

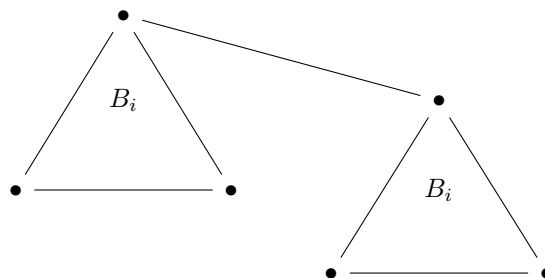
send ~~memes~~ comments to tomachello3@gmail.com

**amortised cost** example. you're given an  $n$  bit register, and the operation  $inc : x \mapsto x + 1$ . it's reasonable to say  $\text{cost}(inc, x) = \#\{\text{lsb bits} = 1\}$ . so at worst  $\text{cost} = n$ . on the other hand,  $\mathbb{E}\text{cost} = \sum k/2^k$  is small. but also note the following: if you had  $\text{cost} = k$  then it will take an exponential time for cost to equal  $k$  again. [one costly operation for a bunch of cheap ones.](#)

**Tarjan's potential method** let  $\phi : S \mapsto \mathbb{Z}_{\geq 0}$  assign structures a non-negative integer. if  $a = t + \Delta\phi$  where  $t$  is the time it takes to make the change  $\Delta S$ . then  $\sum a + \phi(S_0) \geq t_{\text{total}}$ .  
above example contd. let  $\phi = \text{total \#1's in register}$ . if  $\text{cost} = k$  then  $t = k$  but  $\Delta\phi = 1 - k$  (or  $-k$  if  $k = n$ ) so that  $a = 1$  and  $t_{\text{total}} \leq \# \text{operations} + n$ .

**credit method** give each item some dollars. if an operation costs  $t$ , it has to be paid for by the items. if we can make this work,  $t_{\text{total}} \leq \text{starting total money}$ .

**binomial heap**  $B_0 = \bullet$  and  $B_{i+1} = (1+r)B_i =$



in  $B_n$  we have  $2^n$  vertices and height  $n$ . the root has  $n$  edges and there's  $\binom{n}{k}$  nodes of depth  $k$ . say we have  $N = \sum a_j 2^j$  keys. we keep a  $B_j$  if  $a_j = 1$ . it costs 1 to make two  $B_i$ 's into one  $B_{i+1}$  so it costs  $\log_2 N$  to meld two heaps of size at most  $N$  or insert a new key. it costs  $\log_2 N$  to find min, and also to delete it (how?).

algo2 notes <https://www.cs.technion.ac.il/~hamilis/Algorithms2.pdf>  
data structures 2 notes <https://www.cs.technion.ac.il/~itai/Courses/ds2/lectures/lecture.html>