ficha Alga - 3 - Vetores, retas e planos

1)

a) • $X+Y$
$= (1,-2,1) + (-1,1,0)$
$= (0,-1,1)$

• $3X - 2Y$
$= 3(1,-2,1) - 2(-1,1,0)$
$= (3,-6,3) + (2,-2,0)$
$= (5,-8,3)$

b) Para serem perpendiculares:
• $X \cdot Y = 0$
$\Leftrightarrow (1,-2,1) \cdot (-1,1,0) = 0$
$\Leftrightarrow -1 - 2 + 0 = 0 \Leftrightarrow -3 \neq 0 \Rightarrow$ Não são perpendiculares

Para serem colineares, então:

• $X = KY$
$\Leftrightarrow (1,-2,1) = K(-1,1,0)$

$\Leftrightarrow \begin{cases} 1 = -K \\ -2 = K \\ 1 \neq 0 \end{cases} \rightarrow$ Não são colineares

c)
i) $X = (1,-2,1)$ $Y = (-1,1,0)$

$\|X\| = \sqrt{1^2 + (-2)^2 + 1}$
$= \sqrt{2+4}$
$= \sqrt{6}$

$\|Y\| = \sqrt{1+1+0}$
$= \sqrt{2}$

$\cos(X \wedge Y) = \dfrac{X \cdot Y}{\|X\| \cdot \|Y\|} = \dfrac{(1,-2,1) \cdot (-1,1,0)}{\sqrt{2} \cdot \sqrt{6}}$

$= \dfrac{-1-2}{\sqrt{12}} = \dfrac{-3}{\sqrt{12}} = -\dfrac{\sqrt{3}}{2} \Rightarrow (X \wedge Y) = \dfrac{5\pi}{6}$

d) $X = (1, -2, 1)$ $\qquad \|X\| = \sqrt{6}$

Vetor unitario $\dfrac{X}{\|X\|} = \left(\dfrac{1}{\sqrt{6}} (1; -2; 1)\right)$ //

g) Os vetores perpendiculares a X e Y são do tipo $v = (a, b, c)$

$\begin{cases} X \cdot v = 0 \\ Y \cdot v = 0 \end{cases} \Leftrightarrow \begin{cases} (1; -2; 1) \cdot (a, b, c) = 0 \\ (-1, 1, 0) \cdot (a, b, c) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} a - 2b + c = 0 \\ -a + b = 0 \end{cases} \Leftrightarrow \begin{cases} b - 2b + c = 0 \\ a = b \end{cases} \Leftrightarrow \begin{cases} c = b \\ a = b \end{cases}$

$v = (b; b; b) = b(1, 1, 1), \ b \in \mathbb{R}$

h) Para v ser perpendicular a X, temos que

$X \cdot v = 0$

$\Leftrightarrow (1, -2, 1) \cdot (a, b, c) = 0$

$\Leftrightarrow a - 2b + c = 0$

$\Leftrightarrow a = 2b - c$

fiz em função de b e c, podiam fazer em função de ambos..

$v = (2b - c, b, c)$

$= b(2; 1; 0) + c(-1; 0, 1) \ ; b, c \in \mathbb{R}$

3) $X = (1, 0, 0)$ $\qquad Y = (a, b, c)$

- $(X \wedge Y) = \pi/3 \Rightarrow (\cos(X \wedge Y)) = 1/2$ (*)

$\|X\| = 1 \qquad \|Y\| = \sqrt{a^2 + b^2 + c^2}$

(*) $1/2 = \dfrac{X \cdot Y}{\|X\| \cdot \|Y\|} \Leftrightarrow 1/2 = \dfrac{a}{\sqrt{a^2 + b^2 + c^2}}$

$\Leftrightarrow a = \dfrac{\sqrt{a^2 + b^2 + c^2}}{2} \Leftrightarrow a^2 = \dfrac{a^2 + b^2 + c^2}{4}$

$$4a^2 = a^2 + b^2 + c^2$$

$$\Leftrightarrow 3a^2 = b^2 + c^2 \Leftrightarrow a = \sqrt{\frac{b^2+c^2}{3}} \Leftrightarrow a = \frac{1}{3}\sqrt{3b^2+3c^2}$$

