



Soluções da Ficha de Exercícios 2

1. (a) $x^3 + \frac{5}{2}x^2 + 7x + c, \quad c \in \mathbb{R}$
(b) $\frac{3}{4}\sqrt[3]{x^4} + c, \quad c \in \mathbb{R}$
(c) $\frac{x^7}{7} + \frac{x^4}{2} + x + c, \quad c \in \mathbb{R}$
(d) $\frac{(\operatorname{arctg} x)^2}{2} + c, \quad c \in \mathbb{R}$
(e) $\ln |1 + x^3| + c, \quad c \in \mathbb{R}$
(f) $-\frac{1}{6x^6} + c, \quad c \in \mathbb{R}$
(g) $\frac{1}{8} \ln(2 + 4x^2) + \frac{\sqrt{2}}{4} \operatorname{arctg}(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
(h) $\operatorname{sen} x^4 + c, \quad c \in \mathbb{R}$
(i) $-\sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
(j) $-\frac{\cos^6 x}{6} + c, \quad c \in \mathbb{R}$
(k) $-\ln |\cos x| + c, \quad c \in \mathbb{R}$
(l) $\frac{(\ln x)^2}{2} + c, \quad c \in \mathbb{R}$
(m) $e^{\operatorname{tg} x} + c, \quad c \in \mathbb{R}$
(n) $\frac{1}{2 \ln 7} 7^{x^2} + c, \quad c \in \mathbb{R}$
(o) $-\frac{\sqrt{2}}{2} \cos(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
(p) $\frac{x^2}{2} + \ln |x| + c, \quad c \in \mathbb{R}$
(q) $-\frac{1}{5\sqrt{7+5x^2}} + c, \quad c \in \mathbb{R}$
(r) $\frac{1}{4} \operatorname{arctg}(x^4) + c, \quad c \in \mathbb{R}$
(s) $\frac{5}{3} \operatorname{arcsen}(x^3) + c, \quad c \in \mathbb{R}$
(t) $\frac{\sqrt{7}}{7} \operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right) + c, \quad c \in \mathbb{R}$
(u) $\frac{1}{2} \operatorname{arctg}\left(\frac{x+1}{2}\right) + c, \quad c \in \mathbb{R}$
(v) $\frac{1}{2} \operatorname{arctg}(x^2) + c, \quad c \in \mathbb{R}$
(w) $\frac{1}{2} \operatorname{arcsen}(x^2) + c, \quad c \in \mathbb{R}$
(x) $-\frac{1}{2} \sqrt{1 - x^4} + c, \quad c \in \mathbb{R}$
2. (a) $e^{\operatorname{arcsen} x} + c, \quad c \in \mathbb{R}$
(b) $\operatorname{tg} x - x + c, \quad c \in \mathbb{R}$
(c) $\operatorname{sen}(\ln x) + c, \quad c \in \mathbb{R}$
(d) $-\frac{3}{\ln^2(4x)} + c, \quad c \in \mathbb{R}$
(e) $-\frac{1}{15(e^{3x}-2)^5} + c, \quad c \in \mathbb{R}$
(f) $\frac{\operatorname{tg}^2 x}{2} + \ln |\cos x| + c, \quad c \in \mathbb{R}$
(g) $\operatorname{arcsen}(\ln x) + c, \quad c \in \mathbb{R}$
(h) $\frac{2}{3} \sqrt{(1 + e^x)^3} + c, \quad c \in \mathbb{R}$
(i) $\ln |\ln x| + c, \quad c \in \mathbb{R}$

- (j) $2e^{\sqrt{x}} + c, \quad c \in \mathbb{R}$
- (k) $\ln |x + \operatorname{sen} x| + c, \quad c \in \mathbb{R}$
- (l) $\frac{e}{2} \ln(e^{2x} + 3) + c, \quad c \in \mathbb{R}$
- (m) $-\frac{\cos(x^6)}{6} + c, \quad c \in \mathbb{R}$
- (n) $-\frac{1}{2}(\arccos x)^2 + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
- (o) $\frac{1}{2}\operatorname{sen}(\ln(x^2)) + c, \quad c \in \mathbb{R}$

