Ficha 6 - Cónicas e quádricas

1)

a) $\cdot\ x^2+y^2-2xy+2x+4y+5=0$

$A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ $B=\begin{bmatrix}2 & 4\end{bmatrix}$ $\mu=5$

$\cdot$ Valores próprios de A $\quad D=\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix}$

$\cdot\ \det(A-\lambda I)=0$

$\Leftrightarrow \begin{vmatrix}1-\lambda & -1\\ -1 & 1-\lambda\end{vmatrix}=0 \Leftrightarrow (1-\lambda)^2-1=0$

$\Leftrightarrow 1-2\lambda+\lambda^2-1=0$

$\Leftrightarrow \lambda(-2+\lambda)=0$

$\Leftrightarrow \lambda=0 \vee \lambda=2$

$\cdot$ Se $X=P\hat{X}$ $\qquad \cdot\ \hat{B}=BP$

$\cdot$ Determinação da matriz P

$\cdot$ Para $\lambda=0$

$\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\sim\begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix} \Rightarrow x_1=x_2 \Rightarrow X_1=(1;1)$

$\|X_1\|=\sqrt{2}$

Para $\lambda=2$

$\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}\sim\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} \Rightarrow x_2=-x_1 \Rightarrow X_2=(+1;-1)$

$\|X_2\|=\sqrt{2}$

$P=\begin{bmatrix}1/\sqrt{2} & \sqrt{2}/2\\ -1/\sqrt{2} & \sqrt{2}/2\end{bmatrix}=\begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2\\ -\sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}$

$\hat{B}=BP=\underset{1\times 2}{\begin{bmatrix}2 & 4\end{bmatrix}}\cdot\underset{2\times 2}{\begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2\\ -\sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}}=\begin{bmatrix}\sqrt{2}-2\sqrt{2} & \sqrt{2}+2\sqrt{2}\end{bmatrix}$

$=\begin{bmatrix}-\sqrt{2} & 3\sqrt{2}\end{bmatrix}$

$\bullet\ \tilde{x}^T D . \tilde{x} + \tilde{B}\tilde{x} + \mu = 0$

$\Leftrightarrow \underset{1\times 2}{\begin{bmatrix} x & y \end{bmatrix}} . \underset{2\times 2}{\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}} . \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\sqrt{2} & 3\sqrt{2} \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} + 5 = 0$

$\Leftrightarrow \begin{bmatrix} 2x & 0 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\sqrt{2}x + 3\sqrt{2}y \end{bmatrix} + 5 = 0$

$\Leftrightarrow 2x^2 - \sqrt{2}x + 3\sqrt{2}y + 5 = 0$

$\Leftrightarrow 2.(x^2 \sqrt{2}/2 x + 1/8 - 1/8) + 3\sqrt{2}y + 5 = 0$

$\Leftrightarrow 2.(x - \sqrt{2}/4)^2 - 2/8 + 3\sqrt{2}y + 5 = 0 \quad , \tilde{x} = x - \sqrt{2}/2$
$\quad , \tilde{y} = y$

$\Leftrightarrow 2.\tilde{x}^2 + 3\sqrt{2}y + 1/4 + 5 = 0$

$\Leftrightarrow 2x^2 + 3\sqrt{2}y + 5{,}25 = 0$

$\Leftrightarrow y = \dfrac{-2x^2 - 5{,}25}{3\sqrt{2}} \rightarrow$ é uma parábola

b). $4xy - 2x + 6y + 3 = 0$

$\bullet\ A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 6 \end{bmatrix} \qquad \mu = 3$

$\bullet\ \det(A - \lambda I) = 0 \qquad\qquad D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

$\Leftrightarrow \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = 0 \quad \Leftrightarrow \lambda^2 = 4$
$\quad \Leftrightarrow \lambda = -2 \ \vee\ \lambda = 2$