Logo,

$$v = (1/3 \sqrt{3b^2+3c^2} ; b ; c) \text{ , } b, c \in \mathbb{R}$$

5)

a) $X = (2, -1, 1)$ $\quad Y = (0, 2, -1)$

• $X \times Y = \begin{vmatrix} i & 2 & 0 \\ j & -1 & 2 \\ k & 1 & -1 \end{vmatrix} = w(a_1, a_2, a_3)$

$a_1 = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1 - 2 = -1$

$a_2 = -\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = -(-2) = 2$

$a_3 = \begin{vmatrix} 2 & 0 \\ -1 & 2 \end{vmatrix} = 4 - 0 = 4$

logo, $X \times Y = (-1; 2; 4)$

b) Para se verificar a condição, então

$$\begin{cases} (X \times Y) . X = 0 \\ (X \times Y) . Y = 0 \end{cases} \Leftrightarrow \begin{cases} (-1, 2, 4) . (2, -1; 1) = 0 \\ (-1, 2, 4) . (0, 2, -1) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -2 - 2 + 4 = 0 \\ 0 + 4 - 4 = 0 \end{cases} \Leftrightarrow \begin{cases} 0 = 0 \\ 0 = 0 \end{cases}$$

$X \times Y$ é ortogonal a X e Y

8)

a) $X = (1, 2, 0)$
$Y = (1, -1, 1)$

Os vetores ortogonais a X e Y são: $v = (a, b, c)$

$$\begin{cases} X \cdot v = 0 \\ Y \cdot v = 0 \end{cases} \Leftrightarrow \begin{cases} (1,2,0) \cdot (a,b,c) = 0 \\ (1;-1;1) \cdot (a,b,c) = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} a + 2b = 0 \\ a - b + c = 0 \end{cases} \Leftrightarrow \begin{cases} a = -2b \\ -2b - b + c = 0 \end{cases} \Leftrightarrow \begin{cases} a = -2b \\ c = 3b \end{cases}$$

$$v = (-2b, b, 3b)$$
$$= b(-2, 1, 3), \ b \in \mathbb{R}$$

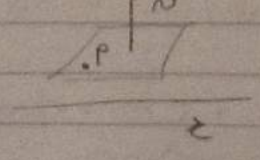
Retas e planos

11)

a) $P_0 (x_0, y_0, z_0)$
$v = (v_x, v_y, v_z) \neq 0$

$$r: \frac{x - x_0}{v_x} = \frac{y - y_0}{v_y} = \frac{z - z_0}{v_z}, \quad \text{com } v_x, v_y, v_z \neq 0$$

b) $u_1 = (0, -v_z, v_y)$
$u_2 = (v_z; 0; -v_x)$
$u_3 = (-v_y; v_x; 0)$



Para a reta estar contida no plano, P tem de pertencer ao plano e à reta, e o vetor diretor da reta tem de ser perpendicular ao vetor normal ao plano.

$$\begin{cases} u_1 \cdot v = 0 \\ u_2 \cdot v = 0 \\ u_3 \cdot v = 0 \end{cases} \Leftrightarrow \begin{cases} (0; -v_z; v_y) \cdot (v_x, v_y, v_z) = 0 \\ (v_z, 0; -v_x) \cdot (v_x, v_y, v_z) = 0 \\ (-v_y; v_x; 0) \cdot (v_x, v_y, v_z) = 0 \end{cases} \Leftrightarrow \begin{cases} 0 = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Provado!