3. Resolvido

4. $F(x) = 2 \ln |x| - \frac{3}{x} - 2$

5. $\frac{\pi}{8}(\sqrt{2} - 2)$

6. $F(x) = -\frac{1}{x} + x - \frac{3}{2}$

7. (a) Resolvido

- (b) $x \operatorname{sen} x + \cos x + c, \quad c \in \mathbb{R}$
- (c) $x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + c, \quad c \in \mathbb{R}$
- (d) $-\frac{2x+3}{3}e^{-3x} - \frac{2}{9}e^{-3x} + c, \quad c \in \mathbb{R}$
- (e) $x(\ln^2 x - 2 \ln x + 2) + c, \quad c \in \mathbb{R}$
- (f) $x \ln x - x + c, \quad c \in \mathbb{R}$
- (g) $x \ln(x^2 + 1) - 2(x - \operatorname{arctg} x) + c, \quad c \in \mathbb{R}$
- (h) $\frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2}(x - \operatorname{arctg} x) + c, \quad c \in \mathbb{R}$
- (i) $\frac{x}{2} \cos(\ln x) + \frac{x}{2} \operatorname{sen}(\ln x) + c, \quad c \in \mathbb{R}$
- (j) $\frac{-e^{2x} \cos x + 2e^{2x} \operatorname{sen} x}{5} + c, \quad c \in \mathbb{R}$
- (k) $\frac{x \operatorname{sen}(\ln x) - x \cos(\ln x)}{2} + c, \quad c \in \mathbb{R}$
- (l) $x \operatorname{arcsen} x + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
- (m) $\frac{x^2}{2} \operatorname{arcsen}(x^2) + \frac{1}{2}\sqrt{1-x^4} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{2}e^{x^2}(x^2 - 1) + c, \quad c \in \mathbb{R}$
- (o) $x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (p) $x \operatorname{arctg} \frac{1}{x} + \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (q) $\frac{2}{3}\sqrt{x^3} \ln x - \frac{4}{9}\sqrt{x^3} + c, \quad c \in \mathbb{R}$
- (r) $\frac{\cos x \cos(3x) + 3 \operatorname{sen} x \operatorname{sen}(3x)}{8} + c, \quad c \in \mathbb{R}$
- (s) $\frac{\cos x \operatorname{sen} x + x}{2} + c, \quad c \in \mathbb{R}$
- (t) $\frac{\sec x \operatorname{tg} x + \ln |\sec x + \operatorname{tg} x|}{2} + c, \quad c \in \mathbb{R}$
- (u) $\frac{x}{\sqrt{1-x^2}} - \operatorname{arcsen} x + c, \quad c \in \mathbb{R}$

8. (a) Resolvido

- (b) $\frac{2}{3}\sqrt{x^3} - x + 2\sqrt{x} - 2 \ln |\sqrt{x} + 1| + c, \quad c \in \mathbb{R}$
- (c) $-\frac{\sqrt{1-x^2}}{x} + c, \quad c \in \mathbb{R}$
- (d) $-\frac{\sqrt{4+x^2}}{4x} + c, \quad c \in \mathbb{R}$
- (e) $\frac{1}{\sqrt{5}} \arccos\left(\frac{\sqrt{5}}{x}\right) + c, \quad c \in \mathbb{R}$
- (f) $-\frac{2}{3}(1-x)\sqrt{1-x} - \frac{2}{7}(1-x)^3\sqrt{1-x} + \frac{4}{5}(1-x)^2\sqrt{1-x} + c, \quad c \in \mathbb{R}$
- (g) $2 \operatorname{arcsen} \frac{x}{2} - \frac{x(2-x^2)\sqrt{4-x^2}}{4} + c, \quad c \in \mathbb{R}$