$\bullet$ Para $\lambda = -2$

$\begin{bmatrix} +2 & 2 \\ 2 & 2 \end{bmatrix} \quad u - x_1 = +x_2 \qquad X_1 = (1 ; -1)$

$\qquad ||X_1|| = \sqrt{2}$

• Para $\lambda = 2$

$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$, $x_1 = x_2$, $V_2 = (1,1)$
$\|V_2\| = \sqrt{2}$

$P = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}$ • $\hat{B} = BP$, $X = P\hat{X}$

• $\hat{B} = \underset{1\times 2}{\begin{bmatrix} -2 & 6 \end{bmatrix}} . \underset{2\times 2}{\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}} = \begin{bmatrix} -\sqrt{2}+3\sqrt{2} & -\sqrt{2}-3\sqrt{2} \end{bmatrix}$
$= \begin{bmatrix} 2\sqrt{2} & -4\sqrt{2} \end{bmatrix}$

• $\hat{X}^t . D . \hat{X} + \hat{B}\hat{X} + \mu = 0$

$\Leftrightarrow \begin{bmatrix} x & y \end{bmatrix} . \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2\sqrt{2} & -4\sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 3 = 0$

$\Leftrightarrow \begin{bmatrix} 2x & -2y \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2\sqrt{2}x - 4\sqrt{2}y \end{bmatrix} + 3 = 0$

$\Leftrightarrow 2x^2 - 2y^2 + 2\sqrt{2}x - 4\sqrt{2}y + 3 = 0$

$\Leftrightarrow 2(x^2 + \sqrt{2}x) - 2(y^2 + 2\sqrt{2}y) = -3$

$\Leftrightarrow 2.(x^2 + \sqrt{2}x + 1/2 - 1/2) - 2(y^2 + 2\sqrt{2}y + 2 - 2) = -3$

$\Leftrightarrow 2(x + \sqrt{2}/2)^2 - 1 - 2(y + \sqrt{2})^2 + 4 = -3$

$\Leftrightarrow 2(x + \sqrt{2}/2)^2 - 2(y + \sqrt{2})^2 = -6$, $\hat{x} = x + \sqrt{2}/2$
, $\hat{y} = y + \sqrt{2}$

$\Leftrightarrow \frac{(x + \sqrt{2}/2)^2}{-3} + \frac{(y + \sqrt{2})^2}{3} = 1$

$\Rightarrow \frac{(\hat{x})^2}{-3} + \frac{(\hat{y})^2}{3} = 1 \Leftrightarrow \frac{\hat{y}^2}{3} - \frac{\hat{x}^2}{3} = 1$

$\lambda_1 = \frac{1}{3}$, $\lambda_2 = -\frac{1}{3}$

Como λ tem sinais contrários, e $\mu \neq 0 \Rightarrow$ hipérbole

c) • $x^2+2x+y^2-4y=0$

$\Leftrightarrow x^2+2x+1-1+y^2-4y+4-4=0$

$\Leftrightarrow (x+1)^2+(y-2)^2=5$, $\tilde{x}=x+1$
, $\tilde{y}=y-2$

$\Leftrightarrow \tilde{x}^2+\tilde{y}^2=5$

$\Leftrightarrow \frac{\tilde{x}^2}{5}+\frac{\tilde{y}^2}{5}=1$ → elipse, pois a equação característica da elipse é $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

2)
a) • $x^2-y^2-z^2+4x-6y-9=0$

$\Leftrightarrow x^2+4x+4-4-(y^2+6y+9-9)-z^2=9$

$\Leftrightarrow (x+2)^2-(y+3)^2-4+\cancel{9}-z^2=\cancel{9}$

$\Leftrightarrow (x+2)^2-(y+3)^2-z^2=4$, $\tilde{x}=x+2$
$\tilde{y}=y+3$
$\tilde{z}=z$