12) $R: \begin{cases} x+y-z=2 \\ x-y+z=0 \end{cases}$

Equação vetorial de R —— // ——

- Determinar um ponto de R
  • Se $x=y=1$, então $P_1=(1,1,0)$

- Vetor diretor da reta:
  • $\begin{cases} x = 2-y+z \\ 2-y+z-y+z=0 \end{cases} \Leftrightarrow \begin{cases} x=2-y+z \\ 2-2y+2z=0 \end{cases} \Leftrightarrow \begin{cases} - \\ 1-y+z=0 \end{cases}$

$R: z=y-1 \Leftrightarrow \frac{z-0}{1} = \frac{y-1}{1} \wedge x=a$

Logo, $\vec{r}=(0,1,1)$//

$r: (x,y,z)=(1,1,0)+k(0,1,1),\ k \in \mathbb{R}$

Equações gerais do Plano —— // ——

$P_0(2,2,1)$, Contem R → S contem r, então um vetor perpendicular a R é normal ao plano ($\vec{n}$)

$\vec{n}=(0,1,-1)$, α é o plano

$\alpha: ax+by+cz+d=0$
$\Leftrightarrow 0x+y-z+d=0$, $P_0(2,2,1)$
$\Leftrightarrow 2-1+d=0$
$\Leftrightarrow d=-1$

$\therefore \alpha = y-z=1$//

13)

a) Plano $\in$ $\begin{cases} A=(1,1,1) \\ B=(0,1,0) \\ C=(0,0,1) \end{cases}$

$P_{a,b}: ax+y+z=b$

- Vetor normal ao plano: $\vec{n}=?$

$\vec{AB}=B-A$
$=(0,1,0)-(1,1,1)$
$=(-1,0,-1)$

$\vec{BC}=C-A$
$=(0,0,1)-(1,1,1)$
$=(-1,-1,0)$

$\begin{cases} \vec{AB}\cdot\vec{n}=0 \\ \vec{BC}\cdot\vec{n}=0 \end{cases} \Leftrightarrow \begin{cases} (-1,0,-1)\cdot(a,b,c)=0 \\ (-1,-1,0)\cdot(a,b,c)=0 \end{cases}$

$\Leftrightarrow \begin{cases} -a-c=0 \\ -a-b=0 \end{cases} \Leftrightarrow \begin{cases} b=c \\ -a=b \end{cases} \Leftrightarrow \begin{cases} c=b \\ a=-b \end{cases}$ $\vec{n}=(-b;b;b)$

Se $b=-1 \Rightarrow \vec{n}=(+1,-1,-1)$

- $P: x-y-z=b$, $C(0,0,1)$
$\Leftrightarrow 0-0-1=b$
$\Leftrightarrow b=-1 \Rightarrow P: x-y-z+1=0$ //

b) $\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ a & 1 & 1 & b \end{array}\right]$

São concorrentes

$a=0$

$a\neq 0$ ↑ $a\neq 0 \wedge a\neq -1$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1 & 1 & b \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ a & 1 & 1 & b \end{array}\right] \sim$ $L_2=L_2-aL_1$

Para $a=0$,
$car(A)=car(A|B)=2$
↓

$\sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1+a & 1+a & b+a \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ a & 1 & 1 & b \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1+a & 1+a & b+a \end{array}\right]$ $L_2 = L_2 - aL_1$

- $a = -1$ $\qquad a \neq -1$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & b-1 \end{array}\right]$

Se $b = 1$ $\qquad b \neq 1$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\qquad \left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & b-1 \end{array}\right]$

$\downarrow \qquad \downarrow$

$car(A) = car(A|B) = 1$ $\qquad car(A) < car(A|B) = 1$

$\downarrow \qquad \downarrow$

Sao coincidentes $\qquad$ Estritamente paralelo se $a = -1 \wedge b \neq 1$

- $a \neq -1$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1+a & 1+a & b+a \end{array}\right]$ $\qquad car(A) = car(A|B) = 2$

Os planos sao concorrentes para $a \neq -1$

14) $R_a: \begin{cases} x + ay + z = 2 \\ x + ay + 2z = 3 \end{cases}$

$P_a: bx + by + z = 2$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 1 & a & 2 & 3 \\ b & b & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & b-ab & 1-b & 2-2b \end{array}\right] \overset{L_2 \leftrightarrow L_3}{\sim}$$

$$\sim \left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & b-ab & 1-b & 2-2b \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$b(1-a)=0$