- (h) $\arccos \frac{1}{x} + c, \quad c \in \mathbb{R}$
- (i) $-\frac{1}{2} \ln \left| \frac{\sqrt{x^2+4}}{x} + \frac{2}{x} \right| + c, \quad c \in \mathbb{R}$
- (j) $-\frac{\sqrt{9-x^2}}{9x} + c, \quad c \in \mathbb{R}$
- (k) $2 \operatorname{arcsen} \frac{x+1}{\sqrt{2}} - \frac{(x+1)\sqrt{2-(x+1)^2}}{2} + 2\sqrt{2-(x+1)^2} + c, \quad c \in \mathbb{R}$
- (l) $\frac{\sqrt{x^2-7}}{7x} + c, \quad c \in \mathbb{R}$
- (m) $\frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \operatorname{arctg} \sqrt[6]{x} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{48}(2x+5)^{12} - \frac{5}{44}(2x+5)^{11} + c, \quad c \in \mathbb{R}$
- (o) $-\frac{3}{8} \left(\frac{1-x}{1+x} \right)^{\frac{4}{3}} + c, \quad c \in \mathbb{R}$
- (p) $2e^{\sqrt{x}}(\sqrt{x}-1) + c, \quad c \in \mathbb{R}$
- (q) $\frac{2}{3}(\sqrt{1+\ln x})^3 - 2\sqrt{1+\ln x} + c, \quad c \in \mathbb{R}$
- (r) $2\sqrt{\operatorname{tg} x - 1} + c, \quad c \in \mathbb{R}$
9. (a) Resolvido
- (b) $-\ln |x-2| + \frac{5}{4} \ln |x-3| - \frac{1}{4} \ln |x+1| + c, \quad c \in \mathbb{R}$
- (c) $\frac{1}{8} \ln |x-1| - \frac{1}{8} \ln |x+1| + \frac{1}{4(x+1)} + \frac{1}{4(x+1)^2} + c, \quad c \in \mathbb{R}$
- (d) $\frac{1}{12} \ln |x+2| - \frac{1}{24} \ln(x^2-2x+4) + \frac{\sqrt{3}}{12} \operatorname{arctg} \left(\frac{x-1}{\sqrt{3}} \right) + c, \quad c \in \mathbb{R}$
- (e) $\frac{x^7}{7} - \frac{x^5}{5} + \frac{x^3}{3} - x + \operatorname{arctg} x + c, \quad c \in \mathbb{R}$
- (f) $-\ln |x| - \frac{1}{2x^2} + \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (g) $-\frac{2}{x} - \operatorname{arctg} \left(\frac{x}{2} \right) + c, \quad c \in \mathbb{R}$
- (h) $\frac{x^3}{3} + \frac{x^2}{2} + 4x + 2 \ln |x| + 5 \ln |x-2| - 3 \ln |x+2| + c, \quad c \in \mathbb{R}$
- (i) $\ln |x-1| - \frac{2}{x-1} - \frac{1}{2(x-1)^2} + c, \quad c \in \mathbb{R}$
- (j) $-\frac{3}{4} \ln |x| - \frac{1}{4x} + \frac{13}{16} \ln |x-2| + \frac{15}{16} \ln |x+2| + c, \quad c \in \mathbb{R}$
- (k) $\frac{1}{3}(2 \ln |x-1| - \ln(x^2+x+1)) + c, \quad c \in \mathbb{R}$
- (l) $\frac{1}{4}(4x + \ln |x-1| - \ln |x+1| - 2 \operatorname{arctg} x) + c, \quad c \in \mathbb{R}$
- (m) $\ln |x| - \frac{1}{2} \ln(1+x^2) + \frac{1}{2(x^2+1)} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{2} \ln(x^2+4x+5) - \operatorname{arctg}(x+2) + c, \quad c \in \mathbb{R}$
10. (a) $\frac{1}{2}\theta - \frac{1}{4}\operatorname{sen}(2\theta) + c, \quad c \in \mathbb{R}$
- (b) $\frac{3}{8}x - \frac{1}{4}\operatorname{sen}(2x) + \frac{1}{32}\operatorname{sen}(4x) + c, \quad c \in \mathbb{R}$
- (c) $-\frac{\cos^3 x}{3} + c, \quad c \in \mathbb{R}$
- (d) $-\cos x + \frac{1}{3} \cos^3 x + c, \quad c \in \mathbb{R}$
- (e) $-\frac{\cos^3 x}{3} + \frac{2}{5} \cos^5 x - \frac{\cos^7 x}{7} + c, \quad c \in \mathbb{R}$
- (f) $\operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + c, \quad c \in \mathbb{R}$
- (g) $\ln \left| \sqrt{\frac{2+x^2}{2}} + \frac{x}{\sqrt{2}} \right| + c, \quad c \in \mathbb{R}$
- (h) $-2 \cos \sqrt{x} + c, \quad c \in \mathbb{R}$
- (i) $3 \ln |x-3| - 2 \ln |x-2| + c, \quad c \in \mathbb{R}$
- (j) $\operatorname{arcsen}(x-1) + c, \quad c \in \mathbb{R}$
- (k) $\frac{(1+x^2)^2 \sqrt{1+x^2}}{5} + c, \quad c \in \mathbb{R}$
- (l) $x - 2\sqrt{x} + 2 \ln(1+\sqrt{x}) + c, \quad c \in \mathbb{R}$

