School of Mathematical Sciences

Vocational Financial Mathematics

MATH 4055

Session 2020 – 2021

PORTFOLIO MANAGEMENT

Project 3 – Group 5

Tom Aldridge
Elena Chioti
Troy Da Costa
George Duxberry
Luke Hind
Reshma Nambiar

EXECUTIVE SUMMARY

We have been given 10MM and tasked with creating a means for effective portfolio management over a period of 10 years, which includes regular automatic adjustments to maintain our profits. Our task includes creating a programme which does just this and generates a profitable level of return whilst incurring a minimal amount of risk.

Our team did extensive research about the general stock market, its risks, portfolio management methods and portfolio management risks to construct an effective algorithm to pick our stocks from a given market(s). Our aim was to produce a programme that allows us to enter an initial investment amount and track its progress through forecasting tools and optimization techniques, which we learned about through additional research as experimental work with R, a software for statistical computing and graphics. We used forecasting techniques in R to estimate returns and beta values of the stocks to give an indication of the level of risk that each stock poses. These beta values were extracted from the S&P500 Index in Yahoo! Finance by our team, which were updated during our regular adjustments.

We began testing with a few stocks and used results from the actual returns to check whether our forecasting methods were accurate. We continued to do this and corrected any errors that came about, until we were able to test it with a larger number of stocks and a larger fixed initial investment, allowing our programme to select stocks and generate a comfortable level of return from stocks in the S&P500 Index. As well as forecasting returns, our model also includes transaction fees, 2.5% every time we buy and sell, so a total of at least 5% for every adjustment. We then used our portfolio management research to select three types of portfolio management methods to test which gave the highest level of returns over a period of 10 years, from which we concluded that the most effective method of portfolio management in our case was the Maximum Returns Test, yielding the most profitable portfolio, although it did have some limitations regarding the investor’s risk attitude.

CONTENTS PAGE

1. INTRODUCTION

Our task is to create and manage a stock portfolio for, as a group, our 10MM GBP. The aim being to further grow our wealth over the following 5-to-10-year period. This portfolio must provide a detailed study of the possibilities in the market and we must create and test a prototype for the future portfolio management system.

In order to do this successfully we have decided to use the data from FTSE100, FTSE250 and S&P500.

These markets will give us a capitalisation-weighted index consisting of the top to the 350th largest companies listed on the London Stock Exchange as well as the stock market index that tracks the stocks of 500 large-cap U.S. companies. It represents the stock market's performance by reporting the risks and returns of the biggest companies.

We have used our knowledge of financial markets and of stock behaviour to produce a recipe to pick stocks such that our portfolio is likely to be profitable for some period of time and is to carry low risk of loss of our initial investment and formulate an algorithm for regular adjustments of the portfolio reflecting changes on the market.

For this we have investigated several methods in order to help pick the stocks, optimise the portfolio and forecast its future.

In order to create the prototype, we have decided to use the software on R studio. For this we used Yahoo! Finance for the data on each stock including its initial price and selling price in which we deducted a 2.5% transaction fee from the given price at a specific date. We will then use forecasting methods to adjust and add or remove stocks to improve our portfolio.

2. STOCKS

2.1 What Are Stocks

A stock is a security type investment that represents the ownership of a fraction of a company. Investors who own stocks in a corporation are authorized to a proportion of the firm’s assets and profits, which is equivalent to how many stock they have. Such investors are called shareholders, since they share in the corporation’s profits. In addition, ‘shares’ are the units of stock.

One way that companies can see their funds escalating is through the issuing of stocks. Corporations, more specifically public companies, listed in the stock exchange market sell stocks in order to raise funds so they are able to run their businesses. Investors that purchase stocks, known as shareholders and depending on what type of shares they buy, have access to the assets and earnings of the company. More specifically, the investor is one of the owners of the corporation and can demand a part of its assets and earnings. The number of shares an individual hold in proportion to the total shares issued by the corporation can tell the ownership.

For instance, if XYZ corporation has 50,000 shares of stock issued and an investor holds 7,000 shares, then it can be concluded that the investor owns and can demand the 14% of the corporation’s assets and earnings.

2.2 Common Stock Vs Preferred Stock

Common and preferred stocks are the two types of stocks available, that differ from each other.

A share of a common stock is a type of a security that represents the ownership of a proportion of a corporation. Common stockholders have the right to vote in order to elect the board of directors for a company, and vote on corporate policies. Each year they receive dividends, and they benefit from a potential increase of the price of their stock. Typically, in the long-term horizon this type of equity ownership give investors a higher return. Numerous classes of common stock are available, often referred to as type A, type B etc., but these types are standard for all firms. These types may differ in either the dividends or the voting rights. Investors should know in advance what rights do they have. In the event of a possible bankruptcy of the corporation common stockholders will receive their money last. Creditors, bond holders, preferred shareholders must be paid in full before holders of common stocks. As a result, this makes a share of common stock a riskier investment.

Preferred stock is said to be a special type of stock. It has features from both bonds and common stocks which attract investors. Investors owning a preferred stock are paid fixed dividends, as like bonds and thus its price is stable. Holders of preferred stocks do not have the right for the voting procedure to elect the board of directors. Unlike common shareholders, holders of preferred stocks, in the event of liquidation, will receive their money and claim on corporation’s assets before common shareholders.

A summary table for common stock and preferred stock, regarding dividends, growth, liquidation, right to vote, arrears, and certainty can be seen in Appendix A.

2.3 How Stocks Are Sold

Each business day billions and trillions of shares of stocks are bought and sold in the world. The major characteristics for well-grown and developed markets are the well-ordered flow of information, funds raised by companies through the stock market and the stock ownership. Efficient markets motivate investors in purchasing stocks and consequently provide corporations, with investment opportunities, funds and capital.

Stocks are, traditionally, traded either on an organised stock market exchange or over-the-counter. Lately, the stock market has introduced the electronic trading which grows in daily basis.

- ### Organised Stock Market Exchange

  The organised stock market exchange is a specified place where stocks are bought and sold, mainly. There are a lot of stock market exchanges all over the world. The most notable is the New York Stock Exchange (NYSE) which is ranked first with market value approximately $28.19 trillion as in April 2020. The first trading activities in the NYSE occur in 1792, where around 24 brokers started trading stocks on Wall Street. Buyers and sellers of stocks were arranging meeting in daily basis for trading using an open-outcry method, in which professionals communicate, through shouting and using their hands, in order to transfer information about selling and purchasing a stock on the stock market trading floor. Nowadays, the technological improvements, the increased availability of sophisticated softwares and programs have made the open-outcry model less frequently used. Other best-known organised stock market exchange centre is the London Stock Market located in United Kingdom, which takes the 8th position in the list of the world's largest stock markets with $3.13 trillion market value as in April 2020. The DAX, alternatively the Frankfurt Stock Exchange, is another major organised stock market exchange with market value of €1,017 billion roughly in September 2020.

Stocks listed in an organised stock market exchange need to satisfy specific, standard criteria, which are there to strengthen trading activities, and then the corporation must submit an application form.

In the NYSE, only the biggest and most known companies are included in the list. A firm must have considerable earnings, of more than $10 million, and a market value greater than $100 million. The reason is simple, they want to achieve a high volume of transactions.

In addition, there are stock market exchanges that are specialised in specific products. For example, The London Metal Exchange is the world's largest centre for trading industrial metals and the ICE Futures Europe in derivatives and equities.

- Over-the-Counter

Over-the-Counter is the market, where stocks that are not included on one of the organised stock market exchanges, are traded in. It is not called an 'organised' over-the-counter market because there is not an actual place where trading occurs. Alternatively, trading happens through advanced highly sophisticated telecommunication's networks. The National Association of Securities Dealers Automated Quotation System (NASDAQ) is such an example. The system was launched in 1971 which provided the current bid and ask prices of approximately 3,000 stocks. As years passed by, NASDAQ, became an American stock market exchange, and it currently takes the second place with market value of approximately $12.98 trillion as in April 2020.

The NYSE and NASDAQ both together conquer 46% of the global market value, which indicates the American exceptionalism.

However, private sales of stocks can occur, and the reason is because the corporation cannot be listed in any stock market exchange.
Deals occurring in the stock exchange market, and through private sales are regulated by the government. The aim of the government regulation bodies is to

protect potential investors from fraudulent activities and preserve that processes are maintained.

3. ASSESSING RISK

In daily basis, everybody deals with some kind of risk. This might include either driving on a highway, walking in the park, cooking, or investing in the stock market.

In financial terms, risk refers to the probability that a result or an investment's actual return will vary from the expected return or outcome. In other words, it is the chance of losing some or all of the initial investment. There are various types of investors such as risk-lovers, risk-averse and risk-neutral. A risk-lover investor, is the one that will be willing and seeking to receive a greater additional amount of risk for an investment that has a relatively low potential return. Risk-averse investors, are in favour of getting a lower return for the investments with risks that are known. Lastly, a risk-neutral individual, is the one that is indifferent regarding risk when considering investment opportunities.

An important principle in finance is the link between risk and return. Generally, investors face a trade-off between risk and return. The riskier the investment project for investors, then the reward, considering the greater amount of risk, will be higher.

3.1 Portfolio Risk

Each individual investment is exposed to its own risk of loss. A portfolio is the combination of all of the investment projects, therefore portfolio risk indicates the total risk of such a portfolio. Differentiated elements of a portfolio and their weightings are factors that determine the extent to which a portfolio is exposed to several risks.

Market and other systematic risks are crucial when dealing with portfolio risk. The major task for a financial advisor is to try and manage these types of risks in order to ensure that the portfolio meets its financial objectives. Before an advisor can successfully manage the portfolio risk, he must firstly quantify it.

3.2 Risk Tolerance

Constructing a portfolio is not an easy task. Not everyone can handle the idea of losing all or at least a proportion of their money while dealing with investments. Before considering to construct portfolio, potential investors must calculate and be aware of their risk tolerance. Risk tolerance is the amount that an investor can afford to lose, both in financial and psychological terms. In other words, it refers to how psychologically and financially ready the investor is to experience a possible potential lose.
The fact of losing a huge amount of money can result in either, a never recovering portfolio or an individual with stress and illogical decision making.

A financial advisor is the one who can properly figure out the risk tolerance of individual investors. Components such as portfolio value, monthly income and expenses, time horizon and reliability of income can help investors have an idea of their risk tolerance.

Risk tolerance is not a fixed value. It can vary over time and depending on the financial position of an individual. It can be rated as high, moderate or low. For example, a younger investor with at least 20 years off retirement age might have higher risk tolerance and willing to invest in riskier assets. This is because they have the time to recover from possible losses or in a case of a market crash. However, if you are an older investor close to retirement age, losing money late in your career, might lead to not being able to recover, thus older investors prefer less riskier investments consequently having lower risk tolerance.