$\Leftrightarrow b=0 \lor 1-a=0$

Para $b=0$ $\lor$ $a=1$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 1-b & 2-2b \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$car(A) < car(A|B)$
$\;\; \| \; 2$

para $b=0$, são estritamente paralelos

$b=1$ $\lor$ $b \neq 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 1-b & 2-2b \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$car(A)=2$
$=car(A|B)$

$R \subset$ Plano

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 1-b & 2-2b \\ 0 & 0 & 0 & -1+b \end{array}\right]$$

$car(A|B) > car(A) = 2$

Estritamente paralelos

Para $b \neq 0$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & b-ab & 1-b & 2-2b \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Para $a \neq 0$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & b-ab & 1-b & 2-2b \\ 0 & 0 & 1 & 1 \end{array}\right] \to car(A|B) = car(A) = 3$$

O plano e a reta são concorrentes se $a \neq 0 \land b \neq 0$

15) $R: \begin{cases} x = 1 \\ 2y + z = 1 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ 2y + z = x \end{cases} \Leftrightarrow \begin{cases} \\ -x + 2y + z = 0 \end{cases}$

$r_{a,b}: (x, y, z) = (a, 0, 1) + s(0, 2, b), \quad s \in \mathbb{R}$

Passando para eq. paramétricas

$\left(\frac{x - a}{0}\right) = \frac{y - 0}{2} = \frac{z - 1}{b}$ $\quad -2 - \frac{b}{2}$

$\Leftrightarrow x = a \wedge \frac{y - 0}{2} = \frac{z - 1}{b} \Leftrightarrow x = a \wedge by = 2z - 1$

$\Leftrightarrow x = a \wedge by - 2z = -1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ -1 & 2 & 1 & 0 \\ 1 & 0 & 0 & a \\ 0 & b & -2 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & b & -2 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & b & -2 & -1 \\ 0 & 0 & 0 & a-1 \end{array}\right] \quad L_3 = L_3 - \frac{bL_2}{2}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -\frac{b}{2} & -1 \\ 0 & 0 & 0 & a-1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -4-b & -2 \\ 0 & 0 & 0 & a-1 \end{array}\right]$$

• $-b = 4$

$\Leftrightarrow b = -4$

• Para $b = -4$

$a \neq 1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & a-1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & a-1 \end{array}\right]$$

$car(A|B) > car(A) = 2$

$\rightarrow$ Estr. paralelas

$a \neq 1 \wedge b = -4$

• $a = 1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rightarrow car(A|B) = 3 > car(A) = 2$

$\rightarrow$ SCI ...

Se $b \neq -4$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -4-b & -2 \\ 0 & 0 & 0 & a-1 \end{array}\right]$$

Para $a = 1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -4-b & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\downarrow$

$car(A) = car(A|B) = 3$

$\downarrow$

São concorrentes para $b \neq -4 \wedge a = 1$

Para $a \neq 1$

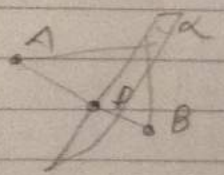
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -4-b & -2 \\ 0 & 0 & 0 & a-1 \end{array}\right]$$

$\downarrow$

$car(A|B) = 4 > car(A) = 3$

$\downarrow$

São enviesadas para $b \neq -4 \wedge a \neq 1$

16) 

$A\,(-1, 0, 2)$
$B\,(1, -1, 1)$

$\overrightarrow{AB} = B - A$
$= (1, -1, 1) - (-1, 0, 2)$
$= (2; -1; -1)$

Ponto médio de $\overline{AB}$

$$M_{AB} = \left(\frac{-1+1}{2}, \frac{0-1}{2}, \frac{2+1}{2}\right) \to \left(0, -\frac{1}{2}, \frac{3}{2}\right) = P$$

O vetor $\overrightarrow{AB}$ vai ser o vetor normal ao plano α.