- (m) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{4}x - \frac{1}{8} \ln(e^{2x} + 4) + \frac{1}{2} \operatorname{arctg} \frac{e^x}{2} + c, \quad c \in \mathbb{R}$
- (o) $x \operatorname{tg} x + \ln |\cos x| + c, \quad c \in \mathbb{R}$
- (p) $-\frac{1}{2(1-\cos x)^2} + c, \quad c \in \mathbb{R}$
- (q) $(\frac{2}{3}x^3 + 3x) \operatorname{arctg} x - \frac{1}{3}x^2 - \frac{7}{6} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (r) $\ln \left| \frac{x+1+\sqrt{(x+1)^2-4}}{2} \right| + c, \quad c \in \mathbb{R}$
- (s) $2\sqrt{1+e^x} + \ln |\sqrt{1+e^x} - 1| - \ln(\sqrt{1+e^x} + 1) + c, \quad c \in \mathbb{R}$
- (t) $2 \operatorname{arctg} \sqrt{e^x - 1} + c, \quad c \in \mathbb{R}$
- (u) $\frac{1}{12} \operatorname{sen}(6x) + \frac{1}{8} \operatorname{sen}(4x) + c, \quad c \in \mathbb{R}$
- (v) $-2\sqrt{\cos x} + \frac{2}{5} \sqrt{\cos^5 x} + c, \quad c \in \mathbb{R}$
- (w) $-\frac{1}{3} \cos^3 x + \frac{2}{5} \cos^5 x - \frac{1}{7} \cos^7 x + c, \quad c \in \mathbb{R}$
- (x) $\frac{1}{2} \ln(\ln^2 x + 1) + c, \quad c \in \mathbb{R}$

11. $F(x) = -\ln |\cos x| + 3$

12. $f(x) = 2 \ln(e^x + 3) - \ln 4$

13. $f(x) = 2x^3 + 2x + 1$

14. (a) $-\frac{1}{2}e^{\cos(2x)} + c, \quad c \in \mathbb{R}$

(b) $4 \left(\frac{\sqrt{x}}{2} + \sqrt[4]{x} + \ln(\sqrt[4]{x} - 1) \right) + c, \quad c \in \mathbb{R}$

(c) $\frac{\sqrt{x^2-9}}{9x} + c, \quad c \in \mathbb{R}$

(d) $\frac{x^3}{3} \operatorname{arctg} x - \frac{x^2}{6} - \frac{1}{6} \ln(1+x^2) + c, \quad c \in \mathbb{R}$

(e) $\frac{1}{2} \ln |x| - \frac{1}{4} \ln(4+x^2) + \frac{1}{2} \operatorname{arctg} \left(\frac{x}{2} \right) + c, \quad c \in \mathbb{R}$

(f) $\frac{2}{3} \sqrt{1+x^3} + c, \quad c \in \mathbb{R}$

(g) $-\frac{\sqrt{1+x^2}}{x} + c, \quad c \in \mathbb{R}$

(h) $\frac{1}{2}(\ln(x^2+1) - 2 \ln |x| + 6 \operatorname{arctg} x) + c, \quad c \in \mathbb{R}$

15. $f(x) = \ln \left(\frac{x^2-2x+2}{x^2} \right) + 3 \operatorname{arctg}(x-1) - \frac{3\pi}{2}$