tomachello

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introduction to the tomachello project

goal: creating a comprehensive mathematics book, pedagogically suited for gifted youth.
motivation: to write the book I needed growing up.
list of guidelines:
1. the reader has the job of discovering the math, while my job is to choose the exercises, guide the discussion, simplify and clarify the ideas and technique, and most importantly challenge the reader and make them love math.
2. in presenting an argument I must focus on the key idea, taking all details for granted - mathematical exposition should not be confusing to follow.
3. if a reader feels I did not follow one of the above guidelines they should let me know.

words

problem #bvo

if two words commute, they must both be powers of a third word.

$$\{(w_1, w_2) \in \textsc{words}^2 \textbf{ s.t. } w_1 w_2 = w_2 w_1\} = \{(w^\alpha, w^\beta) \textbf{ s.t. } w \in \textsc{words},\ \alpha, \beta \in \mathbb{Z}_{\geq 0}\}$$

problem #att

the set of all periods of a given word is closed under gcd, generally speaking.

$$\alpha, \beta \in \texttt{periods}(\mathit{word}),\ \alpha+\beta-\texttt{gcd}(\alpha,\beta) \leq \texttt{length}(\mathit{word}) \implies \texttt{gcd}(\alpha,\beta) \in \texttt{periods}(\mathit{word})$$

problem #fll

suppose you found two equal consecutive sub words of some *word* of length $\texttt{minimal period}(\mathit{word}) - 1$. then they differ in placement by a period of *word*.

algorithms

problem #upl

generate random permutation using random number generator

problem #ocj

longest common sub sequence of two words

problem #uhv

finding largest $j-i$ where $i<j$ and $a_i > a_j$ in a long sequence

information

problem #see

alice tells bob n, as well as the elements of a sphere $S = S(x)$ in the hamming cube Q_n. how should bob find x?

problem #zgj

thirteen objects are displayed to the audience. they choose two, and tell them to bob. then alice walks in the room. bob now tells alice that a certain bulb is not one of the two chosen ones. now alice gets four attempts at finding the two chosen ones out of the twelve. how is the trick done?

problem #aqd

on each of the 64 squares of a chessboard is a coin- some heads, some tails. the audience chooses a certain square on the board and tell bob. bob then flips one coin, after which alice comes in the room, looks only at the chessboard, and after some thought reveals the audiences square. how is the trick done?

problem #qoj

there's n people in a row, and everyone has a hat with a different number from 1 to $n+1$. everyone sees only those in front of them. from the end of the line, they need to guess their number, hearing the answers of the people before them. they cannot guess an answer that was already said. how can they maximize the number of correct answers?

problem #yfg

there's n people in a row, and everyone has a hat with a number from 1 to k (start with $k=2$). everyone sees only those in front of them. from the end of the line, they need to guess their number, hearing the answers of the people before them. how can they maximize the number of correct answers?

problem #uzb

there's n people in a circle, and everyone has a hat with a number from 1 to n. (start with $n = 2$) everyone sees everyone but themselves. they all simultaneously guess their color. how can they guarantee that at least one person gets it right?

problem #hrm

!!haven't solved yet!! there's 9^n coins, all weighing the same except for one which is lighter. you have three scales (you can put some coins on one side and some on the other and it'll tell you which side weighs more) but one scale is broken and can output randomly/adversely. how to find lighter coin in $3n + 1$ steps?

problem #yuq

there's n people, some good and some bad. you don't know which is which, only that there are more good than bad. you can ask any if any other is good or bad, and they'll tell you. if they are bad, they can lie. how can you find out which is which in, say, $2n - 2$ questions?

probability

problem #ifb

monty hall

combinatorics and algebra

problem #lyr

eads: at a certain mathematical conference, every pair of mathematicians are either friends or strangers. At mealtime, every participant eats in one of two large dining rooms. Each mathematician insists upon eating in a room which contains an even number of his or her friends. Prove that the number of ways that the mathematicians may be split between the two rooms is a power.

Euclidean geometry

problem #fou

two circles intersect at A. two points, starting simultaneously from A, trace the circles, both in the same orientation and constant angular speed [they meet at A again after each revolved once around its circle] prove the existence of a point P, such that the two points are always equidistant from P.