Logo, $\alpha: 2x - y - z + d = 0$, $P(0; -1/2; 3/2)$
$\Leftrightarrow 0 + 1/2 - 3/2 = -d$
$\Leftrightarrow -2/2 = -d \Leftrightarrow d = 1$

$\alpha: 2x - y - z + 1 = 0$ //

17)

a) $A(3; 1/2; -7/2)$ Uma reta ortogonal a α tem o mesmo vetor de $\vec{n}$.

$P: y + z = -1$

$r: (x, y, z) = (3, 1/2, -7/2) + k(0, 1, 1), k \in \mathbb{R}$

b) $d(A, P) = \dfrac{|0 \times 3 + 1 \times 1/2 - 7/2 + 1|}{\sqrt{1^2 + 1^2 + 0^2}}$ 1/2

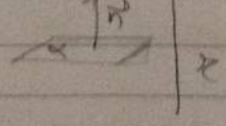
$= \dfrac{|1/2 - 7/2 + 1|}{\sqrt{2}} = \dfrac{|-6/2 + 2/2|}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

O outro processo é criarem uma reta que passa em A e intercetar com o plano, e fazer a norma do vetor AB, sendo B o ponto em que a tal reta interceta o plano.

18)

a) $P_0(-1, 1, 2)$ $R: \begin{cases} A_0(1, 0, 0) \\ B_0(0, 0, 1) \end{cases}$

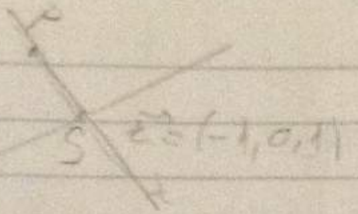
O vetor de R é dado por:



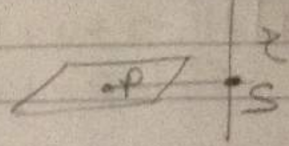
$\overrightarrow{AB} = B - A$
$= (0, 0, 1) - (1, 0, 0)$
$= (-1, 0, 1) = \vec{n}_\alpha$

$\alpha: -x + z + d = 0$, $P_0(-1, 1, 2)$
$\Leftrightarrow 1 + 2 + d = 0$
$\Leftrightarrow d = -3$

• $\alpha: -x + z - 3 = 0$
$\Leftrightarrow x - z + 3 = 0$

b) 

$\vec{t} = (1; 0; +1)$



$t: (x,y,z) = (-1,1,2) + k(1,0,1),\ k \in IR$

Ponto genérico $(-1+k; 1; 2+k)$

$r: (x,y,z) = (1,0,0) + k(-1,0,1),\ k \in IR$ ✓

O ponto S é a intercesão do plano com a reta r.

Ponto genérico da reta $(1-k, 0, k)$

• Intercesão com o plano

• $1 - k - k + 3 = 0$
$\Leftrightarrow -2k = -4 \Leftrightarrow k = 2$

$S(-1; 0; 2)$

Logo, $d(P, r) = \|\overrightarrow{SP}\|$

$\overrightarrow{SP} = P - S$
$= (-1,1,2) - (-1,0,2)$
$= (-1+1; 1; 0)$
$= (0,1,0) \Rightarrow \|\overrightarrow{SP}\| = 1$ //

19)

a) $P: x+y+2z=3$
$P_{a,b}: ax+2y+4z=b$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ a & 2 & 4 & b \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 2-a & 4-2a & b-3a \end{array}\right]$$

• Se $a=2$ ——— // ———

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & b-6 \end{array}\right]$$

• $b=6$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$car(A) = car(A|B) = 1$

Os planos são coincidentes para $a=2$ e $b=6$

$b \neq 6$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & b-6 \end{array}\right]$$

$car(A|B) = 2 > car(A) = 1$

São estritamente paralelos para $a=2$ e $b \neq 6$

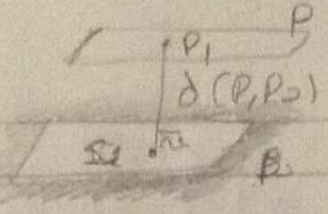
• Se $a \neq 2$ ——— // ———

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -a+2 & 4-2a & b-3a \end{array}\right]$$

