Assignment 4
Kawasaki Z650

Multiple-DOF vibration model of the motorbike frame

(20 points)

MECA0155 – Dynamics of mechanical systems

Academic year 2024-2025

Objective

Multiple-DOF vibration model of the motorbike frame

The aim of the assignment is to

- build a multiple-degree-of–freedom model of the motorbike frame in bending in the vertical plane (“pitch” modes) using the Rayleigh-Ritz method;
- compute an estimation of the first 4 natural frequencies in bending and the corresponding mode-shapes;
- compare the theoretical predictions with the experimental results.

Structural modelling

- The model simulates a constant speed motion
  - The frame is represented as a beam whose characteristics are listed in the general presentation of the project
  - Two concentrated masses are located at points B and D
    - B: represents the engine and the auxiliaries, located at mid-distance between the fork (A) and the swingarm (C) and at 250 mm below the frame (linked to the beam with a rigid link)
    - D: represents the driver, assumed to be on the frame
  - The frame is supported by the fork and the swingarm
    - Both are modeled as linear and rotational springs with adequate stiffness

$$M_B = M_{\text{mot}}$$
$$M_{dv} = 75 \text{ kg}$$
$$J_B = 1 \text{ kg·m}^2$$
$$J_D = 10 \text{ kg·m}^2$$

$$k_{1l} = 10^4 \text{ N/m}$$
$$k_{1r} = 10^9 \text{ Nm/rad}$$
$$k_{2l} = 10^5 \text{ N/m}$$
$$k_{2r} = 10^4 \text{ Nm/rad}$$

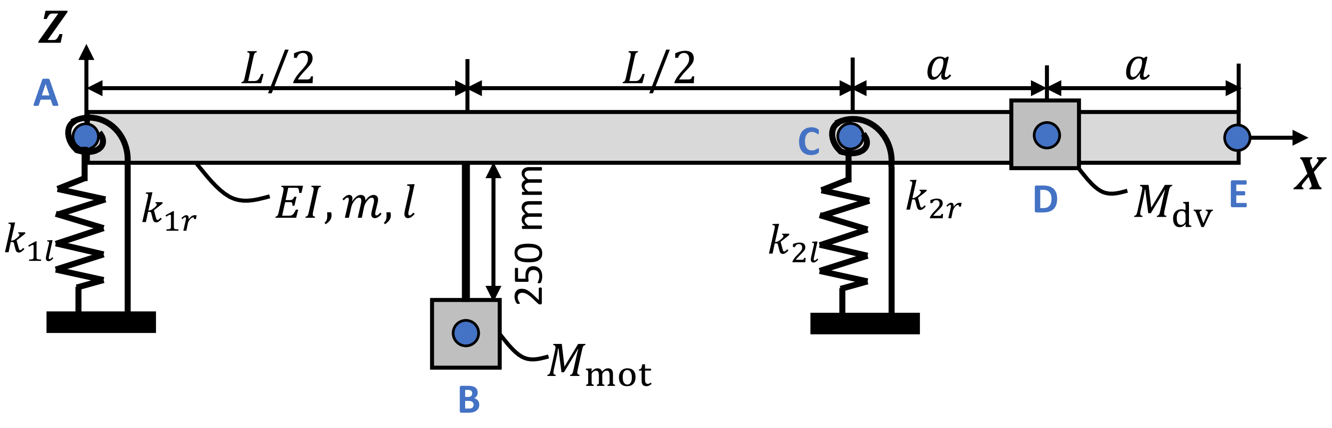
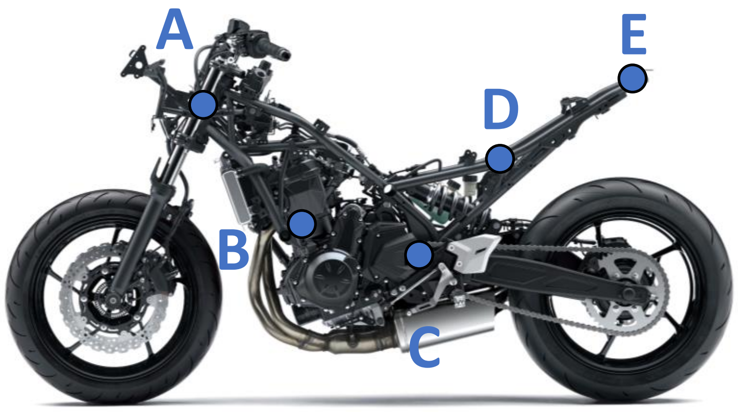


Figure 1 – SDOF model with $l = L + 2a$, $a = \frac{L}{4} = 0.2$ m.



