

Language modeling

CS 685, Spring 2023

Advanced Natural Language Processing

<http://people.cs.umass.edu/~miyyer/cs685/>

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Impending deadlines

- **2/17:** HW 0 due
- **2/17:** Final project group assignments due
 - Google Form for project teams to follow
- **3/8:** Project proposals due

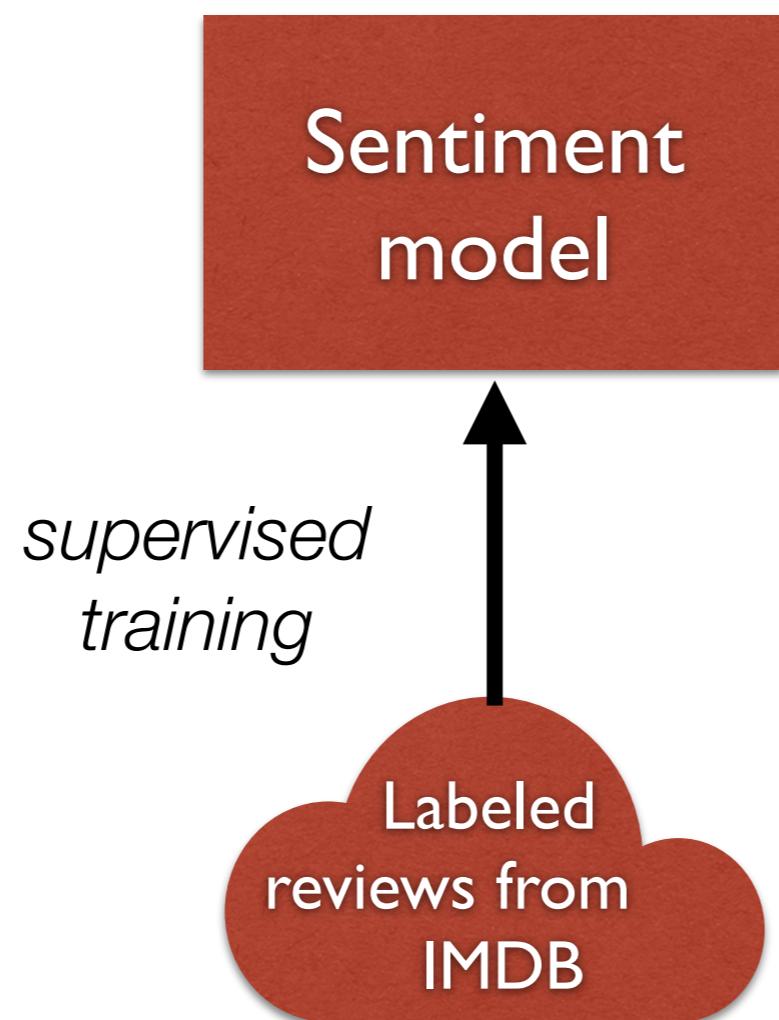
Earn extra credit!

- This semester, there is a weekly remote NLP seminar on Tuesdays from 11:30am-12:30pm.
- If you attend a talk and submit a short summary of its contents, you'll receive extra credit (Overleaf template to be released).
- You can receive extra credit for up to 3 talks.
- Talks will be recorded so you can watch them later if that time doesn't work.
- Details to be announced on Piazza, first talk is next Tuesday (2/14)!