3.3 Type Of Portfolio Risks

A lot of different types of risks are available, both at the portfolio level and individual security level. The two most important risks investors face at the portfolio level are:

3.3.1 Market Risk

Market Risk, or in other words systematic risk, is the greatest risk investors face at the portfolio level. It refers to the possibility that they will experience losses due to macro-economic factors that affect the entire market at the same time. Changes in interest rates, inflation, recessions, currency fluctuations political turmoil, natural disasters and terrorist attacks are the main sources of market risk. The fact that market risk is a systematic risk, it makes it difficult to be eliminated through diversification.

3.3.2 Specific Risk

Specific risk is the risk not correlated with the market. It is called specific because it applies to a particular company or industry. Specific risk is also known as unsystematic, diversifiable or residual risk. Business risk and financial risk are examples of a company-specific risks. Unlike market risk, specific risk can be eliminated through the diversification of their portfolio. According to the economists Lawrence Fisher and James H. Lorie, holding approximately 30 securities decreases the specific risk.

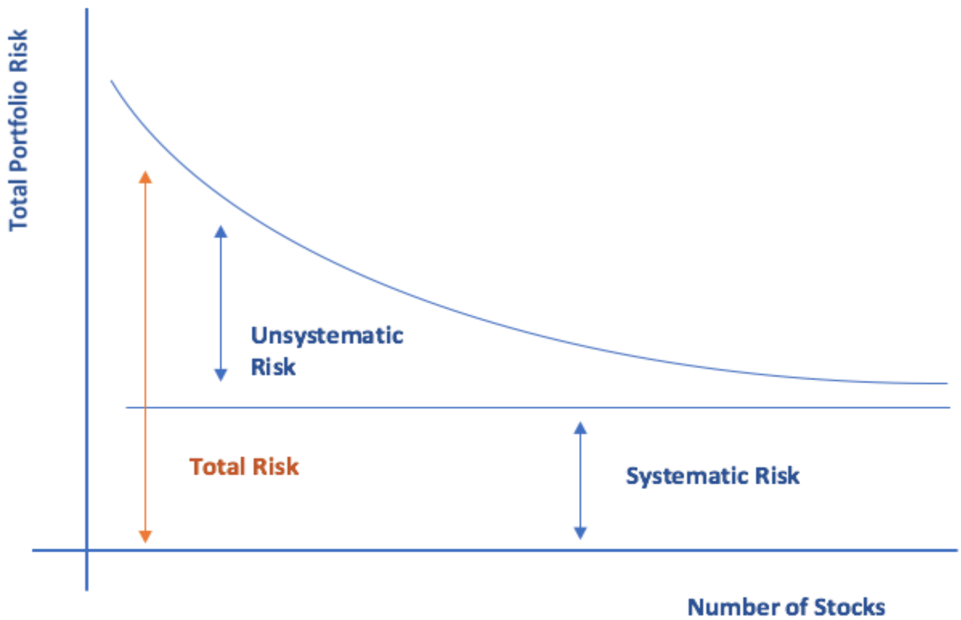


Figure 1: Total Risk = Systematic Risk + Unsystematic Risk.

3.4 Quantifying Risk

Numerous ways are available for quantifying the risk of a portfolio, which each method has its own advantages and disadvantages. Let us consider each method separately.

3.4.1 Volatility And Covariance

Volatility is a statistical tool which computes the dispersion of returns for a particular security. It determines the uncertainty and risk associated with the change in the value of the security, making it the most common proxy. Generally, the riskier the security it is, then the greater the volatility. Standard deviation and variance are the numerical instruments used to calculate volatility.

In general, the variance of a random variable X is given by,

$$\sigma_X^2 = E[X - E(X)]^2 \quad (1)$$

$$= \sum_{s=1}^{S} p_s[x_s - E(X)]^2 \quad (2)$$

$$= p_1[x_1 - E(X)]^2 + p_2[x_2 - E(X)]^2 + \cdots + p_s[x_s - E(X)]^2$$

$$where, \qquad E(X)\ is\ the\ expected\ value\ of\ random\ variable\ X$$

$$\sigma_X^2\ is\ the\ variance\ of\ random\ variable\ X$$

$$and\ p_s = \frac{1}{S} where\ S\ is\ the\ outcomes.$$

Thus, the standard deviation of a random variable X is given by,

$$\sigma_X = \sqrt{E[X - E(X)]^2} = \sqrt{\sum_{s=1}^{S} p_s[x_s - E(X)]^2} = \sqrt{\sigma_X^2} \quad (3)$$

In portfolio analysis, finding the portfolio risk using volatility requires several steps. Firstly, we need to consider portfolio weights. Portfolio weights, w_i, refers to the proportion of the total portfolio value that will be invested in security i. Suppose that all funds will be considered in the portfolio, then:

$$\sum_{i=1}^{n} w_i = 1 \quad (4)$$

This is the balance-sheet identity, which demonstrates that n portfolio weights, w_i, invested in n different stocks must equal to 1. The constraint cannot be violated or be changed.

For example, let us consider a portfolio m consisting of 4 stocks:

Stock Name	Weights
Stock A	0.3
Stock B	0.2
Stock C	0.1

Stock D	0.4
Total	1

Table 1: Portfolio m and weights.

Furthermore, the portfolio's expected return needs to be computed. Assuming that r_m is the return of portfolio m which contains n number of individual stocks. Then, the expected return of portfolio m is given by:

$$E(r_m) = E(\sum_{i=1}^{n} w_i r_i) = \sum_{i=1}^{n} w_i E(r_i) \quad (5)$$

$$where, \qquad w_i \ indicates \ the \ portfolio \ weight$$

$$E(r_i) \ gives \ the \ expected \ return \ of \ stock \ i.$$

Particularly, the expected return of portfolio m is the weighted average of the expected returns of each individual security i that is included in the portfolio.

Stock Name	Weights	Expected return of each stock
Stock A	0.3	13%
Stock B	0.2	24%
Stock C	0.1	10%
Stock D	0.4	15%
Total	1	-

Table 2: Weights and Expected return of each stock.

Using the example above, the portfolio's expected return is given by,

$$E(r_m) = E(\sum_{i=1}^{n} w_i r_i) = \sum_{i=1}^{n} w_i E(r_i) \quad (6)$$
$$= (0.3)(13\%) + (0.2)(24\%) + (0.1)(10\%) + (0.4)(15\%)$$

$$= 15.7\%$$

Portfolio risk, denoted by, σ_m^2, is the variance of the return from portfolio m. The mathematical expression for σ_m^2 is given in terms of the variances and covariances of all stocks in the portfolio.

Covariance is a statistical concept, which measures the association between two random variables. This statistical measure is used by investors to examine whether a change in price of a security is related to another security. The covariance between two securities, i and j, is given by,

$$\sigma_{ij} = Cov(r_i, r_j) = E\{[r_i - E(r_i)][r_j - E(r_j)]\} = \rho_{ij}\sigma_i\sigma_j \quad (7)$$
$$where, \qquad \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i\sigma_j} \ is \ the \ correlation \ of \ returns$$
$$\sigma_i \ is \ the \ standard \ deviation \ of \ asset \ i$$
$$\sigma_j, is \ the \ standard \ deviation \ of \ asset \ j.$$

Therefore, the variance of returns of portfolio m which consists of n assets is calculated by,

$$\sigma_m^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \, \sigma_{ij} = \sum_{i=1}^{n} w_i^2 \, \sigma_i^2 + \sum_{\substack{i=1 \\ for}}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} w_i w_j \sigma_{ij} \quad (8)$$

Portfolio weights, standard deviation or variance of each asset and covariance or correlation between assets are the three ingredients needed to quantify the portfolio risk.

3.4.2 The Sharpe Ratio

The Sharpe Ratio is used by investors since it helps them compare the performance of an investment in relation to a risk-free asset. It is the risk-adjusted return of a financial portfolio, and the formula is given by:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \quad (9)$$

$$where, \qquad R_p\ is\ the\ return\ of\ portfolio$$
$$R_f\ is\ the\ risk\ free\ rate$$
$$\sigma_p\ is\ the\ standard\ deviation\ of\ the\ portfolio's\ excess\ return$$

The approach determines the return for every *dollar* of risk taken or in other words, how much additional returns can be made by taking on additional risk. The greater the ratio, the more desirable the return (adjusted for risk) is. Although this is ideal for isolating definitive portfolios with respect to the performance and risk, the process is backward looking and simply cannot predict future risk and return. In addition, when using standard deviation as a denominator it may create some limitations, theory implies it assumes returns from the portfolio are normally distributed and that price movements are equally risky in either direction, however, in practise this may not apply and may skew the data in slight favour of one portfolio from another.

3.5 Managing Portfolio Risk

Historically, the stock market has generated the greatest returns, but at the same time it has experienced the highest volatility. Investors can use numerous of methods to limit portfolio risk, and in most of the cases more than one technique is considered.

The first step that should be considered by investors is the diversification of their portfolio. Diversification is a method used that minimizes the exposure to risk, by sharing out the investments across several asset classes, industries and other categories. This process aims at maximizing returns for investors by investing in various distinct sectors that they will react in a different way to an unexpected event. A diversified portfolio might include either stocks, which offer the greatest long-term returns, bonds, which give a predictable amount of income, or cash, which supply instantaneous liquidity.

Although, diversification does not guarantee that investors will not experience any losses, according to investment professionals, diversification technique is an important factor to be considered in order to reach financial goals while at the same time minimizing the risk.

4. ASSUMPTIONS

Due to the sheer volume of information regarding stocks and the stock market, we collectively decided to make a few assumptions to guide our process and make it easier to draw conclusions from our model. These were made for the initial selection of stocks for the portfolio as well as for the ease of maintaining the portfolio through regular adjustments to make the algorithm simpler to use.

Firstly, we are only using stocks from the S&P500 Index, as it has a wide range of stocks among various markets, ensuring that we can sufficiently diversify our portfolio. During our initial prototype, we used the stocks from this index as these it was easiest to scrape from Yahoo! Finance. Although there are fewer stocks in our prototype than there would be in a 'normal' market, we assumed that it was fine to assume that only these stocks exist in the market, as we only need to check that the algorithm works properly in order to produce a profitable portfolio and maintain this through adjustment, as the task requires. If we assume that the market contains only this index, it will ensure that the number of stocks we have available are not extremely limited, which would be the case if we chose to only select stocks from the FTSE100 index, say.

