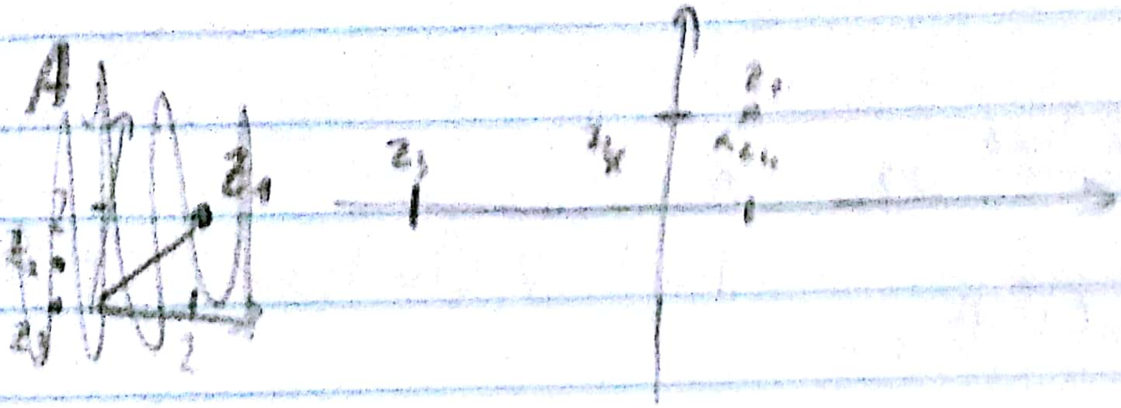
Ficha 1

1.

a)



$z_4 = \sqrt{3}\, e^{j\frac{\pi}{3}}$

$= \sqrt{3}\left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)$

$= \sqrt{3}\left(\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2} + \frac{3}{2}j$

b) z_3 / z_4

c) i) $z_1 + z_3 = 2+j2-4 = -2+2j$

ii) $z_2 = -1+j$

$r = \sqrt{(-1)^2+1^2} = \sqrt{2}$

$\varphi = \operatorname{arctg}\left(\frac{1}{-1}\right) = \frac{3}{4}\pi$

$z_2 = \sqrt{2}\, e^{j\frac{3}{4}\pi}$

$\frac{z_4}{z_2} = \frac{\sqrt{3}\, e^{j\frac{\pi}{3}}}{\sqrt{2}\, e^{j\frac{3\pi}{4}}} = \sqrt{\frac{3}{2}}\, e^{j\left(-\frac{5\pi}{12}\right)}$

iii) $(z_2)^4 = \left(\sqrt{2}\, e^{j\frac{3\pi}{4}}\right)^4 = 4\, e^{j3\pi}$

iv) $\sqrt{z_3} = \sqrt{(-4)} = 2j$

$z_1 + z_2 - \sqrt{z_3} = 2+j2-1+j-2j = 1+j$

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2

a) $e^{j(100\pi t+\frac{\pi}{3})}+e^{j(100\pi t+\frac{\pi}{4})}$

$= e^{j100\pi}\left(e^{j\frac{\pi}{3}}+e^{j\frac{\pi}{4}}\right)$

$= e^{j100\pi}\left(\cos\left(\frac{\pi}{3}\right)+j\sin\left(\frac{\pi}{3}\right)+\cos\left(\frac{\pi}{4}\right)+j\sin\left(\frac{\pi}{4}\right)\right)$

$= e^{j100\pi t}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j\right)$

$= e^{j100\pi t}\left(\frac{1+\sqrt{2}}{2}+\frac{\sqrt{3}+\sqrt{2}}{2}j\right)$.