Let's say I want to train a model for *sentiment analysis*

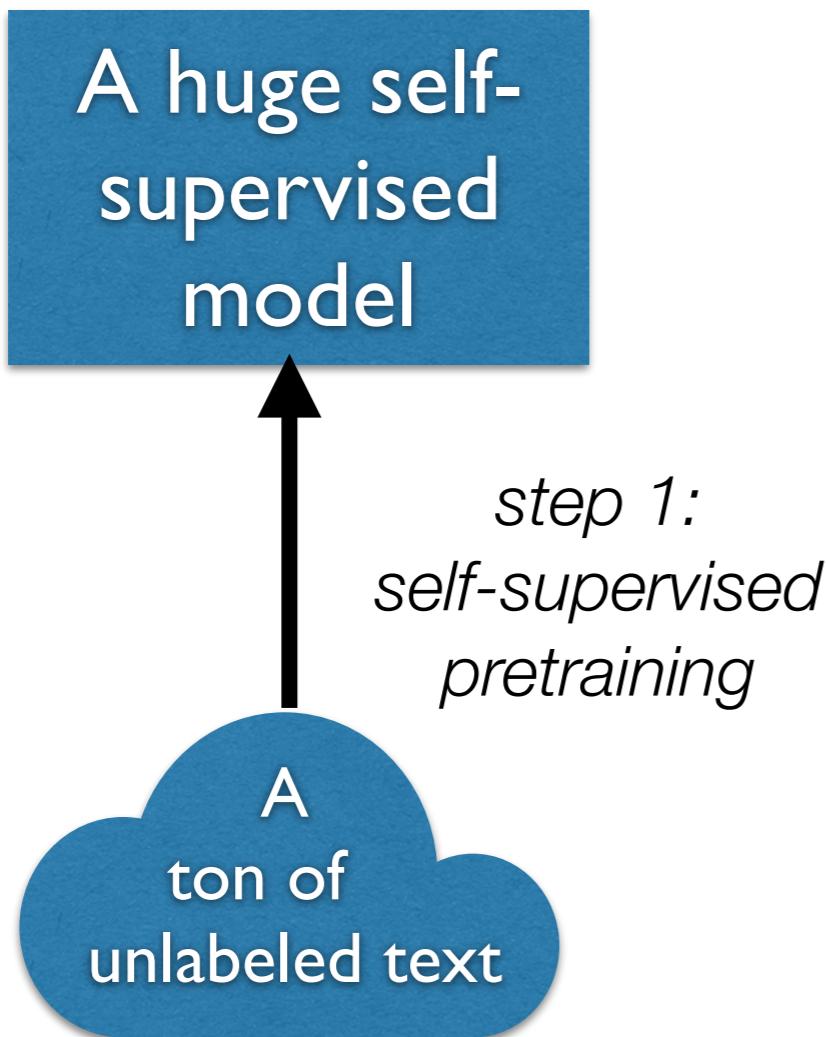
Let's say I want to train a model for *sentiment analysis*

In the past, I would simply train a *supervised* model on labeled sentiment examples (i.e., review text / score pairs from IMDB)



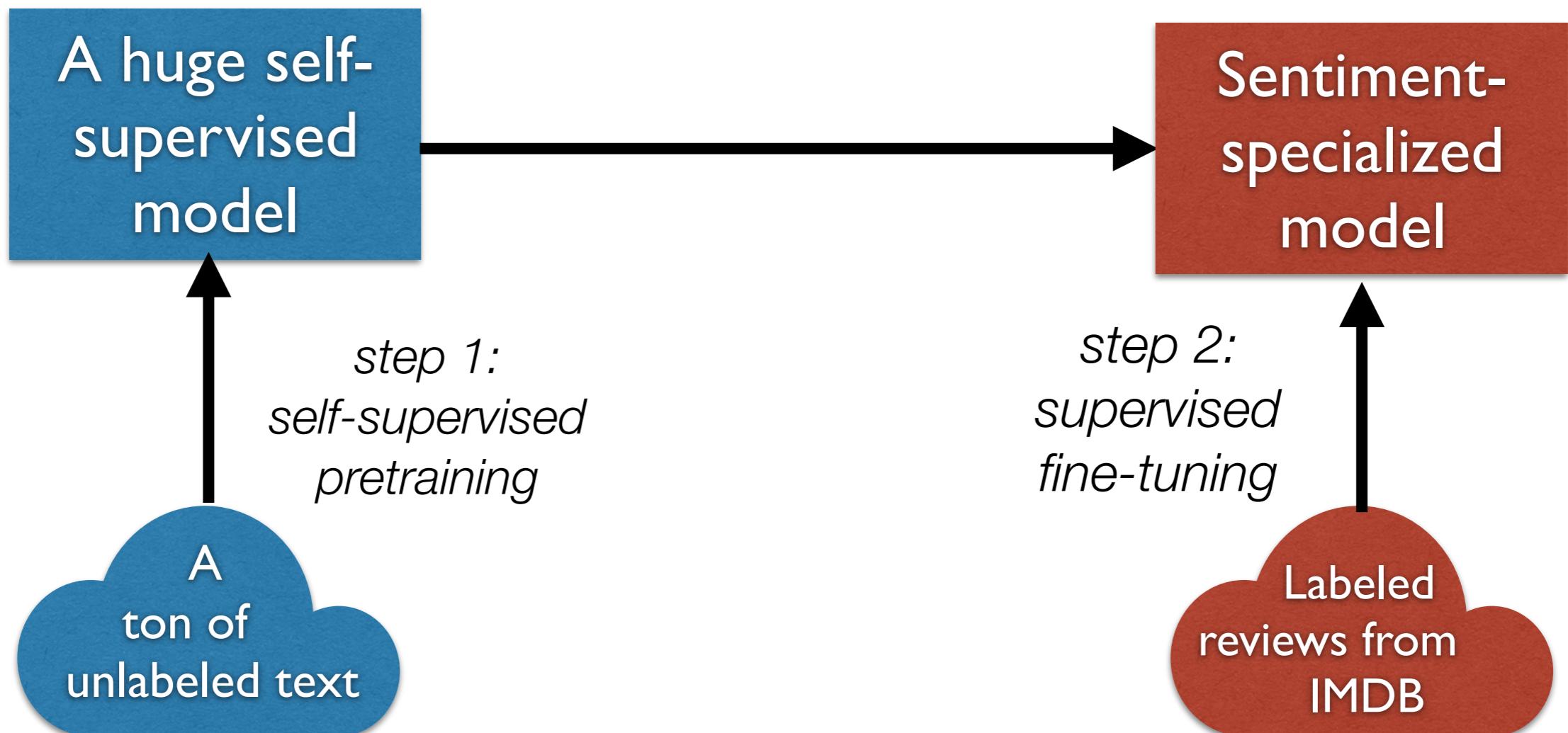
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Nowadays, however, we take advantage of *transfer learning*:



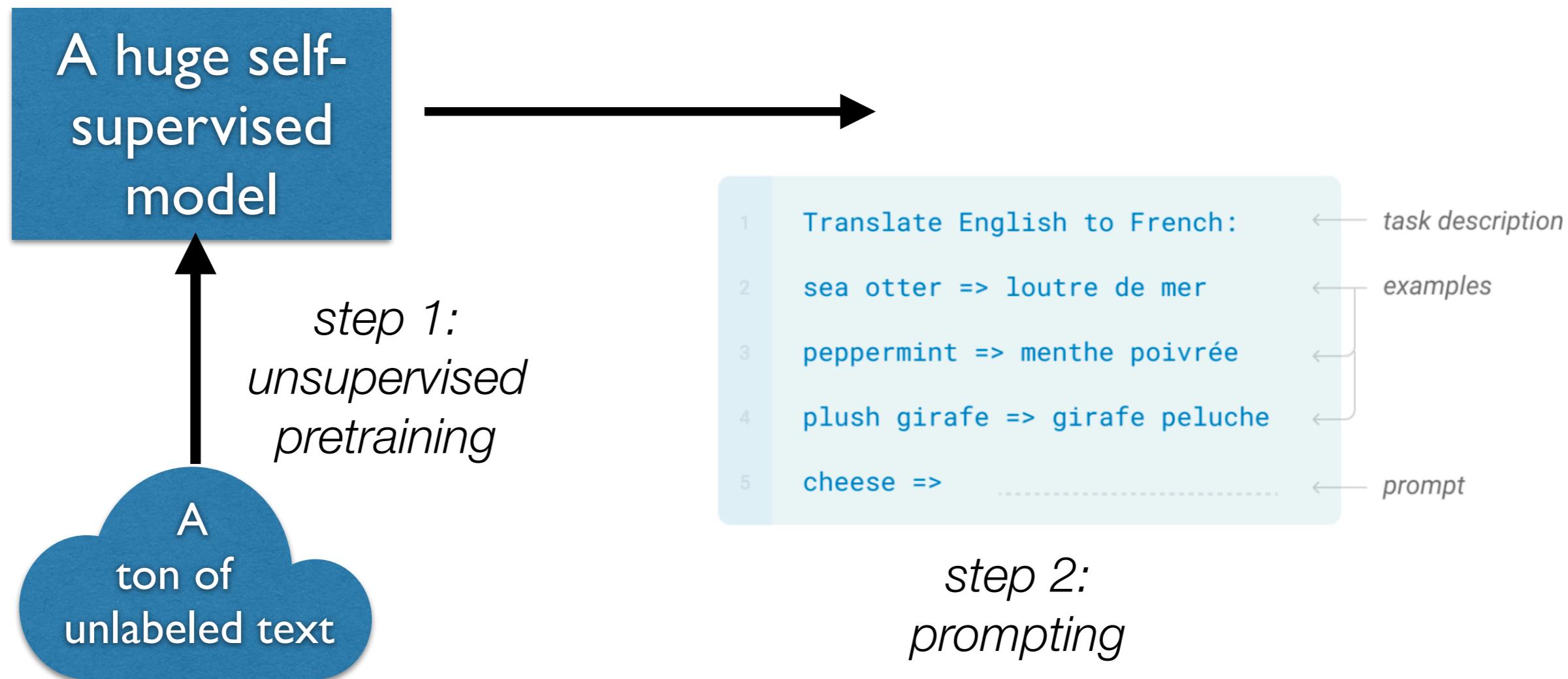
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