Secondly, we have assumed that 20 stocks will remain in our portfolio over the course of our investment period of 10 years. Our research has shown some contradicting results about how many stocks an average investor should keep in their portfolio for it to be considered diversified and profitable. We concluded that, although there is no consensus answer, there does exist a reasonable range for the ideal number of stocks that an investor should hold in their portfolio. Some sources claim that a well-diversified portfolio of randomly chosen stocks should ideally contain at least 30 stocks for a borrowing investor and 40 for a lending investor. On the other hand, we have seen that this contradicts the widely accepted notion that, the benefits of diversification are negligible when a portfolio contains more than 10 stocks. We decided that, as we are not 'randomly' choosing our stocks and are filtering the stocks in our initial step, as well as during the regular adjustment reviews of the algorithm, we will always have 20 stocks in our portfolio, to generate a comfortable level of return with sufficient diversification.

Additionally, we have not assumed to be investing in any particular type of stock or stocks from a certain market, as the categorisation of the stock is not as significant to us as a factor of profitability in our algorithm, as say, the beta value of the stock is. An implication of this is that we do not look at the company's morals and ethics, so long as their stock fits our selection criteria. We have done this, as our main priority is to build a profitable portfolio, not necessarily one that only invests in companies with an acceptable level of ethics in their operations. To do this would require a whole other selection process, which is highly subjective as is the field of ethics, which is not a requirement in our task of maximising profitability, whilst minimising risk.

Finally, we have assumed that the investor keeps some capital in the bank account to be able to pay the transaction fees that they incur. In particular, we assumed that they will keep 30% of the total initial amount ($3 000 000 in our case) to account for this.

5. THE RECIPE OF PICKING STOCKS

This section is related to the statistical methods used in our model to select stocks from the user's input. The two instruments that are covered is the covariance and the beta coefficient.

5.1 Covariance

As previously stated, the covariance is a statistic used to calculate the relationship between the variability in price of two assets. A positive covariance implies the two assets move together whereas a negative covariance implies the two moves inversely (a covariance of 0 means there is no linear relationship between the variability). This has proven useful in the model since it allows investors to create a diverse mix of distinct securities, in order to diversify the portfolio and reduce risk. By including assets that have a negative covariance, the volatility in the price changes is greatly reduced. However, a drawback to using covariance is that the statistic is susceptible to unreliable results in the case of outliers. If there was one extreme price change in a security, it would skew the final value.

For example, if a portfolio included the following 5 stocks 'Apple', 'Lloyds', 'Tesco', Rolls Royce' and 'Right Move' over the period 2016-2021 the covariances of these assets can be calculated and expressed as seen in the table below.

Covariance Matrix	APPL	LLOY.L	RMV.L	RR.L	TSCO.L
AAPL	0.0907	0.0267	0.0226	0.0183	0.0134
LLOY.L	0.0267	0.1227	0.0431	0.0787	0.0293
RMV.L	0.0226	0.0431	0.0782	0.0375	0.0181
RR.L	0.0183	0.0787	0.0375	0.2749	0.0225
TSCO.L	0.0134	0.0293	0.0181	0.0225	0.0716

Table 3: Covariance Matrix of a portfolio including 5 stocks.

Each of the assets are calculated against every other asset in the portfolio, where the covariances in the diagonal are the variances of that asset, due to the property:

$$Cov(X, X) = Var(X) \quad (10)$$

This naturally highlights that stocks in 'Tesco' are the least risky with 'Rolls Royce' being the riskiest in this portfolio (measured using their variances). On the other hand, comparison of covariances suggests 'Lloyds' has the highest average value and 'Tesco' has the lowest. Typically to reduce risk in this scenario (and create a diversified portfolio) an investor would invest more in 'Tesco' stocks and less in the 'Lloyds'.

Notably this instrument is primarily used in; the global minimum variance portfolio which aims to minimize the risk of the portfolio and, the highest Sharpe Ratio portfolio which uses the standard deviation to assess the efficiency of the portfolio. This instrument in the model allows the user to identify and isolate stocks which are ideal for creating large portfolio (minimal covariance).

5.2 Beta Coefficient

The beta coefficient is a measure of the systematic risk of an individual asset in comparison to the market as a whole and is mainly used in the CAPM (Capital Asset Pricing Model). Systematic risk refers to risk inherent within the market and cannot be reduced through diversification. Essentially, the coefficient approximates how much risk a security will add to the total for a portfolio, helping investors gauge their risk exposure if they invest in this security. The calculation is shown below:

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)} \quad (11)$$

$$where, \quad r_i \; is\; the\; return\; on\; the\; individual\; security$$
$$r_m \; is\; the\; return\; on\; the\; whole\; market\; (market\; must\; be\; related\; to\; security)$$
$$Cov\; is\; the\; Covariance\; function$$
$$Var\; is\; the\; Variance\; function$$

In theory, beta shows the volatility of the share's returns when responding to market fluctuations. This would therefore be useful in selecting shares for a portfolio since it indicates whether the security moves similarly with the market. The theoretical meaning of each type of beta value and its corresponding effect on the portfolio is as follows:

$$\beta_i < 1\; ;\; stock\; is\; less\; volatile\; than\; the\; market, makes\; portfolio\; less\; risky$$
$$\beta_i > 1\; ;\; stock\; is\; more\; volatile\; than\; the\; market, makes\; portfolio\; more\; risky$$
$$\beta_i = 1\; ;\; stock\; is\; strongly\; correlated\; with\; the\; market, portfolio\; unaffected$$
$$\beta_i < 0\; ;\; stock\; is\; negatively\; correlated\; with\; the\; market$$

Beta assumes that asset returns are normally distributed, whereas realistically markets can fluctuate and can be vulnerable to industry shocks. However, beta is still useful in portfolio management, since it can be used to evaluate whether individual stocks are worth investing in or to select a suitable portfolio out of numerous options. The coefficient should be considered along with the other methods used in our model. For example, a high beta may mean more risk, but there is potential for greater returns to be made.

Despite its advantages, there are some drawbacks to using the beta coefficient in portfolio management. Firstly, beta is based on historical data, meaning it is not a completely accurate statistic to predict future behaviour and volatility. Also, beta may prove inefficient in relation to long-term investments, since the volatility of stocks can vary across many years as the companies grow.

6. PORTFOLIO SELECTION

This section this is related to modern portfolio theory (and the efficient frontier) used within our model to select a portfolio ideal to the user. The three main portfolio criterions covered are: the global minimum variance portfolio (section 6.1); highest return portfolio (section 6.2); the highest Sharpe Ratio portfolio (section 6.3) and forecasting methods (section 6.4).

6.1 Global Minimum Variance Portfolio

The global minimum variance portfolio is the portfolio that has the lowest possible risk (minimum volatility) in return, of all portfolios containing a certain amount of assets. The benefit of using this to select the optimal portfolio can be illustrated through the minimum variance frontier, with expected return on the y-axis and standard deviation (risk) on the x-axis. All possible portfolios with different combinations of assets can be plotted on this graph, each of which have their own risk-level and expected returns. The shape of the minimum variance frontier will always be a parabola as shown in the diagram below.

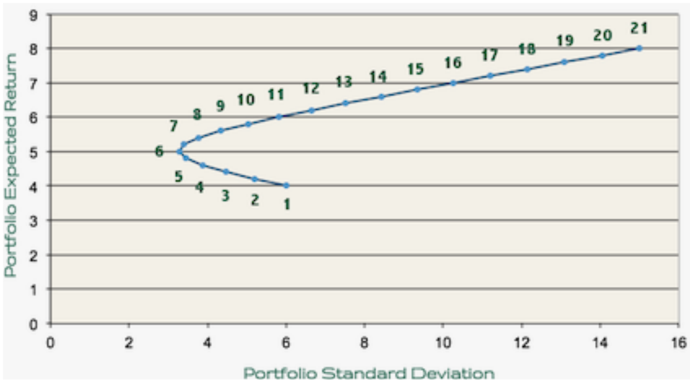


Figure 2: Minimum variance frontier.

The global minimum variance portfolio is always found at the left-most side of the parabola (point 6), such that it is the minimum bound for all risk (standard deviation) since it has the least variance, and hence the least risk, when compared to all achievable portfolios along the curve. Anything below this point (points 1 to 5) are considered inefficient, since one can get a higher return for the same level of risk by picking portfolios from 7 to 11. The line above the global minimum variance portfolio is known as the efficient frontier since the higher returns come with a higher risk.

In finance this portfolio is calculated by:

$$\xi g \;=\; \frac{1}{\alpha} 1V^{-1} \quad (12)$$

$$where, \qquad \alpha = \; 1V^{-1}1^T$$

$$V = (v_{ij}) = Cov(R_i, R_j) = E[(R_i - r_i)(R_j - r_j)]$$

$$Or\ Alternatively, \quad \min_W \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(R_i, R_j) \quad (13)$$

$$Where{:}\ \sum_{i=1}^{n} w_i = 1\,, W = (w_1, w_2, \dots, w_n)$$

Equation (12), represents the modern portfolio theory of calculating the global minimum variance portfolio. This formula uses the intuition that we can calculate the covariance matrix from assets in the portfolio (section 5.1) and utilises this to create a simplistic method by implementing the inverse of the covariance matrix. Alternatively, (13) is an optimization problem using the variance portfolio equation (8) and minimises it with respect to weights of the portfolio (where W is the vector of portfolio weights, w_i,). This allows our model to output two results, the value of global minimum risk (used to assess the overall portfolio) and the respective weights used to achieve this portfolio. Furthermore, in our model this is recalculated again once any adjustments are made, i.e. transaction fees included, to make sure the portfolio is still optimized.

Attitude towards risk differs from investor to investor which is why in modern portfolio theory there are many decisions which alter the characteristics of the portfolio. In this example the global minimum variance portfolio minimises risk and typically would be suited to an investor who is risk averse (disinclined to take risks). Since this portfolio is low risk it presents some stability, meaning the return is minimal yet the security on this is maximised.

6.2 Highest Return Portfolio

The highest return portfolio is solely orientated around the expected return of a portfolio. This unique portfolio is located to the top of the efficient frontier such that it is the maximum bound for all return of achievable portfolios. In finance this can be calculated by the optimization problem:

$$\max_{W} \sum_{i=1}^{n} w_i E(R_i) \quad (14)$$

$$Where\colon \sum_{i=1}^{n} w_i = 1\,, W = (w_1, w_2, \ldots, w_n)$$

This portfolio is calculated by adapting the expected return portfolio, equation (14), to maximise the output which forms this optimization problem. Similarly, to the other optimization problems, our model uses the random generated portfolio weights to find the optimal portfolio constrained to maximising return. After finding the solution to this problem the model outputs two results, the value of highest return portfolio (used to assess the overall portfolio) and the respective weights used to achieve this portfolio. Furthermore, in our model this is recalculated again once any adjustments are made, i.e. transaction fees included, to make sure the portfolio is still optimized.