Modal testing

- The frame is instrumented by a total of 14 accelerometers
  - 13 accelerometers are distributed equally along the frame and 1 is located at the engine point (B)
    - Among the 13 equally distributed accelerometers, the fork (A), the swingarm connection (C), the driver seat (D) and the passenger seat (E) have their own accelerometer.

Reference point	Distance from the fork along the frame [mm]	Reference point	Distance from the fork along the frame [mm]
P1	0 (A)	P8	600
P2	100	P9	700
P3	200	P10	800 (C)
P4	300	P11	901
P5	400 (B)	P12	1000 (D)
P6	400	P13	1101
P7	500	P14	1200 (E)

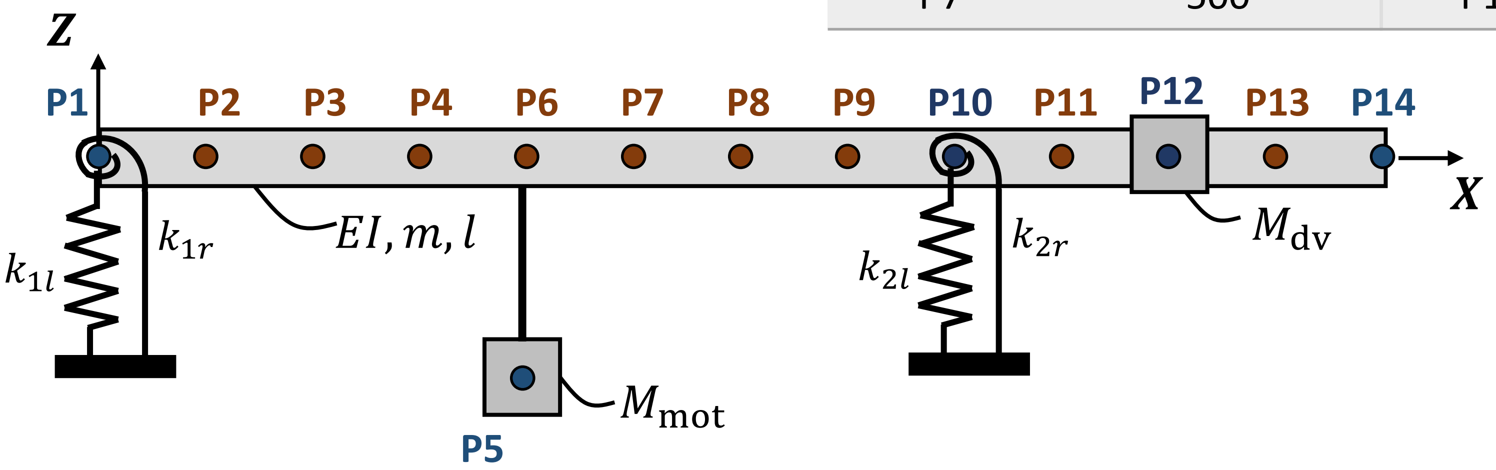


Figure 1 – SDOF model with $l = L + 2a$, $a = L/4$. Use $a = 0.2$ m.

Modal testing

- Numerical data available on eCampus
- The file ‘P2024_f_Part4.txt’ contains 4 identified frequencies corresponding to the first ‘pitch’ modes (in the XZ-plane).

Frequency [Hz]
…

- The file ‘P2024_Modes_Part4.txt’ contains the amplitudes **[mm]** of the corresponding identified mode-shapes (columns 1 to 4) under the format:

Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
…	…	…	…
…	…	…	…
…	…	…	…

14 lines corresponding to locations P1 to P14

The measured mode-shapes are mass-normalized.