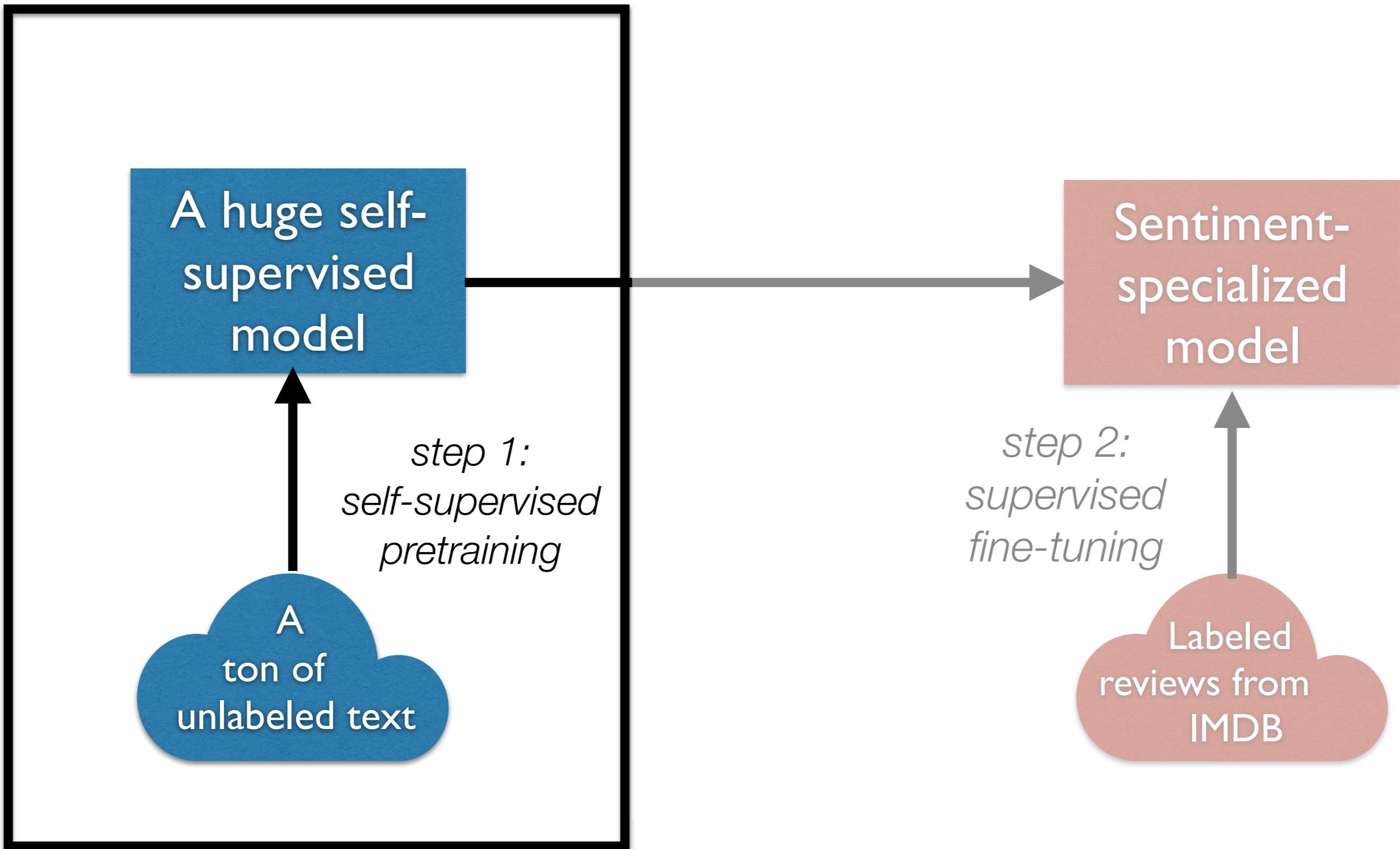