Since this portfolio maximises return regardless of the risk of the portfolio, it can be assumed that this selection criteria are adopted by investors who is risk loving (inclined to take risks). Since they are willing to take on additional risk for the extreme high potential pay outs even if the potential loss is larger.

6.3 Max Sharpe Ratio Portfolio

As previously stated, this ratio is used to compare total risk to the return on an investment, or more specifically it is the “average return earned in excess of the risk-free rate per unit of total risk”.

In modern portfolio theory the max Sharpe Ratio portfolio illustrates that an investor can diversity a portfolio without sacrificing return. For many investors, this makes it more desirable compared to other portfolio criteria such like, global minimum variance portfolio (since this sacrifice return for security) and the highest return (since this sacrifice security for return), whereas this maximises return with minimum risk. However, since it is backword looking when implemented into forecasting return, clear assumptions must be outlined when using the portfolio weights, these being: risk, weight, number, and named stocks all remain constant over the forecasting period. This consequently raises concerns with truthfulness, however all methods which are involved in forecasting are not definite.

In this scenario, the model calculates various portfolios and aims to find the portfolio which maximises this Sharpe Ratio. This unique portfolio is located on the efficient frontier at the point which maximises return for minimum risk taken (usually to the top left of the efficient frontier curve). This calculation in the model is represented by the optimization problem:

$$\max_{W} \frac{\left[\sum_{i=1}^{n} w_i E(R_i)\right] - R_f}{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov\left(R_i, R_j\right)}} \quad (15)$$

$$Where: \sum_{i=1}^{n} w_i = 1\,, W = (w_1, w_2, \dots, w_n)$$

This optimization problem is an expansion of the original Sharpe Ratio seen in (9) which is optimized by finding the weights of the portfolio which maximise the expected return of the portfolio whilst simultaneously minimising the standard deviation of the portfolio (calculated by taking the square root of the variance). This consequently outputs two results, the value of the Sharpe Ratio (used to assess the overall portfolio) and the respective weight used to achieve this portfolio. Furthermore, in our model this is recalculated again once any adjustments are made (transaction fees included) to make sure the portfolio is still optimized.

6.4 Holt’s Trend Method vs ARIMA – Forecasting

Holt’s Trend Method is a ‘smoothing model’ (a model that removes noise from the data, only allowing the significant patterns to remain in order to predict future trends) that is used to forecast data. There are three equations that make up the forecast:

$$Forecast\ Equation: \hat{y}_{t+h|t} = \ell_t + hb_t \quad (16)$$
$$Level\ Equation: \ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1}) \quad (17)$$
$$Trend\ Equation: b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \quad (18)$$

$$\ell_t = Estimate\ of\ the\ level\ of\ the\ series\ at\ time\ t$$
$$b_t = Estimate\ of\ the\ trend\ (slope)\ of\ the\ time\ series\ at\ time\ t$$
$$0 \le \alpha \le 1 : The\ smoothing\ parameter\ for\ the\ level$$
$$0 \le \beta^* \le 1 : The\ smoothing\ parameter\ for\ the\ trend$$

The trend equation is a basic smoothing equation, which modifies the last period’s trend value. The trend is then updated over a period by using the level equation and finally the forecast equation shows the final forecast.

Autoregressive Integrated Moving Average (ARIMA), is another method used for forecasting too. It is a class of models that that 'explains' a given time series "based on its own past values, that is, its own lags and the lagged forecast errors, so that equation can be used to forecast future values". A lag plot is when the time series is compared with a 'lagged' (delayed) time series, so for example, the most commonly used lag is a first-order lag, which is a shift in the time series by 1. Lag plots are useful in finding outliers, randomness and serial correlation; therefore, this helps to capture relationships in the data to help forecast future points.

The Arima instrument in R, fits the best model for the (sample) data by using different criterion which access the balance between having a good fit and not having too many parameters (it also identifies any trends or seasonality, which is vital). To achieve this, it fits the models using maximum likelihood through the criterions AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion) and AICc (Corrected AIC). Formally the best models minimise the criterions since each involve penalty parameters when overfitting occurs, which are expressed by the equations below:

$$AIC = -2\log\{maximised\ likelihood\} + 2k \quad (19)$$
$$BIC = -2\log\{maximised\ likelihood\} + (\log n)k \quad (20)$$
$$AICc = AIC + \frac{2k(k+1)}{n-k-1} \quad (21)$$
$$where\ 2k\ , (\log n)k\ and\ \frac{2k(k+1)}{n-k-1}\ are\ the\ penalty\ parameters\ respectively$$

Clearly, BIC has a stronger penalty on the number of parameters, and so results in smaller number of parameters in the best fitted model than the choice made by AIC, although AICc's penalty is much similar. From the point when the model has been formulated by ARIMA(p ,d, q) for any of the criterions, it can be utilised to forecast the (future) data for a given period. The standard notation is as follows:

$$ARIMA(p, d, q)$$
$$Where, \qquad p\ is\ the\ number\ of\ lag\ observations\ included$$
$$d\ is\ the\ number\ of\ times\ raw\ observations\ are\ differenced\ (*)$$
$$q\ is\ the\ size\ of\ the\ moving\ average\ window$$
$$(*)\ when\ the\ data\ have\ had\ past\ values\ subtracted$$

For all forecasting instruments there are doubts of the accuracy and reliability of the models and values formulated. "All models are wrong, but some are useful" (Box, G.E.P, 1976). This quote illustrates that modelling and forecasting will not be correct, but it justifies using these as a point of understanding to make a coherent prediction based on available data. Such that using back-testing on sample data allows us to get a clear representation of which models and forecasting methods identify current data's trends, seasonality and which may be more useful in the future. After some independent testing on available stocks in our markets, the decision was made to implement ARIMA forecasting since it is more transparent and gave a comprehensive forecast.

7. APP FUNCTIONALITY

7.1 Complete Outline For The Application

The model relies on the following .csv files, each of which have their own purpose:

- stocks - contains the current portfolio, the name of each stock, the quantities owned and the price and date when a share is bought.
- beta - this file entails a list of all stock betas for the S&P500.
- wallet - includes the cash amount the investor has, not including portfolio value.
- forecast - contains the forecasted value for each stock in the portfolio in 6 months' time.
- port_info - contains information about the portfolio invested in, such as type (minimum variance, max returns, etc.), the weights of each stock in the portfolio, the risk & return values and the Sharpe ratio.

These files are stored on Dropbox and can be downloaded locally when needed to write.

7.1.1 Buying And Selling Stocks

The process for buying stocks starts with the current portfolio being read into the system. We then get the price of the stock at a specific date from Yahoo finance and multiply the cost (price multiplied by quantity) by the transaction fee (2.5%) to work out the total cost of the procedure. This cost is deducted from the wallet as well as being saved and recorded, then the stock is saved to the portfolio file.
As for selling stocks, the application acquires the price of the stock being sold at the specified date, which is then multiplied by the quantity to sell. Once again, the transaction fee is added, this stock is removed from the portfolio (or the amount sold is removed from the total owned) and the wallet is reimbursed with the fee, minus the transaction cost.

7.1.2 User Interface

The app.r file is where the current portfolio table is displayed, as well as: the profit or loss for each stock, the total portfolio balance and manual buttons for buying, selling and inspecting stocks.

7.1.3 Initial Stock Selection

A table displaying all stocks and their betas from the S&P500 is displayed in the beta.r file. The user can then filter this using a slider to return stocks that are between their specified beta range. Next, the user specifies how many stocks they want to invest in, which the application then finds the stocks to invest in by calculating their covariances. The stocks with the lowest covariances are then selected to minimise the systematic risk in the portfolio.

A large number of randomly generated portfolios that each contain a random mix of assets are then created. These are then analysed for their specific risk, return and Sharpe ratio. All the portfolios are then displayed on a mean-variance portfolio frontier, with the key portfolios that have minimal variance and maximum returns displayed as red dots. The user then must specify the amount to invest in their chosen portfolio, then the application carries out the rest.

Each stock is then forecasted for any potential adjustments that may need to be made in the future. Information about these forecasted values and the chosen portfolio itself are stored in files.

7.1.4 Calculating Covariances

The logarithmic daily returns for each stock are taken from Yahoo finance in order to calculate the covariance matrix. Next, the lowest values within this matrix are selected and then removed so they are not reselected later. The full list of stocks with the lowest covariance are then returned.

7.1.5 Generating Randomly Weighted Portfolios

Our application generates ‘n’ random portfolios by randomly assigning weights (that sum to 1) to each stock. The returns, risk and the Sharpe ratio are then calculated for each random portfolio. All these random portfolios are stored in a table so that they can be chosen, depending on what the investor chooses. Such criteria to choose from includes minimum variance, greatest returns, highest Sharpe ratio or user-selected requirements. For this process to work, the recommended number of portfolios to generate should be greater than 20,000.

7.1.6 Forecasting

For each stock in the portfolio, the model pulls their adjusted returns to date from Yahoo finance. A time series is constructed from this information and we find the best ARIMA model using automated functions. Next, we forecast the stocks using the best ARIMA model over six months and extract the low 80% and low 95% values.

7.1.7 Adjusting the portfolio

Firstly, we compare the current prices to our forecasted values from 6 months ago and if the current price falls below the 95% low forecast for the stock, we prepare to remove that stock. We note the beta of that stock, then the market is then checked to see if any stocks have a similar beta and the application removes any that are already included in the portfolio. After, we find the covariances of these new stocks when paired with the stocks within our portfolio and note which pairing has the lowest total covariance. This new stock with the lowest covariance will replace the stock that fell below the forecast. The old stock is sold, and an equal amount of the new stock is then bought to replace it. This process is repeated for each stock in the portfolio.

Secondly, we then generate a large number (25000) of new portfolios. The old portfolio type is read into the system and depending on what portfolio was initially invested in, the algorithm will check for different factors. For example, if we initially invested in the minimum variance portfolio, the algorithm would look for the new minimum variance portfolio generated and compare its risk and return level. The application will first compare the current risk to the new risk and if the new risk is lower, then we look at the cost of switching to the new portfolio. If the cost is greater than the increase in returns that we will receive by switching, then it does not proceed.

Otherwise, the model then calculates the difference in portfolio weights between the old and new portfolio. This is converted into the quantity of stocks to buy or sell for each stock, and then the system completes the transaction. Similar procedures are followed for max returns and max Sharpe ratio by using each in place of risk.

7.2 Data/Test Results

The application was tested using a starting investment of $7,000,000, allowing for $3,000,000 in the bank to cover any transaction fees or large stock readjustments that occur. Exploratory analysis was performed on how the beta range selected produces a different Markowitz mean-variance frontier, displayed in the table below.