Project statement

- Write the analytical expressions of the kinetic and potential energies of the system (slide 3).
    - Fully consider the contribution of the concentrated masses.
- Compute the first 4 natural frequencies in bending using the Rayleigh-Ritz method.
    - Study the convergence of the results by increasing the number of approximation functions from 1 to at least 6.
    - Compute the relative errors on the frequencies between analytical results and numerical data. Comment your results.
- Describe the results in terms of mode-shapes and compare with the numerical data.
- Compute the MAC matrix between the two sets of mode-shapes as explained hereafter. Comment your results.

Comparison of the theoretical and experimental mode-shapes

Defining

- Ψ_x as a numerical mode shape, and
- Ψ_a as an analytical eigenvector (predicted by a model),

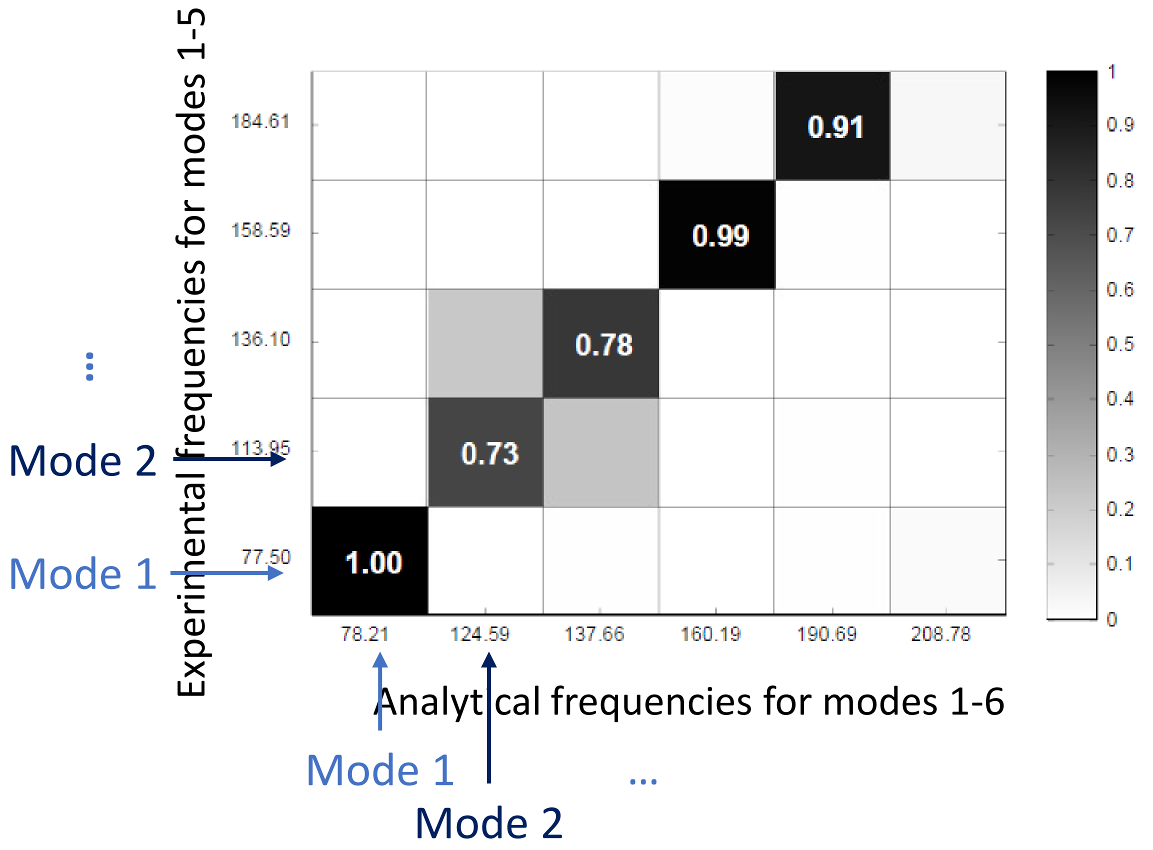
the Modal Assurance Criterion (denoted MAC) matrix is defined as

$$MAC = \frac{\left|{\Psi_x}^T \Psi_a\right|^2}{\left({\Psi_x}^T \Psi_x\right)\left(\Psi_a^T \Psi_a\right)} \quad \text{with } 0 \leq MAC \leq 1$$

The MAC allows to quantify the degree of correlation between two sets of mode-shapes. When it is close to 1, the mode-shapes are well correlated, otherwise they do not correspond one to each other.