Let's say I want to train a model for *sentiment analysis*

Or just rely entirely on the self-supervised model via prompting...



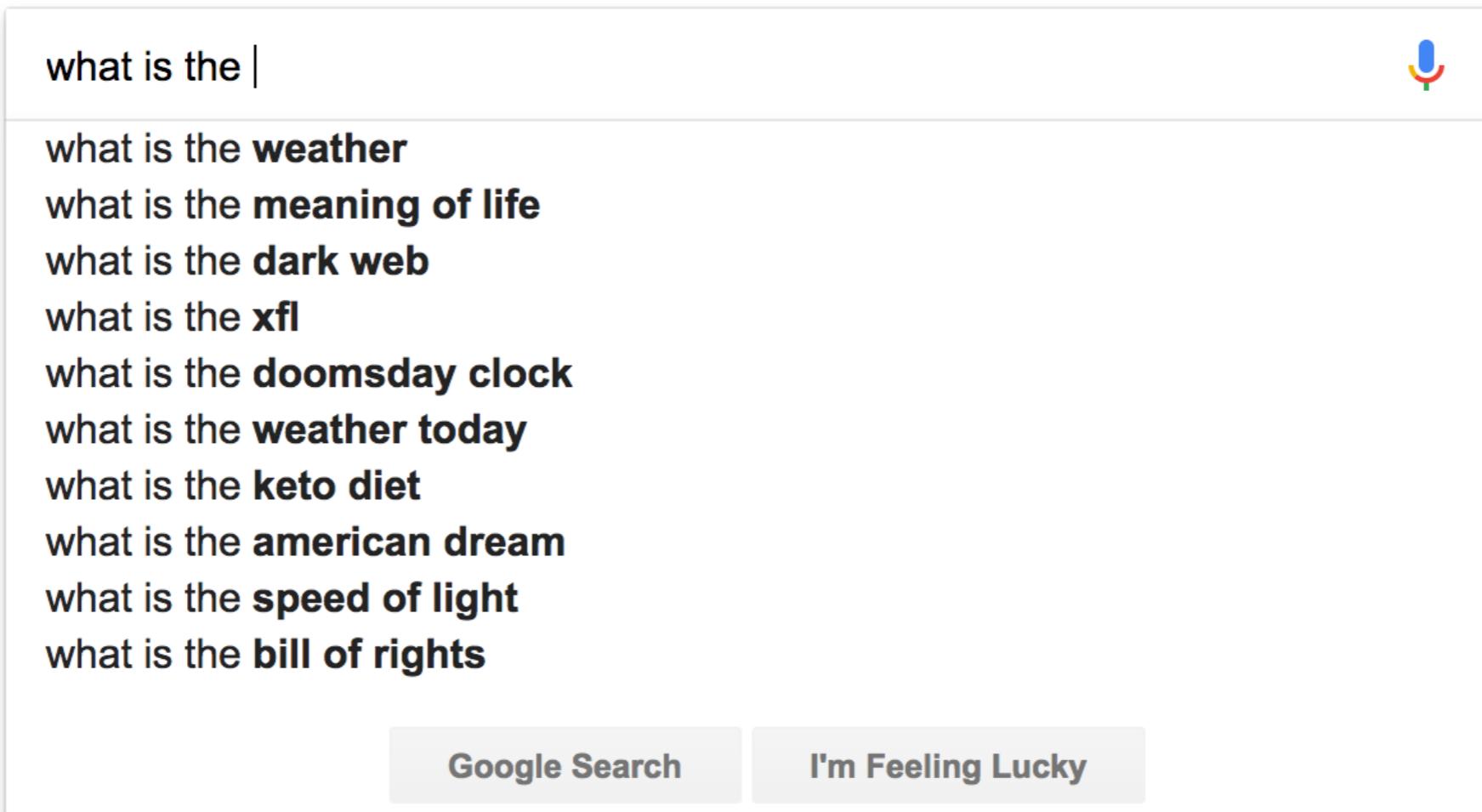
This lecture: **language modeling**, which forms the core of most self-supervised NLP approaches



Language models assign a probability to a piece of text

- why would we ever want to do this?
- translation:
 - $P(i \text{ flew to the movies}) <<<< P(i \text{ went to the movies})$
- speech recognition:
 - $P(i \text{ saw a van}) >>>> P(\text{eyes awe of an})$

You use Language Models every day!



A screenshot of a Google search interface. The search bar at the top contains the text "what is the |". To the right of the search bar is a microphone icon. Below the search bar, a list of suggested search queries is displayed, each starting with "what is the" followed by a bolded term. At the bottom of the interface are two buttons: "Google Search" and "I'm Feeling Lucky".

- what is the **weather**
- what is the **meaning of life**
- what is the **dark web**
- what is the **xfl**
- what is the **doomsday clock**
- what is the **weather today**
- what is the **keto diet**
- what is the **american dream**
- what is the **speed of light**
- what is the **bill of rights**

Google Search I'm Feeling Lucky

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

$$P(W) = P(w_1, w_2, w_3, w_4, w_5 \dots w_n)$$

- Related task: probability of an upcoming word:

$$P(w_5 | w_1, w_2, w_3, w_4)$$

- A model that computes either of these:

$P(W)$ or $P(w_n | w_1, w_2 \dots w_{n-1})$ is called a **language model** or **LM**

How to compute $P(W)$

- How to compute this joint probability:
 - $P(\text{its, water, is, so, transparent, that})$
- Intuition: let's rely on the Chain Rule of Probability

Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$P(B|A) = P(A,B)/P(A) \quad \text{Rewriting: } P(A,B) = P(A)P(B|A)$$

- More variables:

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

- The Chain Rule in General

$$P(x_1, x_2, x_3, \dots, x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2)\dots P(x_n|x_1, \dots, x_{n-1})$$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

$$\begin{aligned} P(\text{"its water is so transparent"}) &= \\ P(\text{its}) \times P(\text{water} | \text{its}) \times P(\text{is} | \text{its water}) \\ \times P(\text{so} | \text{its water is}) \times P(\text{transparent} | \text{its water is so}) \end{aligned}$$