Beta Range	Risk%	Yearly Return%
0.85-0.9	16.5	17.8
0.85-1	16.3	19.5
0.9-1.1	18	13
1.1-1.2	18	13.5
1.2-1.3	19	17

After generating 25000 portfolios with each beta range selected, the risk and return were calculated. From this, it can be seen that a beta range of 0.85 to 1 yields not only the highest yearly return but also the lowest risk. We have decided to test each type of portfolio selection we have implemented, including minimum variance (lowest risk), maximum returns and highest Sharpe ratio, and we used the same beta range for all of these. This will allow for comparison between each type of portfolio which may influence the user's choice.

The initial date of the portfolio was the 1st of January 2015 (01/01/2015). Adjustments were decided to be made every 6 months starting on the 1st of July 2015 (01/07/2015). Adjustments are computationally heavy tasks and can take quite some time to complete, so this was chosen to simplify the results gathering process. As we started the portfolios at the same date with the same range of betas for each portfolio, the application selected the same set of stocks each time, although with different quantities. The initial investment portfolios are displayed in table 4.

Stock	Initial Price	Quantity in Min Var	Quantity in Max Returns	Quantity in Max Sharpe Ratio
ADBE	74.129997	9110.116098	6401.91403	3968.946423
AES	14.37	12085.4102	10620.83247	8895.098945
AVB	167.440002	4143.741747	1377.263585	4121.497209
CAT	93.709999	1137.732036	2954.761295	86.54726305
DE	90.139999	2779.014332	3988.235673	3515.629995
EL	77.400002	6426.462228	4008.921914	479.0986933
FMC	49.982655	8898.579369	583.27191	315.3638005

HUM	144.479996	921.8188419	1443.439272	208.9064784
J	45.040001	5441.17438	2149.903265	369.144712
KEYS	33.91	22252.56824	4369.196343	14482.47488
LDOS	43.970001	11455.99993	18276.21604	14035.53083
LMT	195.639999	1405.059717	1377.248517	3538.904654
MPWR	50.060001	1334.80608	15079.66912	10455.48525
NKE	48.650002	10032.74831	1618.093885	3987.03599
NRG	28.16	4125.178403	6996.683594	1852.30455
NWL	38.400002	4052.163923	6668.942634	17794.93709
SBUX	41.189999	13638.25297	9422.060322	14820.99999
TGT	75.529999	6479.289204	12171.63082	7670.566351
TSCO	78.690002	1075.053329	9478.101663	7438.084663
WST	53.77	5281.902774	5575.596847	8082.632861

Table 4: Initial Investment Portfolios

We will discuss results on a case-by-case basis, including the results from each portfolio with and without adjustments.

7.2.1 Minimum Variance Test (No Adjustments)

Our minimum variance portfolio had an estimated risk/return profile of 9.8% risk, 80.3% returns. From our initial investment of $7,000,000, at the end of the testing period (11/03/2021) the portfolio value was $20,517,100.47 with $3,000,000 left in the bank, resulting in a total value of $23,517,100.47 from our $10,000,000 total cash pool. Our total return on investment was 193.10%, with average yearly returns coming out at 19.6%. As we can see, our return estimate for this specific portfolio was over half the actual returns. However, our yearly returns are very close to our estimate from table 3, 19.5%.

7.2.2 Minimum Variance Test

The portfolio with adjustments made began the same as the portfolio above, but as stocks were to be added and removed, getting an accurate risk/return estimate proved difficult. The final value of the portfolio was $16,385,117 with $2,736,205.89 left in the bank. This results in 134.07% total returns, with average yearly returns of 15.23%. There were a total of 9 adjustments made, meaning 9 stocks were sold and 9 stocks were bought. The timeline of these adjustments can be seen in table 5.

Date	Sold	Bought	Portfolio Value at Adjustment Time	Portfolio Value at End Date
01/01/2015	Initial	Initial	7000000	20517100
01/07/2015	KEYS	MMM	7210671	18387865
01/01/2016	NRG	SPGI	7600335	18376931
01/07/2016	N/A	N/A	7398896	18376931
01/01/2017	EL, TSCO	CSCO, GOOGL	7969755	17357717
01/07/2017	TGT	GOOG	8872803	16929738
01/01/2018	N/A	N/A	10040454	16929738
01/07/2018	SBUX	FIS	10271013	16410709

01/01/2019	FMC, GOOGL	V, NOW	9734282	16481629
01/07/2019	N/A	N/A	11940656	16481629
01/01/2020	N/A	N/A	13330718	16481629
01/07/2020	AVB	IFF	14026898	16385117
01/01/2021	N/A	N/A	16798648	16385117

Table 5: The timeline of the adjustments.

This table shows which stock(s) were bought and sold, as well as the value of the portfolio at the adjustment time and the value of the portfolio at the end date, given that the current portfolio stays constant. It can be seen that as each adjustment is made, the total value of the portfolio at the end date decreases. A graph showing this relationship can be seen in figure 3.

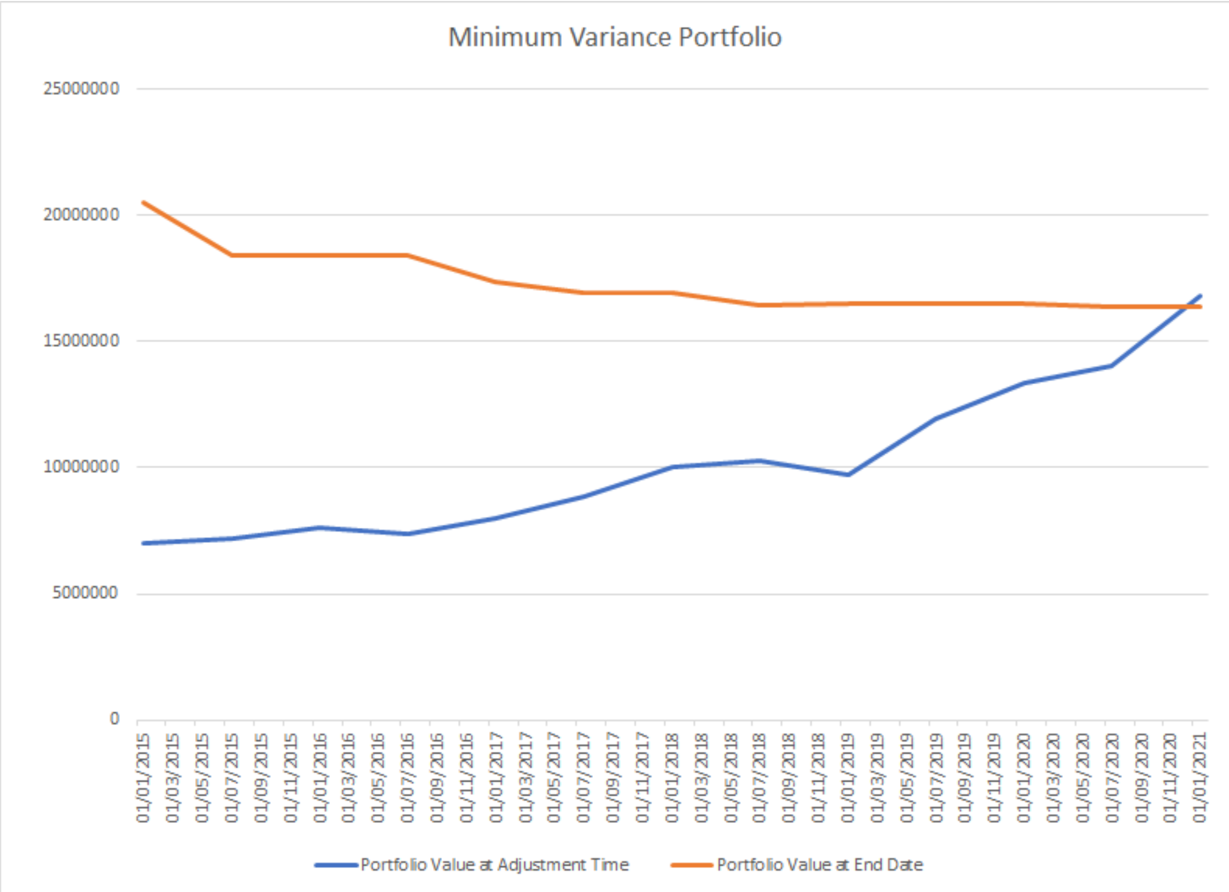


Figure 3

The drop in total portfolio seems strange but can be explained. It is possible that the initial stock selection was very lucky and resulted in very high returns, and adjusting it simply made the portfolio more realistic. The adjustments would have also mitigated the risk of the portfolio, and so the higher risk portfolio achieved higher rewards while the lower risk portfolio yielded lower returns, as can be expected.

7.2.3 Maximum Returns Test (No Adjustments)

Our maximum returns portfolio without adjustments was estimated to have a risk of 12%, with 127.8% total returns. At the end of the period, the portfolio was valued at

$22,103,669.39. This yielded 215.77% total returns and 21.10% average yearly returns. This is once again higher than the expected returns.

7.2.4 Maximum Returns Test

Only 3 adjustments were made to the maximum returns portfolio over the period. These can be seen in table 6.

Date	Sold	Bought	Portfolio Value at Adjustment Time	Portfolio Value at End Date
01/01/2015	Initial	Initial	7000000	22103669.39
01/07/2015	KEYS	MMM	7344198	21672920
01/01/2016	N/A	N/A	7736474	21672920
01/07/2016	N/A	N/A	7578445	21672920
01/01/2017	TSCO	GOOGL	8217985	21856555
01/07/2017	N/A	N/A	9061694	21856555
01/01/2018	N/A	N/A	10495210	21856555
01/07/2018	N/A	N/A	11128732	21856555
01/01/2019	N/A	N/A	10414617	21856555
01/07/2019	N/A	N/A	12759045	21856555
01/01/2020	N/A	N/A	15478965	21856555
01/07/2020	AVB	GOOG	16449218	21904181
01/01/2021	N/A	N/A	22071532	21904181

Table 6

At the end of the period, our final portfolio value was $21,904,181 with $2,799,023.79 left in the bank. Once again, the total value of the portfolio fell as adjustments were made, but this time the total drop was only 0.9%. Returns came out as 212.92%, with average yearly returns at 20.94%. We can see that the maximum returns portfolio has achieved higher returns than the minimum variance portfolio, as would be expected. The graph showing movement over time can be seen below in figure 4.