$car(A) = car(A|B) = 2$

Os planos são concorrentes para $a \neq 2$ e $b \in \mathbb{R}$

b) $d(P_1, P_2)$



$2 - 3 = -2z$
$\Rightarrow -1 = -2z$
$\Rightarrow z = 1/2$

$\alpha: x + y + 2z = 3$
$\beta: 2x + 2y + 4z = 2$ $\rightarrow \begin{cases} x + y + 2z = 3 \\ x + y + 2z = 1 \end{cases}$

$\vec{n} = (1, 1, 2)$

Ponto qualquer $\alpha = (1, 1; 1/2) = P_1$

$$d(P_1, P_2) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \quad ; \quad d = -1$$

$$= \frac{|1 \times 1 + 1 \times 1 + 2 \times 1/2 + (-1)|}{\sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{|1 + 1 + 1 - 1|}{\sqrt{6}} = \frac{2}{\sqrt{6}}$$

2.)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & -2 & 0 & -1 \\ 0 & 1 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & -1 & -1 & -2 \\ 0 & 1 & 1 & 3 \end{array}\right] \sim$$

$L_2 = L_2 - L_1$

$L_2 = -L_2$

$L_3 = L_3 + L_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$car(A|B) = 3 > car(A) = 2$
$\downarrow$
Eles são estritamente paralelos

P - ponto qualquer da reta: Se $x = 1 \Rightarrow y = 1$
$P = (1, 1, 2)$ $\quad \vec{v} = (1, -1, 1)$

$$d(P, \text{Plano}) = \frac{|1 - 1 + 2 - 1|}{\sqrt{1 + 1 + 1}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

2.1)

a) $P_k: y + kz = 1$

$R: x = 2y = z - 1 \rightarrow \begin{cases} x - 2y = 0 \\ x - z + 1 = 0 \end{cases}$

$$\left[\begin{array}{ccc|c} 0 & 1 & k & 1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & k & 1 \\ 1 & 0 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & k & 1 \\ 0 & 2 & -1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & k & 1 \\ 0 & 0 & -1-2k & -3 \end{array}\right]$$

$\cdot\ -1 = 2k$

$\Leftrightarrow k = -1/2$

Se $k = -1/2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & k & 1 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$\downarrow$

$car(A|B) > car(A) = 2$

$\downarrow$

Estritamente paralelos

$k \neq -1/2$

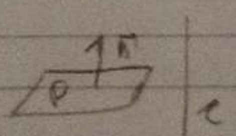
$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & k & 1 \\ 0 & 0 & -1-2k & -3 \end{array}\right]$$

$\downarrow$

$car(A) = car(A|B) = 3$

$\downarrow$

Concorrentes

b) 

$r: \dfrac{x-0}{1} = \dfrac{y-0}{1/2} = \dfrac{z-1}{1}$

$\vec{r} = (1; 1/2; 1)$

$P: x + \dfrac{y}{2} + z + d = 0 \quad , \ d(P, O) = 1$

$\Leftrightarrow 1 = \dfrac{|0+0+0+d|}{\sqrt{1+1+1/4}}$

$\Leftrightarrow 1 \times (3/2) = d$

$\Leftrightarrow d = \pm \dfrac{3}{2}$ $\rightarrow$ como tem modulo, $d = -3/2$ ou $d = 3/2$

$P: x + \dfrac{y}{2} + z \pm \dfrac{3}{2} = 0$ //

22)

a) r_1: $P_0(1,1,-1)$

$\vec{v} = (-1,2,-1)$

r_2: $P_1(1;-1;0)$

$P_2(0;1;-1)$

Vetor diretor de r_2

$$\overrightarrow{P_2P_1} = P_1 - P_2$$
$$= (1,-1,0) - (0,1,-1)$$
$$= (1;-2;1)$$

• $(-1;2;-1).(1;-2;1) = 0$?