The Chain Rule applied to compute joint probability of words in sentence

In HW0, we refer to this as a “prefix”

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

$$\begin{aligned} P(\text{"its water is so transparent"}) &= \\ P(\text{its}) \times P(\text{water} | \text{its}) \times P(\text{is} | \text{its water}) \\ \times P(\text{so} | \text{its water is}) \times P(\text{transparent} | \text{its water is so}) \end{aligned}$$

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the water is so transparent that}) = \frac{\text{Count}(\text{water is so transparent that})}{\text{Count}(\text{water is so transparent that})}$$

Markov Assumption

- Simplifying assumption:



Andrei Markov (1856~1922)

$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$

- Or maybe

$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$

Markov Assumption

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i | w_{i-k} \dots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-k} \dots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i)$$

Simplest case: Unigram model

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model:

fifth, an, of, futures, the, an, incorporated, a, a,
the, inflation, most, dollars, quarter, in, is, mass

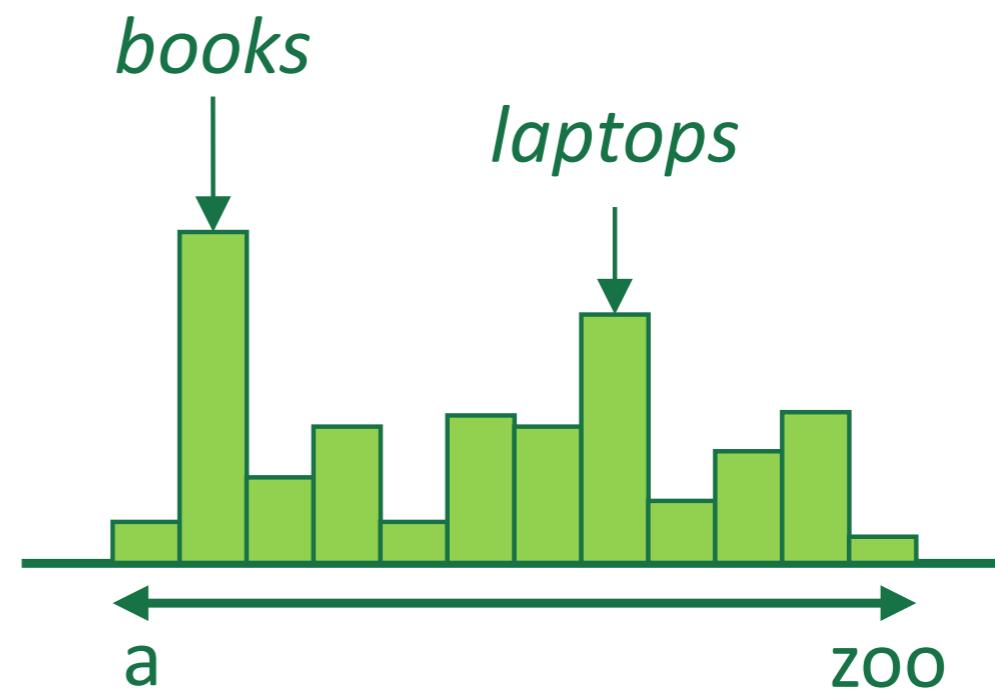
thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the

How can we generate text
from a language model?

Decoding from an LM

Prefix: “students opened their”



**Probability distribution over
next word**

Approximating Shakespeare

- | | |
|-----------|---|
| 1
gram | –To him swallowed confess hear both. Which. Of save on trail for are ay device and
rote life have |
| | –Hill he late speaks; or! a more to leg less first you enter |
| 2
gram | –Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live
king. Follow. |
| | –What means, sir. I confess she? then all sorts, he is trim, captain. |
| 3
gram | –Fly, and will rid me these news of price. Therefore the sadness of parting, as they say,
'tis done. |
| | –This shall forbid it should be branded, if renown made it empty. |
| 4
gram | –King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A
great banquet serv'd in; |
| | –It cannot be but so. |

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
 - In general this is an insufficient model of language
 - because language has **long-distance dependencies**:
- “The computer which I had just put into the machine room on the fifth floor crashed.”