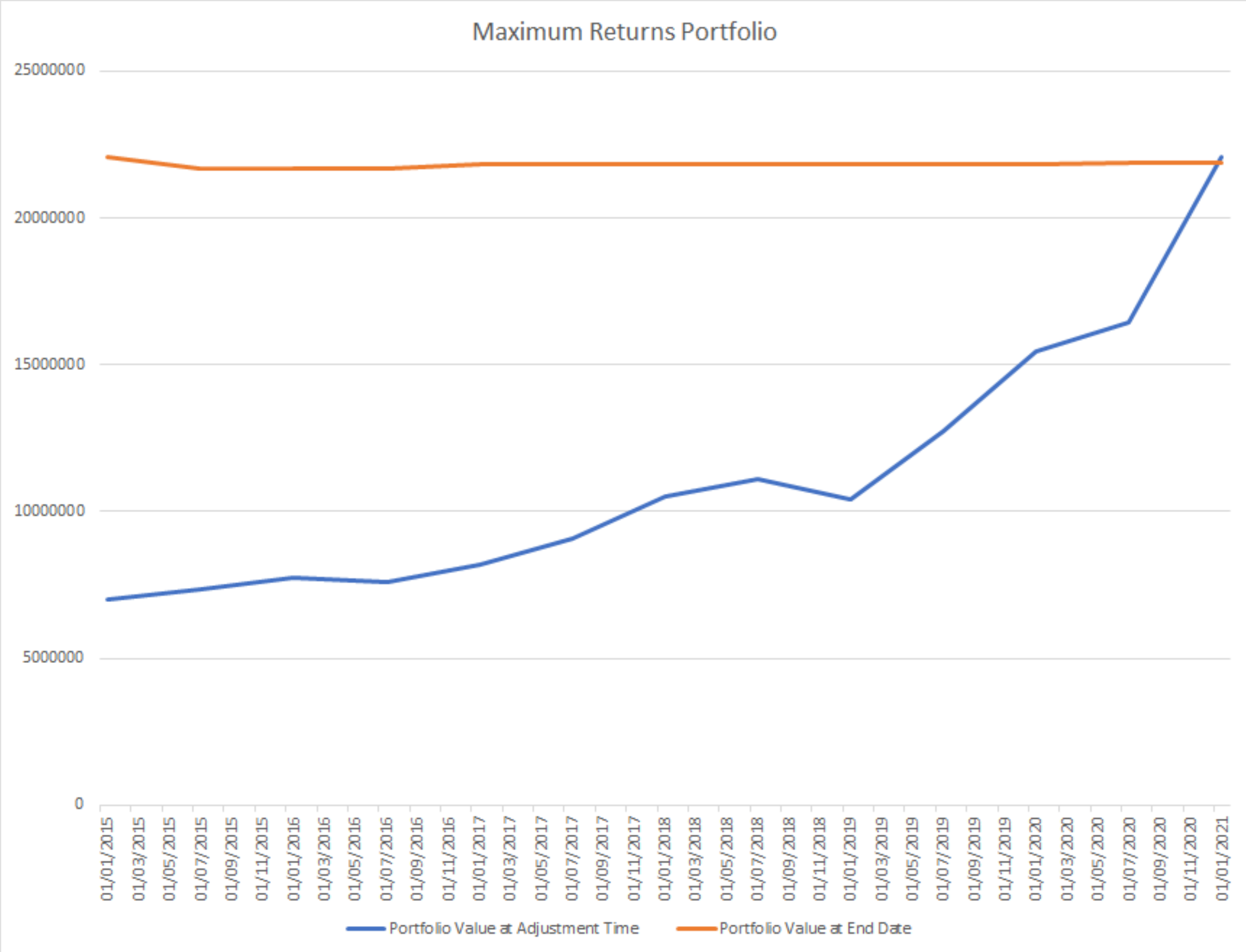


Figure 4

7.2.5 Maximum Sharpe Ratio Test (No Adjustments)

Our expected risk and return for the maximum Sharpe ratio portfolio were 10.66% and 125.12% respectively. This falls between the minimum variance portfolio and maximum return portfolios expected figures. In fact, we see that the final value of the unadjusted portfolio was $19,655,500, generating total returns of 180.79% and average yearly returns of 18.77%. This is lower than both other portfolios, and despite still making better returns than expected, it did not surpass expected returns as much as the other portfolios did.

7.2.6 Maximum Sharpe Ratio Test

Similar to the maximum returns portfolio, the application only made 3 adjustments to this portfolio. The final value of the portfolio was $18,504,338 with $2,784,814.76 left in the bank after adjustments. A breakdown of the portfolio over time can be seen below.

Date	Sold	Bought	Portfolio Value at Adjustment Time	Portfolio Value at End Date
01/01/2015	Initial	Initial	7000000	19655500
01/07/2015	KEYS	MMM	7289464	18227705
01/01/2016	N/A	N/A	7943996	18227705

01/07/2016	N/A	N/A	7844197	18227705
01/01/2017	TSCO	GOOGL	8362912	18371815
01/07/2017	N/A	N/A	9250725	18371815
01/01/2018	N/A	N/A	9903900	18371815
01/07/2018	N/A	N/A	9949294	18371815
01/01/2019	N/A	N/A	9436522	18371815
01/07/2019	N/A	N/A	11497522	18371815
01/01/2020	N/A	N/A	13570018	18371815
01/07/2020	AVB	GOOG	14106509	18504338
01/01/2021	N/A	N/A	18573899	18504338

Table 7

The portfolio value once again fell as adjustments were made, dropping by 5.86%. However, total returns were 164.35% with average yearly returns at 17.59%. This fits in nicely between maximum returns and minimum variance returns, as expected. The graph of the portfolio value over time can be seen below (figure 5).

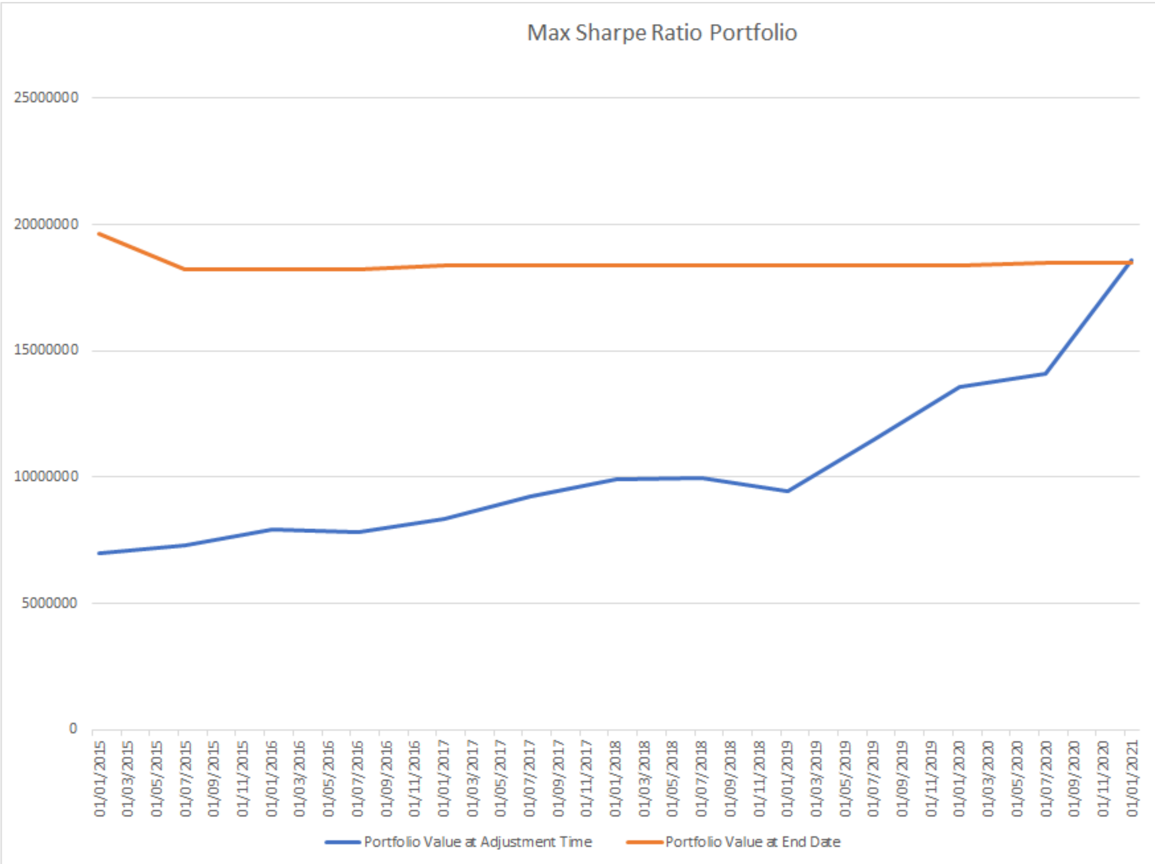


Figure 5

Combining the portfolio value graphs onto one graph allows us to compare the movement of each portfolio against each other. This can be seen in figure 6. We can see that maximum Sharpe ratio and minimum variance followed each other quite closely, with them switching position throughout the period until maximum Sharpe ratio came out on top at the end. Maximum returns also followed closely, until 2018, at which point it started to grow quicker than the rest. All the portfolios experienced a downturn in 2019, but this could be attributed to one stock that each portfolio invested in performing poorly, or a general systematic downturn.