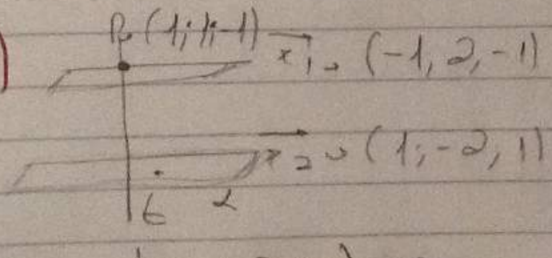
$\Leftrightarrow -1 - 2 - 1 = -4$ // X

• $(1,-2,1) = k(-1,2,-1)$

$$\Leftrightarrow \begin{cases} 1 = -1k \\ -2 = 2k \\ 1 = -1k \end{cases} \Leftrightarrow \begin{cases} k = -1 \\ k = -1 \\ k = -1 \end{cases}$$

As retas são paralelas, podendo, ou não, ser coincidentes.

b)



$\vec{t} = (-1;0,1)$

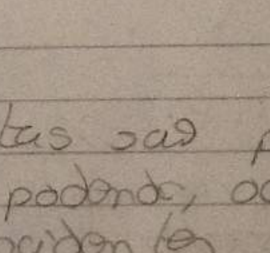
$\alpha: -x + z + d = 0$, $P_1 \to (0,1,-1)$

$\Leftrightarrow -1 = -d$

$\Leftrightarrow d = 1$

$$d(r_1,r_2) = \frac{|-1\times1 + 0\times1 - 1\times1 + 1|}{\sqrt{1^2 + 1^2}}$$

$$= \frac{|-1-1+1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$



23)

a) $R_1: (x,y,z) = (1,2,0) + k(-1,0,1),\ k \in \mathbb{R}$

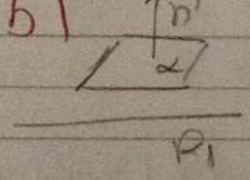
$R_2: (x,y,z) = (0,1,0) + \alpha(0,-1,1),\ k \in \mathbb{R}$

• $(-1,0,1) \cdot (0,-1,1)$
$= 0 + 0 + 1 = 1$ Não são perpendiculares

• $(-1,0,1) = k(0,-1,1)$

$\Leftrightarrow \begin{cases} -1 = 0 \\ 0 = -k \\ 1 = k \end{cases}$ Não são colineares

logo, ~~não~~ são enviesadas

b) 

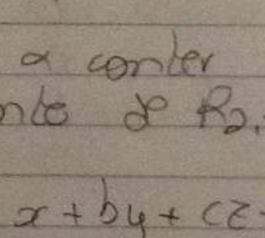
Como α é paralelo a R_1, logo
$\vec{n} = (1;0,1) \to$ (perpendicular)

Para α conter R_2, então basta conter um ponto de R_2: $(0,1,0)$

$\alpha: ax + by + cz + d = 0$
$\Leftrightarrow 1x + 1z + d = 0$
$\Leftrightarrow 0 + 0 = d \Leftrightarrow d = 0$

$\alpha: x + z = 0$

Dá diferente das soluções...



