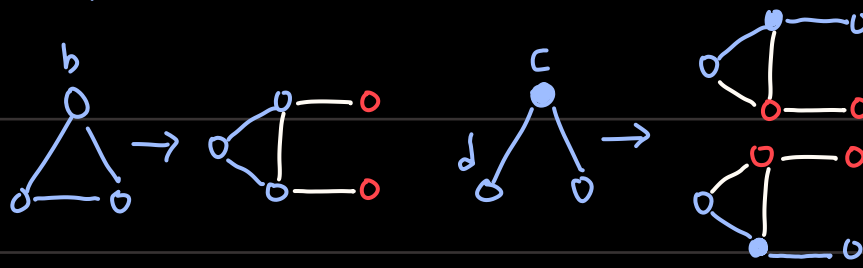
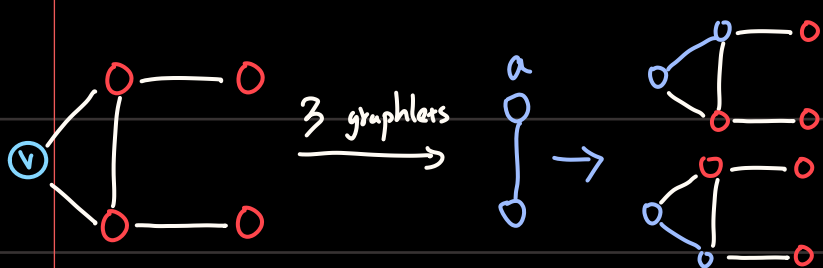
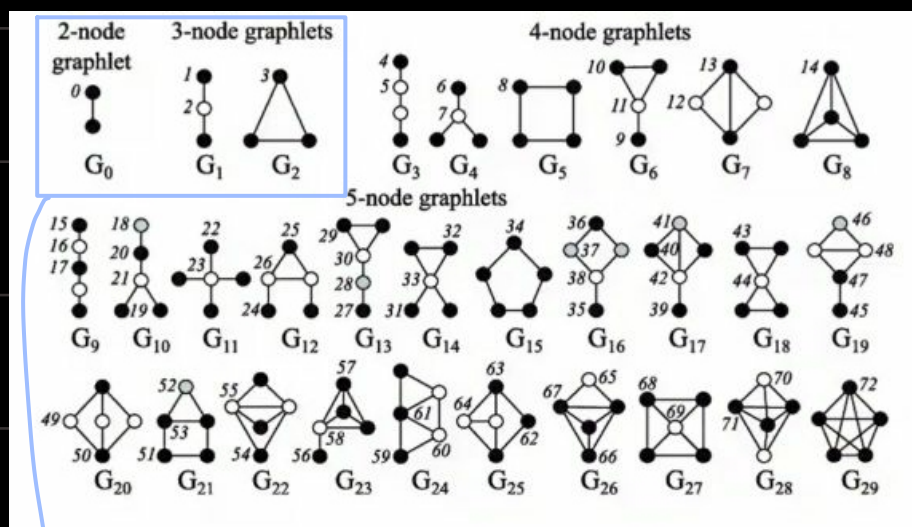


Social Network \rightarrow friend of friend also my friend

Def Graphlets \rightarrow Rooted Connected Non-Isomorphic Subgraphs

Graphlet Degree Vector (GDV): Graphlet-base feature for node
(A count vector of graphlets rooted at a given node)

- Degree counts that a Node touches Edges
- Clustering counts that a Node touches Triangles
- GDV counts that a Node touches Graphlets



GDV of Node v on 2 to 3 nodes: $(a, b, c, d) \rightarrow (2, 1, 0, 2)$

If on 2 to 5 \rightarrow $GDV \in V^{1 \times 13}$

GDV provide a measure of a Node's Local Network Topology

Comparing GDV of 2 Nodes provides a more detailed measure of local topological similarity than Node Degrees or Clustering Coefficient