Estimating bigram probabilities

- The Maximum Likelihood Estimate (MLE)
 - relative frequency based on the empirical counts on a training set

$$P(w_i | w_{i-1}) = \frac{\text{count}(w_{i-1}, w_i)}{\text{count}(w_{i-1})}$$

$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

c – count

An example

$$P(w_i | w_{i-1}) = \frac{\underset{\text{MLE}}{C}(w_{i-1}, w_i)}{C(w_{i-1})} \begin{array}{l} <\!\!s\!\!> \text{ I am Sam } <\!\!/s\!\!> \\ <\!\!s\!\!> \text{ Sam I am } <\!\!/s\!\!> \\ <\!\!s\!\!> \text{ I do not like green eggs and ham } <\!\!/s\!\!> \end{array}$$

$$P(\text{I} | <\!\!s\!\!>) = \frac{2}{3} = .67$$

$$P(<\!\!/s\!\!> | \text{Sam}) = \frac{1}{2} = 0.5$$

$$P(\text{Sam} | <\!\!s\!\!>) = ???$$

$$P(\text{Sam} | \text{am}) = ???$$

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$$P(\text{Sam} | <\!\!s\!\!>) = \frac{1}{3} = .33$$

$$P(\text{Sam} | \text{am}) = \frac{1}{2} = .5$$

$$P(\text{am} | I) = \frac{2}{3} = .67$$

$$P(\text{do} | I) = \frac{1}{3} = .33$$

An example

Important terminology: a word **type** is a unique word in our vocabulary, while a **token** is an occurrence of a word type in a dataset.

$$P(w_i | w_{i-1}) = \frac{\text{MLE } C(w_{i-1}, w_i)}{C(w_{i-1})} \begin{array}{l} <\!\!s\!\!> \text{ I am Sam } <\!\!s\!\!> \\ <\!\!s\!\!> \text{ Sam I am } <\!\!s\!\!> \\ <\!\!s\!\!> \text{ I do not like green eggs and ham } <\!\!s\!\!> \end{array}$$

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Demo

<https://books.google.com/ngrams/>

A bigger example: Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

note: this is only a subset of the (much bigger) bigram count table

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}^{\text{MLE}}$$

- Normalize by unigrams:

• Result:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

$$P(< s > \text{ I want english food } < /s >) =$$

$$P(I | < s >)$$

$$\times P(\text{want} | I)$$

$$\times P(\text{english} | \text{want})$$

$$\times P(\text{food} | \text{english})$$

$$\times P(< /s > | \text{food})$$

$$= .000031$$

these probabilities get super tiny when we have longer inputs w/ more infrequent words... how can we get around this?

logs to avoid underflow

$$\log \prod p(w_i | w_{i-1}) = \sum \log p(w_i | w_{i-1})$$

Example with unigram model on a sentiment dataset:

sentence: I love love love love love the movie

logs to avoid underflow

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Example with unigram model on a sentiment dataset:

sentence: I love love love love love the movie

$$p(\text{i}) \cdot p(\text{love})^5 \cdot p(\text{the}) \cdot p(\text{movie}) = 5.95374181\text{e-}7$$

$$\begin{aligned} \log p(\text{i}) + 5 \log p(\text{love}) + \log p(\text{the}) + \log p(\text{movie}) \\ = -14.3340757538 \end{aligned}$$

What kinds of knowledge?

- $P(\text{english} \mid \text{want}) = .0011$ → about the world
- $P(\text{chinese} \mid \text{want}) = .0065$
- $P(\text{to} \mid \text{want}) = .66$ → grammar – infinitive verb
- $P(\text{eat} \mid \text{to}) = .28$
- $P(\text{food} \mid \text{to}) = 0$ → ???
- $P(\text{want} \mid \text{spend}) = 0$ → grammar
- $P(\text{i} \mid \langle s \rangle) = .25$

Language Modeling Toolkits

- SRILM
 - <http://www.speech.sri.com/projects/srilm/>
- KenLM
 - <https://kheafield.com/code/kenlm/>

Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
 - Assign higher probability to “real” or “frequently observed” sentences
 - Than “ungrammatical” or “rarely observed” sentences?
- We train parameters of our model on a **training set**.
- We test the model’s performance on data we haven’t seen.
 - A **test set** is an unseen dataset that is different from our training set, totally unused.
 - An **evaluation metric** tells us how well our model does on the test set.

Evaluation: How good is our model?

- The goal isn't to pound out fake sentences!
 - Obviously, generated sentences get “better” as we increase the model order
 - More precisely: using maximum likelihood estimators, higher order is always better likelihood on **training set**, but not **test set**

Example: I use a bunch of New York Times articles to build a bigram probability table



train →

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
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evaluate



Now I'm going to evaluate the probability of some *heldout* data using our bigram table

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evaluate



A good language model
should assign a high
probability to heldout text!

Now I'm going to evaluate the probability of some *heldout* data using our bigram table

Training on the test set

- We can't allow test sentences into the training set
- We will assign it an artificially high probability when we set it in the test set
- “Training on the test set”
- Bad science!