$\Leftrightarrow \frac{\tilde{x}^2}{4}-\frac{\tilde{y}^2}{4}-\frac{z^2}{4}=1$

↳ hiperboloide de duas folhas

b) • $x^2+2y^2+4y+z^2-2x=0$

$\Leftrightarrow x^2-2x+1-1+2(y^2+2y+1-1)+z^2=0$

$\Leftrightarrow (x-1)^2+2(y+1)^2-1-2+z^2=0$, $\tilde{x}=x-1$
$\tilde{y}=y+1$
$\tilde{z}=z$

$\Leftrightarrow \tilde{x}^2+2(\tilde{y})^2+\tilde{z}^2=3$

$\Leftrightarrow \frac{\tilde{x}^2}{3}+\frac{\tilde{y}^2}{3/2}+\frac{\tilde{z}^2}{3}=1$ → elipsoide

c) • $x^2 + y^2 + 4x - 6y - z = 0$

$\Leftrightarrow x^2 + 4x + 4 - 4 + y^2 - 6y + 9 - 9 - z = 0$

$\Leftrightarrow (x+2)^2 + (y-3)^2 - z = 13$, $\tilde{x} = x + 2$, $\tilde{y} = y - 3$, $\tilde{z} = z + 13$

$\Leftrightarrow \tilde{x}^2 + \tilde{y}^2 = \tilde{z}$

↳ parabolóide elíptico

d) • $x^2 + 4y^2 + 4xy - 2x - 4y + 2z + 1 = 0$

• $A = \begin{array}{c|c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ $B = [-2 \; -4 \; 2]$

• Cálculo dos valores próprios

• $\det(A - \lambda I) = 0$

$\Leftrightarrow \begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & 4-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0 \Leftrightarrow (1-\lambda).(4-\lambda).(-\lambda) + 8 + 8 - \{(4.(4-\lambda) + 4(1-\lambda) - 4\lambda) = 0$

$\Leftrightarrow \lambda = 0 \lor \lambda = 0 \lor \lambda = 5$

• Para $\lambda = 0$

$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Leftrightarrow x_1 = -2x_2$, $X_0 = (-2x_2; x_2, x_3)$

$X_0 = \{(-2;1;0);(0,0,1)\}$

$\|X_{0_1}\| = \sqrt{5}$, $\|X_{0_2}\| = \sqrt{1} = 1$

• Para $\lambda = 5$

$\begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix} \sim \begin{bmatrix} -2 & 1 & 0 \\ -2 & +1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow 2x_1 = x_2$

$X_5 = (x_1, 2x_1, 0)$

$X_5 = \{(1, 2; 0)\}$ $\|X_5\| = \sqrt{5}$

$$D=\begin{bmatrix}5&0&0\\0&0&0\\0&0&0\end{bmatrix} \qquad P=\begin{bmatrix}\sqrt{5}/5&0&-\frac{2\sqrt{5}}{5}\\ 2\sqrt{5}/5&0&\sqrt{5}/5\\0&1&0\end{bmatrix}$$

- $X = P\hat{X}$, $\hat{B} = BP$

- $\hat{B} = \underset{1\times3}{\begin{bmatrix}-2&-4&2\end{bmatrix}} \cdot \begin{bmatrix}\sqrt{5}/5&0&-\frac{2\sqrt{5}}{5}\\ 2\sqrt{5}/5&0&\sqrt{5}/5\\0&1&0\end{bmatrix}_{3\times3}$

$$=\begin{bmatrix}-\frac{2\sqrt{5}}{5}-\frac{8\sqrt{5}}{5}, & 2, & \frac{4\sqrt{5}}{5}-\frac{4\sqrt{5}}{5}\end{bmatrix}$$