$\varphi = \mathrm{arctg}\left(\frac{\frac{\sqrt{3}+\sqrt{2}}{2}}{\frac{1+\sqrt{2}}{2}}\right) = \mathrm{arctg}\left(\frac{(\sqrt{3}+\sqrt{2})(1-\sqrt{2})}{1-2}\right) \approx \frac{7}{24}\pi$

$= \mathrm{arctg}(\sqrt{3}-\sqrt{6}-\sqrt{2}+2$

$A=\sqrt{\left(\frac{1+\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{3}+\sqrt{2}}{2}\right)^2}\approx 1{,}9$

b) $\cos\left(100\pi t+\frac{\pi}{3}\right)=Re\{e^{j(100\pi t+\frac{\pi}{3})}\}$

$\cos\left(100\pi t+\frac{\pi}{4}\right)=Re\{e^{j(100\pi t+\frac{\pi}{4})}\}$

$\cos\left(100\pi t+\frac{\pi}{3}\right)+\cos\left(100\pi t+\frac{\pi}{4}\right)$

$=Re\{e^{j(100\pi t+\frac{\pi}{3})}\}+Re\{e^{j(100\pi t+\frac{\pi}{4})}\}$

$=Re\{e^{j(100\pi t+\frac{\pi}{3})}+e^{j(100\pi t+\frac{\pi}{4})}\}$

vor a)

c)

$\cos\left(\omega t+\frac{\pi}{6}\right)+\cos\left(\omega t+\frac{\pi}{2}\right)+\cos(\omega t-\pi)$

$=Re\{e^{j(\omega t+\frac{\pi}{6})}+e^{j(\omega t+\frac{\pi}{2})}+e^{j(\omega t-\pi)}\}$

$=Re\{e^{j\omega t}\left(e^{j\frac{\pi}{6}}+e^{j\frac{\pi}{2}}+e^{-j\pi}\right)\}$

$=Re\{e^{j\omega t}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}j\right)+j-1\}$

$=Re\{e^{j\omega t}\left(\frac{2-\sqrt{3}}{2}+\frac{3}{2}j\right)\}$

$A=\sqrt{\left(\frac{2-\sqrt{3}}{2}\right)^2+\left(\frac{3}{2}\right)^2}$

$\varphi=\mathrm{arctg}\left(\frac{\frac{3}{2}}{\frac{2-\sqrt{3}}{2}}\right)$

3

a) $A=2$

$f=\frac{0{,}3\pi}{2\pi}=0{,}15$ $\quad T=\frac{1}{0{,}15}=\frac{100}{15}=\frac{20}{3}$

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b) $\cos(0{,}12\pi t) = Re\{e^{j0{,}12\pi t}\}$

$\cos(0{,}12\pi t + \frac{\pi}{3}) = Re\{e^{j(0{,}12\pi t + \frac{\pi}{3})}\}$

$Re\{e^{j0{,}12\pi t} + e^{j(0{,}12\pi t + \pi/3)}\}$

$= Re\{e^{0{,}12\pi t}(1 + e^{\pi/3 j})\}$

$= Re\{e^{0{,}12\pi t}(1 + \cos(\pi/3) + j\sin(\frac{\pi}{3}))\}$

$A = \sqrt{(1+\cos(\frac{\pi}{3}))^2 + \sin(\frac{\pi}{3})^2}$

$T = \frac{1}{0{,}06}$

d) $A = 1$ $T = \frac{1}{f_0}$

4

a) É uma soma de sinusoides, todas com freq f_0, logo terá o período $\frac{1}{f_0}$

b) $\frac{1}{f_0}$

c) Ver demonstração nos slides

d) É simétrico em relação ao eixo dos números reais
O modulo é $\frac{A_k}{2}$ e a fase é θ_k

e)

$T = \frac{1}{100}$

~~$x(t) = \sum_{m=1}^{2} \frac{1}{2^m} \cos(2\pi m \cdot 100t + (-\frac{1}{2})^m \cdot \pi) + 2$~~

$\frac{A_1}{2} = 0.5 \Leftrightarrow A_1 = 1$ $\frac{A_2}{2} = 0{,}25 \Leftrightarrow A_2 = 0.5$

$\frac{A_0}{2} = 1 \Leftrightarrow A_0 = 2$

$x(t) = \sum_{m=1}^{2} \frac{1}{2^{m-1}} \cos(200\pi m t + (-\frac{1}{2})^m \cdot \pi) + 2$

$x(t) = \sum_{m=-2}^{2} \frac{1}{2^{|m|}} \exp(\frac{|m|}{m} \cdot (-\frac{1}{2})^m \cdot \pi) \cdot \exp(j 2\pi \cdot 100 \cdot |m| \cdot t)$

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