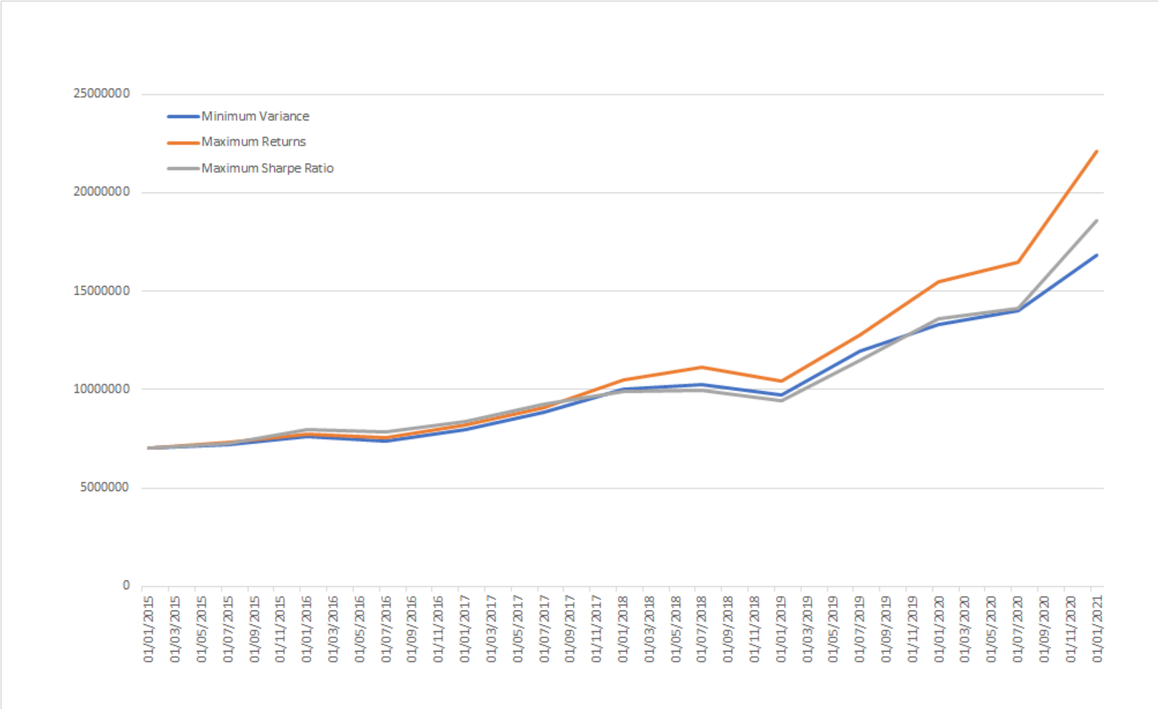


Figure 6

7.3 Limitations And Critique

Throughout testing, limitations with the application and aspects that have been overlooked were brought to attention. Leaving 3 million in the bank to cover transaction fees and spare capital for stock replacement seemed sensible, but the overall value was too high. The most taken from the bank was under $250,000, and so lowering our leftover cash balance would allow for a greater initial investment, leading to higher returns. The biannual adjustment period could also have been shortened, as forecasting for 6 months is a hard task to perform accurately for many reasons. For this reason, sometimes forecasts could be excessive and even forecast the stock dropping deep into negative values. This means that even if the stock dropped significantly, because of the excessive forecast it would never be removed from the portfolio. Shortening the adjustment period would allow for more accurate forecasting and therefore stock adjustment, although increased adjustment would lead to greater transaction fees that might necessitate the increase of the leftover capital balance. Furthermore, adjusting the portfolio can be computationally heavy for a desktop machine if ran too often, but if the app was to run in the background on a server, then this may not be an issue.

The application also does not account for stocks being removed from the market. As this has not occurred in our tests, it is unknown how the app would respond to this. The app also requires a restart sometimes in order to update figures. This makes it difficult to continuously run on a server, so either adjustments must be made for this to take place or the app must be run as a desktop application.

7.4 Further Developments

Initially, we wanted to include functionality so that the user could split their investment between different beta ranges in order to fully customise their risk and return profile (I.e., X% of the portfolio range could have beta range 0.9-1, while the other 100-X% could have range 1.25-1.4). However, this proved difficult and due to time constraints, this could not be completed. The project was also initially conceived to include markets such as the FTSE100 and FTSE250 to increase the number of stocks available. Functions available within libraries within R made it easy to extract all the stock tickers from the S&P500, but no such functionality was available for other markets and as such these would have to be manually entered. Ideally, we would have included these but because of time constraints we were unable to. In future, both aspects could be developed and implemented to improve the application.

An issue that arose during testing that should be improved upon given the time is that because of poor forecasting, stocks could drop significantly and still not be removed from the portfolio. Because of this, a stop-loss feature should be implemented whereby if a stock drops by more than a certain percentage (potentially customisable by the end user) then it will be sold and replaced. Adding a custom period for which to forecast would also be beneficial, as currently the app is limited to 6 monthly forecasts. This would allow for better forecasting and therefore adjusting.

Streamlining the app processes would also be a worthwhile investment. Currently, adjustments are made via user input instead of automatically. This creates issues as the onus is on the user to remember to open the app and check their portfolio, instead of having it managed for them. Furthermore, a full portfolio breakdown page was planned, with the aims of showing the portfolio value graph to date, forecasted values for each stock and the portfolio, as well as a customisable options tab which would include the frequency of adjustments, number of stocks in the portfolio and more.

8. CONCLUSION

Overall, we found our app functioned very successfully in all aspects from the initial selection to the adjustments and results.

To summarise our findings for the three portfolios stated in our reports for the Beta range of 0.85-1 with a risk of 16.3% and yearly return of 19.5%, we found that the maximum returns test yielded the highest returns portfolio. This was valued at $21,904,181 with $2,799,023.79 left in the bank with just 3 adjustments being made. Hence, returns came out as 212.92%, with average yearly returns at 20.94%. Similarly, without adjustments it also yielded the highest value. It is clear that the main problem of this method is centred around its disregard of risk and so would only be suitable for risk loving investors.

As we expected, the minimum variance test at the end of the testing period was significantly lower than the maximum returns test even after 9 adjustments were made. The final value of the portfolio was $16,385,117 with $2,736,205.89 left in the bank which is a 134.07% total returns, with average yearly returns of 15.23%. Without adjustments however, we found a higher return than with the adjustments. This could be down to luck in the initial selection of the stocks and could be due to the higher risk of not having adjustments being made. This is one of the main problems of the minimum variance test as although the low risk it presents some stability, it often means the return is minimal.

Our final portfolio we used the maximum Sharpe ratio test which gave us the lowest returns of the three without adjustments However, the final value of the portfolio was $18,504,338 with $2,784,814.76 left in the bank after adjustments with just the three adjustments being made. This fits in nicely between maximum returns and minimum variance returns, as was expected. Although the Sharpe ratio maximises return with minimum risk it is considered backword looking when implemented into forecasting return since clear assumptions must be outlined when using the portfolio weights. This consequently raises concerns with truthfulness.

In general, these findings were fairly successful however, given more time or being able to do it again in the future we would have liked to make alterations to some of the adjustments we made. This could include padding out the app which would require some technical analysis.
More precisely, some additions we would have liked to have done are being able to split investments across different Beta ranges could help investors to fully customise their risk and return profile and also to be able to use other markets such as the FTSE100 and FTSE250 to increase the number of stocks available to choose from.

These changes, along with others could help us to further improve the portfolios and get them to yield greater returns on our 10MM we had available to invest.

Appendices:

Bibliography:

- Ali, A., 2020. *The World's 10 Largest Stock Markets*. [online] Visual Capitalist. Available at: <https://www.visualcapitalist.com/the-worlds-10-largest-stock-markets/>
- BDF. (2021). Minimum Variance Portfolio. [online] Breaking Down Finance. Available at: < https://breakingdownfinance.com/finance-topics/modern-portfolio-theory/minimum-variance-portfolio/#:~:text=The%20minimum%20variance%20portfolio%20or,using%20only%20measures%20of%20risk.>
- Box, G.E.P. (1976). *Science and Statistics*. Journal of the American Statistical Association, pp. 791-799. doi:10.1080/01621459.1976.10480949
- Bowman, R., 2019. *Portfolio Risk - How to measure & manage risk of your investment portfolio*. [online] Catana Capital. Available at: <https://catanacapital.com/blog/portfolio-risk-measure-manage-investment-portfolio>
- Chen, J. (2020). *Risk Averse*. [online] Investopedia. Available at <https://www.investopedia.com/terms/s/sharperatio.asp>
- CFI Education Inc. (2021). *Portfolio Variance*. [online] Corporate Finance Institute. Available at: < https://corporatefinanceinstitute.com/resources/knowledge/finance/portfolio-variance/#:~:text=Portfolio%20variance%20is%20a%20statistical,concept%20in%20modern%20investment%20theory.&text=CFI's%20Math%20for%20Corporate%20Finance,concepts%20required%20for%20Financial%20Modeling.>
- CFI Education Inc. (2021). *Covariance*. [online] Corporate Finance Institute. Available at: <https://corporatefinanceinstitute.com/resources/knowledge/finance/covariance/>
- Fernando, J. (2020) *Sharpe Ratio*. [online] Investopedia. Available at: <https://www.investopedia.com/terms/s/sharperatio.asp>
- Francis, J.C., Kim, D. Modern portfolio theory: foundations, analysis, and new developments
- Hayes, A., 2020. [online] Available at: <https://www.investopedia.com/terms/s/stock.asp https://capital.com/stock-exchange-definition#:~:text=A%20marketplace%20where%20buyers%20and,currencies%20and%20other%20financial%20instruments.&text=Companies%20wanting%20their%20shares%20to,information%20to%20the%20market%20regularly.>
- Halton, C. (2021). *Risk Lover*. [online] Investopedia. Available at: <https://www.investopedia.com/terms/r/risklover.asp>
- Lioudis, N. (2021) *Understanding the Sharpe Ratio*. [online] Investopedia. Available at: <https://www.investopedia.com/articles/07/sharpe_ratio.asp>

- Statman, M. (1987) “How Many Stocks Make a Diversified Portfolio?,” Journal of Financial and Quantitative Analysis. Cambridge University Press, 22(3), pp. 353–363. doi: 10.2307/2330969.
- Voigt, K., O’Shea A. 2020. *Definition: What Is Stock? - NerdWallet*. [online] Available at: <https://www.nerdwallet.com/article/investing/what-is-a-stock>

Appendix A:

TYPE OF STOCK	COMMON STOCK	PREFERRED STOCK
DIVIDENDS	Dividends are variable, they can increase and decrease at any time.	Dividends are fixed and pre-determined.
GROWTH	Growth relates to the growth of the company.	No growth.
LIQUIDATION	Paid last after creditors, bond holders and preferred shareholders.	Paid before common shareholders.
RIGHT TO VOTE	Have the right to vote.	No voting rights.
ARREARS	Missed dividends do not accrued, they can be paid next year.	Accrued arrears added to next year’s payment.
CERTAINTY	No profit, no dividends.	Paid even if company experiences losses.

Table 1: Common Stock Vs Preferred Stock.

Appendix B:
Minimum Variance Portfolio Example:

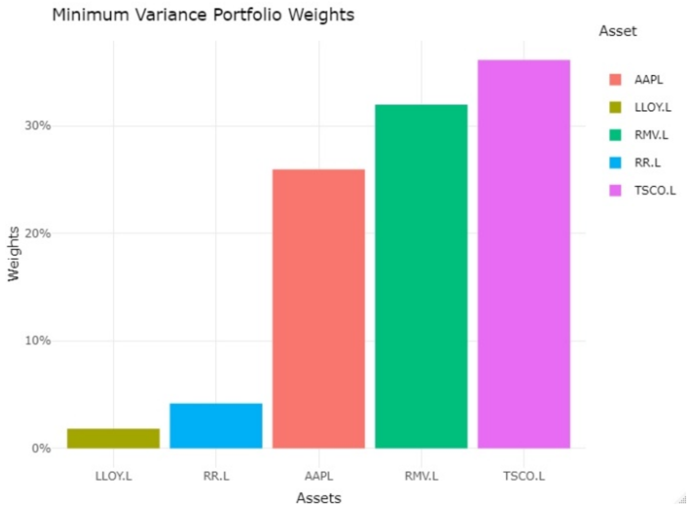


Figure 1: The Minimum Variance Portfolio, using the example from section 5.1.