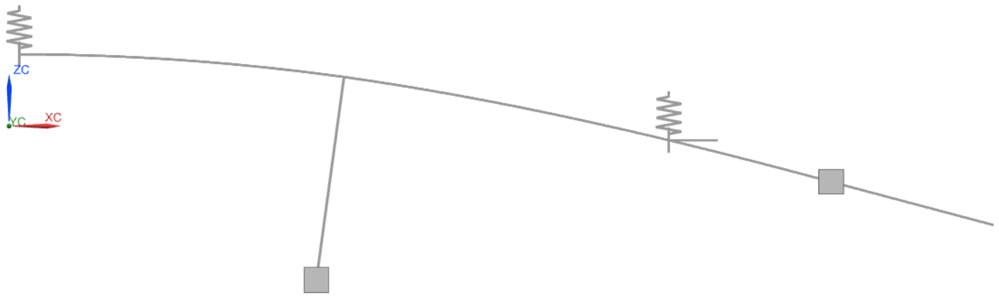
Comparison of the theoretical and experimental mode-shapes

- For the seek of illustration, the figure on the right represents the matrix for two sets of 6 analytical and 5 experimental mode-shapes respectively.
- It can be observed that the correlation is better for modes 1, 4 and 5 than for modes 2 and 3.
  - The cause of bad correlation usually comes from modelling errors (assumptions) and approximations (geometry) or from uncertainties on physical parameters.

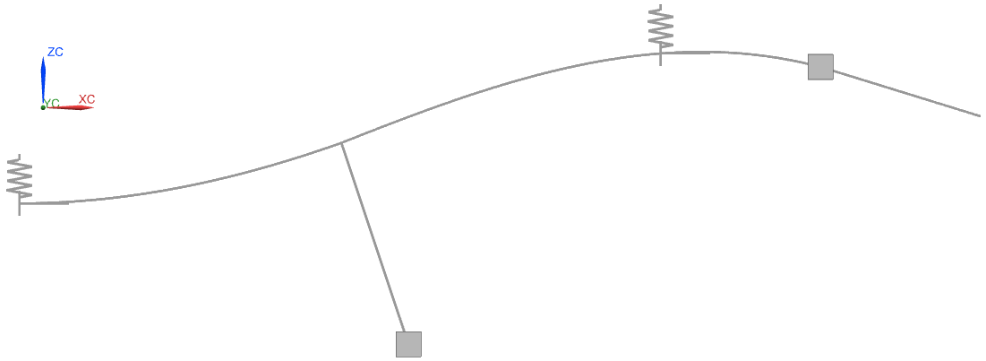


Bending modes

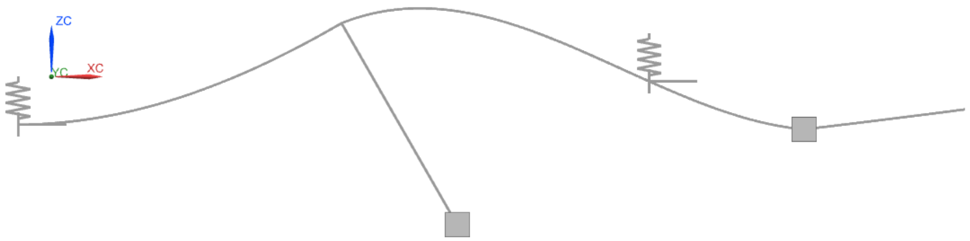
Mode 1: 3.71 Hz



Mode 2: 66.98 Hz



Mode 4: 243.85 Hz



Mode 5: 451.46 Hz

Specific guidelines for assignment 4

1) Pay attention to the clarity of the figures (axes, units, grids, legend, etc.)
2) The length of the report will not exceed 7 pages including figures.
3) You can ask your questions on the dedicated forum on eCampus (preferably).
4) The deadline for the submission of the report (on eCampus platform) is fixed to

December 20, 2024 at 18:00