$$=\begin{bmatrix}-2\sqrt{5} & 2 & 0\end{bmatrix}$$

- $\hat{X}^T D\hat{X} + \hat{B}\hat{X} + \mu = 0$

$$\Leftrightarrow \underset{1\times3}{\begin{bmatrix}x&y&z\end{bmatrix}} \cdot \underset{3\times3}{\begin{bmatrix}5&0&0\\0&0&0\\0&0&0\end{bmatrix}} \cdot \begin{bmatrix}x\\y\\z\end{bmatrix} + \begin{bmatrix}-2\sqrt{5}&2&0\end{bmatrix} \cdot \begin{bmatrix}x\\y\\z\end{bmatrix} + 1 = 0$$

$$\Leftrightarrow \begin{bmatrix}5x&0&0\end{bmatrix} \cdot \begin{bmatrix}x\\y\\z\end{bmatrix} + \begin{bmatrix}-2\sqrt{5}.x & 2y+0\end{bmatrix} + 1 = 0$$

$$\cdot\left(\frac{\sqrt{5}}{5}\right)^2 = \frac{5}{25} = \frac{1}{5}$$

$$\Leftrightarrow 5x^2 - 2\sqrt{5}x + 2y + 1 = 0$$

$$\Leftrightarrow 5\left(x^2 - \frac{2\sqrt{5}}{5}x + 1/5 - 1/5\right) + 2y = -1$$

$$\Leftrightarrow 5.\left(x - \sqrt{5}/5\right)^2 - 1 + 2y = -1$$

$$\Leftrightarrow 5.\left(x - \sqrt{5}/5\right)^2 + 2y = 0 \qquad , \tilde{x} = x - \sqrt{5}/5$$
$$, \tilde{y} = y$$

$$\Leftrightarrow 5.\tilde{x}^2 + 2\tilde{y} = 0$$

$$\Leftrightarrow \tilde{y} = -\frac{5\tilde{x}^2}{2} \rightarrow \text{cilindro parabolico}$$

g) $\cdot -x^2 + y^2 - 2x - 4y + 2 = 0$

$\Leftrightarrow -(x^2 + 2x + 1 - 1) + y^2 - 4y + 4 - 4 + 2 = 0$

$\Leftrightarrow -(x+1)^2 + 1 + (y-2)^2 - 2 = 0$

$\Leftrightarrow -(x+1)^2 + (y-2)^2 = 1$, $\tilde{x} = x+1$
, $\tilde{y} = y-2$

$\Leftrightarrow -(\tilde{x})^2 + (\tilde{y})^2 = 1$

$\Leftrightarrow \tilde{y}^2 - \tilde{x}^2 = 1$ // $\rightarrow$ cilindro hiperbolico

3) $\cdot 5x^2 + 5y^2 + 2xy + 2x - 2y + a = 0$

$A = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \end{bmatrix}$

$\cdot \det(A - \lambda I) = 0$

$\Leftrightarrow \begin{vmatrix} 5-\lambda & 1 \\ 1 & 5-\lambda \end{vmatrix} = 0 \Leftrightarrow (5-\lambda)^2 - 1 = 0$

$\Leftrightarrow 25 - 10\lambda + \lambda^2 - 1 = 0$

$\Leftrightarrow \lambda = 4 \vee \lambda = 6$

$\cdot$ Para $\lambda = 4$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \{ x_1 = -x_2$ $X_4 = (-1, 1)$

$||X_4|| = \sqrt{2}$

$\cdot$ Para $\lambda = 6$

$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \{ \underline{x_1 = x_2}$ $X_6 = (1, 1)$

$||X_6|| = \sqrt{2}$

$P = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$ $D = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$

$\cdot\ X = P\hat{X}$, $\cdot\ \hat{B} = BP$

$= \begin{bmatrix} 2 & -2 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 & -2\sqrt{2} \end{bmatrix}$

$\cdot\ \hat{X}^T . D . \hat{X} + \hat{B}\hat{X} + \mu = 0$

$\Leftrightarrow \begin{bmatrix} x & y \end{bmatrix} . \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & -2\sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \alpha = 0$