Highest Return Portfolio Example:

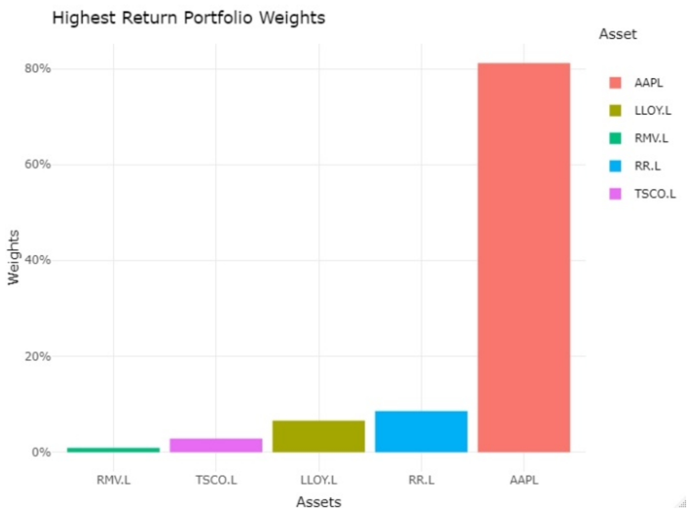


Figure 2: The Highest Return Portfolio, using the example from section 5.1.

Max Sharpe Ratio Portfolio Example:

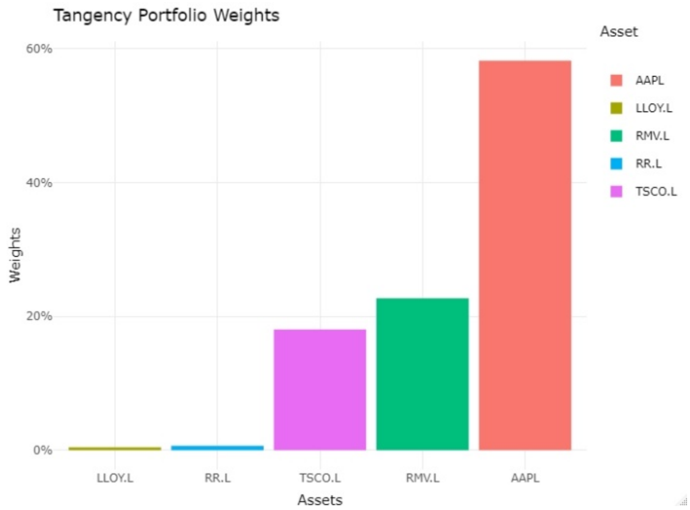
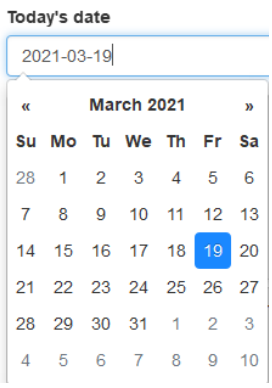
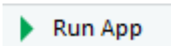


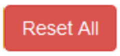
Figure 3: The Max Sharpe Ratio Portfolio, using the example from section 5.1.

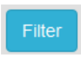
Appendix C:
User Instructions:

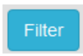
Requires RStudio v1.1.463, R libraries:
shiny, tidyquant, tidyverse, rdrop2, shinycssloaders, DT, XML, fpp2, rvest, shinybusy, timetk.

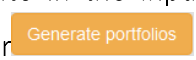


The below instructions are a guide on how to select and invest in a portfolio, as well as how to make adjustments. To begin, open the "beta.r" file and click 

in order to launch the application. Ensure that the portfolio is reset by clicking the 

button. If you are investing in a portfolio for the future, continue to the next step. If not, (i.e. you are testing the application using historical data) ensure that you change the "Today's Date" selection to the date at which you wish to start the portfolio. Once this has been changed, use the slider within the Portfolio Selection tab to filter the beta range that you want the portfolio to use. Pressing 

will then show the stocks that have the requisite beta value in the table to the right, labelled "Filtered Beta Stocks". Next, type in the number of stocks you wish to invest in in the input labelled "Quantity of stocks to select". The minimum value is 2. Clicking 

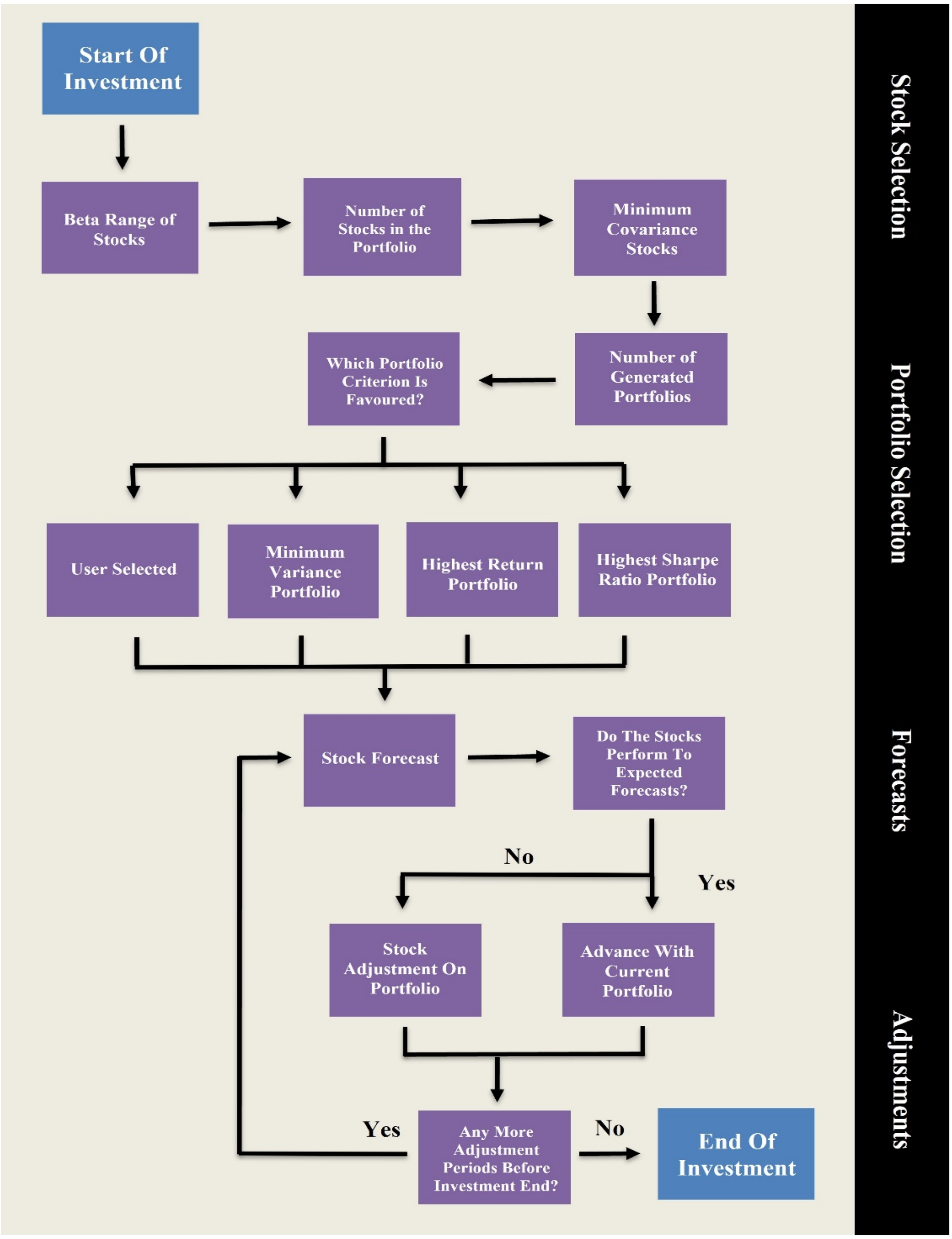
again will display the stocks selected by minimum covariance in the table on the right labelled "Minimum Covariance Stocks". The next step is to specify the number of portfolios to generate in the input below. The recommended value is 20000, but the higher value the better 

will generate the Markowitz mean-variance efficiency frontier, displayed on the graph to the right. The dropdown menu below will allow you to select the type of portfolio to invest in. Clicking on

the graph will display the weights, risk, return and Sharpe ratio of all the portfolios at that point in the table below. Then, enter the value with which you want to invest. The maximum value is $10,000,000. It is recommended to use less than this in order to allow for capital for the transaction fees when making adjustments. Finally, press Invest in order to proceed with the portfolio. Investing in multiple different portfolios is not currently supported.
Next, open the "app.r" file in the same way as before. We must ensure that the application invested correctly. Set the date in the "Today's Date" form to the date when you invested in the portfolio (if in the past) and check that the "Portfolio value" displayed is the same as the amount you invested with. If not, go back to the other page and reset and try again. In order to make adjustments to the portfolio, reopen "beta.r", select the date at which you want to adjust (it is recommended to adjust in 6 month intervals for best results) and press Adjust Portfolio.
On the "app.r" page, you can have manual control over the stocks in the portfolio. Here, you can buy or sell specific stocks as well as inspect a stocks historical price. Furthermore, if you want to reap the rewards of your investment, selecting each stock via the dropdown and pressing Sell all will sell all of that stock and deposit the proceeds in your balance.

Error management: If an error occurs after trying to reset the portfolio where it won't open due to a "string parse error ;", in the R console type in "buyStock("AAPL",1). Then reopen beta.r and reset the portfolio again. This happens if the portfolio is wiped without any data being entered. Ensure the working directory is set to the source file location before doing this.

Appendix D:
Basic App Functionality Flowchart:


Start Of Investment
Beta Range of Stocks
Number of Stocks in the Portfolio
Minimum Covariance Stocks
Number of Generated Portfolios
Which Portfolio Criterion Is Favoured?
User Selected
Minimum Variance Portfolio
Highest Return Portfolio
Highest Sharpe Ratio Portfolio
Stock Forecast
Do The Stocks Perform To Expected Forecasts?
No
Yes
Stock Adjustment On Portfolio
Advance With Current Portfolio
Yes
Any More Adjustment Periods Before Investment End?
No
End Of Investment
Stock Selection
Portfolio Selection
Forecasts
Adjustments