This advice is generally applicable to any downstream task! Do NOT do this in your final projects unless you want to lose a lot of points :)

Intuition of Perplexity

- The Shannon Game:
 - How well can we predict the next word?
I always order pizza with cheese and _____
 - The 33rd President of the US was _____*
 - I saw a _____*
 - Unigrams are terrible at this game. (Why?)
- A better model of a text
 - is one which assigns a higher probability to the word that actually occurs

mushrooms 0.1

pepperoni 0.1

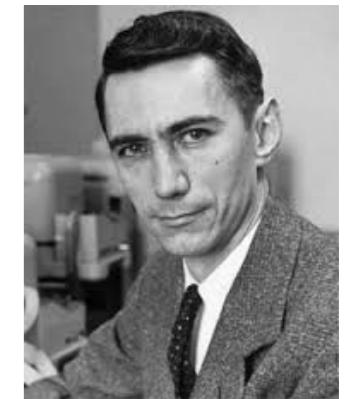
anchovies 0.01

....

fried rice 0.0001

....

and 1e-100



Claude Shannon
(1916~2001)

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest $P(\text{sentence})$

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$\begin{aligned} PP(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}} \end{aligned}$$

Chain rule:

$$PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}$$

For bigrams:

$$PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

Perplexity as branching factor

Let's suppose a sentence consisting of random digits

What is the perplexity of this sentence according to a model that assign $P=1/10$ to each digit?

$$\begin{aligned} \text{PP}(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \left(\frac{1}{10}\right)^{-\frac{1}{N}} \\ &= \frac{1}{10}^{-1} \\ &= 10 \end{aligned}$$

In practice, we use log probs

$$PP(W) = \exp\left(-\frac{1}{N} \sum_i^N \log p(w_i | w_{<i})\right)$$

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$$PP(W) = \exp\left(-\frac{1}{N} \sum_i^N \log p(w_i | w_{<i})\right)$$

Perplexity is the exponentiated *token-level negative log-likelihood*

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, Wall Street Journal

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

Shakespeare as corpus

- $N=884,647$ tokens, $V=29,066$
- Shakespeare produced 300,000 bigram types out of $V^2= 844$ million possible bigrams.
 - So 99.96% of the possible bigrams were never seen (have zero entries in the table)

Zeros

Training set:

- ... denied the allegations
- ... denied the reports
- ... denied the claims
- ... denied the request

• Test set

- ... denied the offer
- ... denied the loan

$$P(\text{"offer"} \mid \text{denied the}) = 0$$

The intuition of smoothing (from Dan Klein)

- When we have sparse statistics:

$P(w | \text{denied the})$

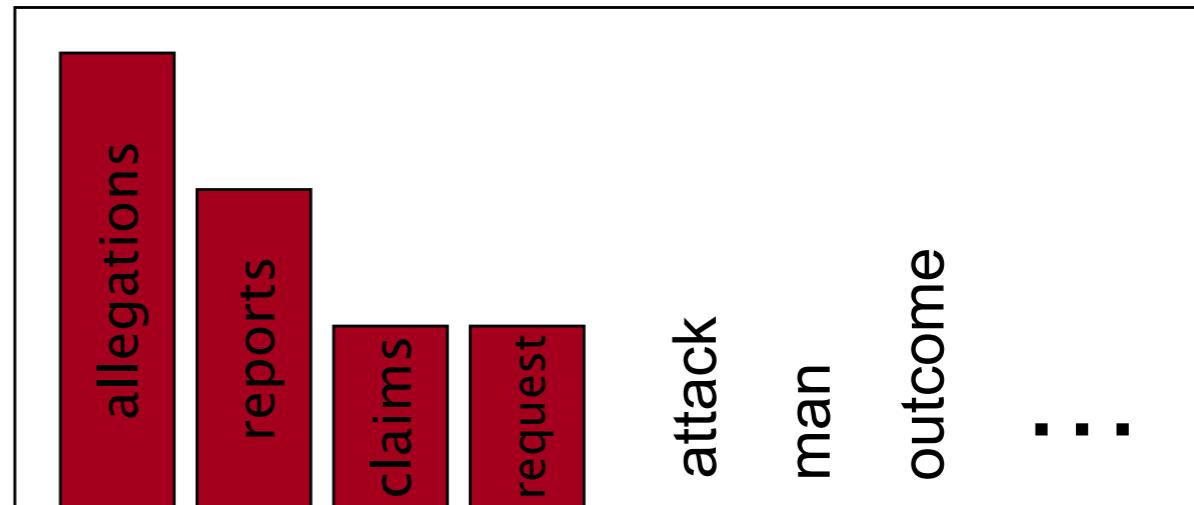
3 allegations

2 reports

1 claims

1 request

7 total



- Steal probability mass to generalize better

$P(w | \text{denied the})$

2.5 allegations

1.5 reports

0.5 claims

0.5 request

2 other

7 total