$\Leftrightarrow \begin{bmatrix} 6x & 4y \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} - 2\sqrt{2}y + \alpha = 0$

$\Leftrightarrow 6x^2 + 4y^2 - 2\sqrt{2}y + \alpha = 0$

$\Leftrightarrow 6x^2 + 4(y^2 - \sqrt{2}/2\, y + 1/8 - 1/8) + \alpha = 0$

$\Leftrightarrow 6x^2 + 4.(y - \sqrt{2}/4)^2 - 1/2 + \alpha = 0$, $\hat{y} = y - \sqrt{2}/4$

$\Leftrightarrow 6x^2 + 4.\hat{y}^2 - 1/2 + \alpha = 0$, $\hat{x} = x$

$\cdot\ \lambda_1 = 6 \qquad \lambda_2 = 4 \qquad \mu = -1/2 + \alpha$

Os valores próprios são positivos
Para ser elipse, μ e λ_1 tem de ter sinais contrários. Logo

$\cdot\ \alpha - 1/2 < 0$
$\Leftrightarrow \alpha < 1/2$ //

4)

a) • Para ser ortogonal, então.

$(\sqrt{2}/2; \sqrt{2}/2) . (-\sqrt{2}/2 ; \sqrt{2}/2) = 0$

$\Leftrightarrow -2/4 + 2/4 = 0 \Leftrightarrow 0 = 0$

• Valores próprios de A

• $\det(A - \lambda I) = 0$

$\Leftrightarrow \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 = 4$
$\Leftrightarrow \lambda = 2 \vee \lambda = -2$

$D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ //

b) • $4xy + x + y = 0$

$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ $\quad B = [1 \quad 1]$

• Seja $X = P\hat{X}$ e $\hat{B} = BP$

• $\hat{X}^T D . \hat{X} + \hat{B}\hat{X} + 0 = 0$ ⊕

• $\hat{B} = BP$

$= [1 \quad 1] . \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

$= [\sqrt{2} \quad 0]$

$\bullet\ [x\ \ y]\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + [\sqrt{2}\ \ 0]\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\Leftrightarrow [2x\ \ -2y]\begin{bmatrix} x \\ y \end{bmatrix} + \sqrt{2}x = 0$

$\Leftrightarrow 2x^2 + \sqrt{2}x - 2y^2 = 0$

$\Leftrightarrow 2\left(x^2 + \frac{\sqrt{2}}{2}x + \frac{1}{8} - \frac{1}{8}\right) - 2y^2 = 0$

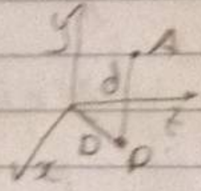
$\Leftrightarrow 2\left(x + \frac{\sqrt{2}}{4}\right)^2 - \frac{1}{4} - 2y^2 = 0$

$\Leftrightarrow \left(x + \frac{\sqrt{2}}{4}\right)^2 - y^2 = \frac{1}{8}$, $\tilde{x} = x + \frac{\sqrt{2}}{4}$, $\tilde{y} = y$

$\Leftrightarrow \tilde{x}^2 - \tilde{y}^2 = \frac{1}{8}$

$\Leftrightarrow \frac{\tilde{x}^2}{1/8} - \frac{\tilde{y}^2}{1/8} = 1 \rightarrow$ hiperbole

5) $A\ (0,1,1)$, $P\ (x,y,z)$



$\bullet\ d = 1 + D$ (*)

$\bullet\ \vec{AP} = P - A$
$= (x,y,z) - (0,1,1)$
$= (x, y-1, z-1)$ $\quad d = \|\vec{AP}\|$

$\bullet\ \vec{OP} = P - O$
$= (x,y,z) - (0,0,0)$ $\quad D = \|\vec{OP}\|$
$= (x,y,z)$

(*) $\|\vec{AP}\| = 1 + \|\vec{OP}\|$.

$\bullet\ \|\vec{AP}\| = \sqrt{x^2 + (y-1)^2 + (z-1)^2}$

$\bullet\ \|\vec{OP}\| = \sqrt{x^2 + y^2 + z^2}$

• $\sqrt{x^2+(y-1)^2+(z-1)^2} = 1 + \sqrt{x^2+y^2+z^2}$

$\Rightarrow \left(\sqrt{x^2+y^2-2y+1+z^2-2z+1}\right)^2 = \left(1+\sqrt{x^2+y^2+z^2}\right)^2$

$\Rightarrow x^2+y^2+z^2-2y-2z+2 = 1+2\sqrt{x^2+y^2+z^2}+x^2+y^2+z^2$

$\Rightarrow -2y-2z+1 = 2\sqrt{x^2+y^2+z^2}$

$\Rightarrow ((-2y-1)-2z)^2 = (2.\sqrt{x^2+y^2+z^2})^2$

$\Rightarrow (-2y-1)^2-4.z(-2y-1)+(-2z)^2 = 4.(x^2+y^2+z^2)$

$\Rightarrow \cancel{4y^2}+4y+1+8zy-4z+\cancel{4z^2} = 4x^2+\cancel{4y^2}+\cancel{4z^2}$

$\Rightarrow +4y+1+8zy-4z-4x^2=0$

$\Rightarrow -4x^2+8zy-4y+4z+1=0$

Há aqui uns sinais trocados, mas o processo é este//

$\Rightarrow 4x^2-8yz+4y+4z-1=0$

↳ hiperbolóide de duas folhas

• Agora tínhamos de fazer o mesmo que os exercícios 1 e 2.

- Determinar matriz A e B
- Valores próprios de A
- Vetores próprios de A
- Ortogonalizar e det matriz P
- Fazer B.P
- Mudar variavel $X = P\hat{X}$, $\hat{B} = BP$
- Resolver a equação

6) $\cdot\ P_{\alpha\alpha}(x, y, z)$ $\qquad \cdot\ d_p(P, A) = \frac{d(\text{Plano}, P)}{3}$

$\cdot\ A_\alpha(0, 0, -2)$

$\cdot\ \alpha: z + 18 = 0 \quad , \quad a = 0,\ b = 0,\ c = 1$

$\cdot\ d_p = \frac{|0 \times 0 + 0 \times 0 + 1 \times z + 18|}{1} = z + 18$ //

$$\begin{array}{r} 6 \\ 18 \\ \times 18 \\ \hline 144 \\ 180 \\ \hline 324 \end{array}$$

$\cdot\ \vec{AP} = P - A$
$= (x, y, z) - (0, 0, -2)$
$= (x; y; z + 2)$

$\cdot\ \|\vec{AP}\| = \sqrt{x^2 + y^2 + (z+2)^2}$

$\cdot\ \|\vec{AP}\| = \frac{d_p}{3} \Leftrightarrow \sqrt{x^2 + y^2 + (z+2)^2} = \frac{z + 18}{3}$

$\Leftrightarrow \left(3 \cdot \left(\sqrt{x^2 + y^2 + (z+2)^2}\right)\right)^2 = (z + 18)^2$

$\Leftrightarrow 9 \cdot (x^2 + y^2 + z^2 + 4z + 4) = z^2 + 36z + 324$

$\Leftrightarrow 9x^2 + 9y^2 + 9z^2 + \cancel{36z} + 36 = z^2 + \cancel{36z} + 324$

$\Leftrightarrow 9x^2 + 9y^2 + 8z^2 - 288 = 0$

$\Leftrightarrow 9x^2 + 9y^2 + 8z^2 = 288$

$\Leftrightarrow \frac{9x^2}{288} + \frac{9y^2}{288} + \frac{8z^2}{288} = 1$

$\Leftrightarrow \frac{x^2}{32} + \frac{y^2}{32} + \frac{z^2}{36} = 1$

$\hookrightarrow$ elipsoide //