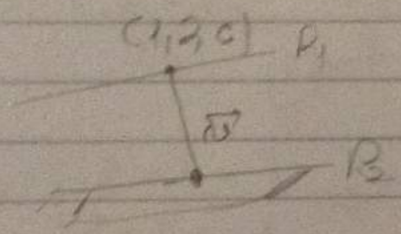
c) Ângulo entre retas

$\cdot\ (-1,0,1)\cdot(0,-1,1)$
$= 0+0+1 = 1$

$\cdot\ \|R_1\| = \sqrt{1+1} = \sqrt{2} \qquad \|R_2\| = \sqrt{2}$

$\cos(R_2 \wedge R_1) = \dfrac{1}{2} \Rightarrow \alpha = \dfrac{\pi}{3}$ //

• Distância entre retas



$d(P, R_2)$

24) $(x,y,z)=(1,1,-1)+s(0,1,-1)+t(4,-1,-1),\ s,t\in\mathbb{R}$

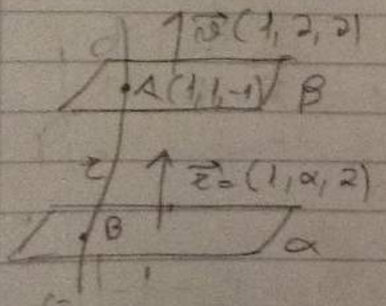
$P: x+ay+2z=\beta$

$\cdot\begin{cases} x=1+4t \\ y=1+s-t \\ z=-1-s-t \end{cases} \Leftrightarrow \begin{cases} \text{—} \\ t=1+s-y \\ z=-1-s-1-s+y \end{cases} \Leftrightarrow \begin{cases} x=1+4t \\ t=1+s-y \\ z=-2-2s+y \end{cases}$

$\Leftrightarrow \begin{cases} x=1+4+4s-4y \\ \text{—} \\ -z-2+y=+2s \end{cases} \Leftrightarrow \begin{cases} x=5+2.(-z-2+y)-4y \\ \text{—} \\ \text{—} \end{cases}$

$\Leftrightarrow \begin{cases} x=5-2z-4+2y-4y \\ = \\ \end{cases} \Leftrightarrow \begin{cases} x=1-2z-2y \\ = \\ \end{cases}$

$\cdot\ \beta: 1x+2y+2z=1 \rightarrow$ Eq geral plano



$r: (x,y,z)=(1,1,-1)+k(1,2,2),\ k\in\mathbb{R}$

$P_g \rightarrow (1+k;\ 1+2k;\ -1+2k)$

Interceção com plano α

$\cdot\ 1+k+\alpha(1+2k)+2(-1+2k)=\beta$

$\Leftrightarrow 1+k+\alpha+2\alpha k-2+4k=\beta$

$\Leftrightarrow -1+5k+\alpha+2\alpha k=\beta$

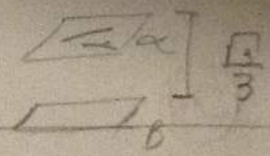
$\Leftrightarrow 2\alpha k+5k=\beta-\alpha+1$

$\Leftrightarrow k(2\alpha+5)=\beta-\alpha+1 \Leftrightarrow k=\dfrac{\beta-\alpha+1}{2\alpha+5}$ //

$B \rightarrow \left(1+\dfrac{\beta-\alpha+1}{2\alpha+5};\ 1+\dfrac{2\beta-2\alpha+2}{2\alpha+5};\ -1+\dfrac{2\beta-2\alpha+2}{2\alpha+5}\right)$

Algo errado!

25) $\alpha: x+y=0$ $\qquad d = \frac{\sqrt{3}}{3}$



$\beta: x+y+z=1$

Os pontos das retas pertendidas são do tipo: $A_\alpha(-y; y; z)$

$\cdot\ d(A,\beta) = \sqrt{3}/3$ $\qquad \vec{n} = (1,1,1)$ $\quad A_\alpha(-y; y; z)$

$d = -1$

$\Leftrightarrow \frac{|-y+y+z-1|}{\sqrt{1+1+1}} = \frac{\sqrt{3}}{3}$

$\Leftrightarrow \frac{|z-1|}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Leftrightarrow |z-1| = \frac{3}{3} \Leftrightarrow |z-1| = 1$

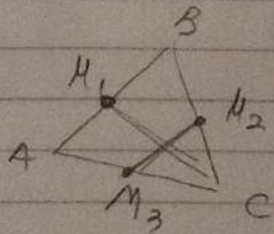
$\Leftrightarrow \begin{cases} z-1=1 \\ z-1=-1 \end{cases} \Leftrightarrow \begin{cases} z=2 \\ z=0 \end{cases}$

Assim, retas são:

$r_1 = \begin{cases} x=-y \\ z=2 \end{cases}$ $\qquad r_2 = \begin{cases} x=-y \\ z=0 \end{cases}$

26)

a) $M_1(2,1,3)$
$M_2(5,3,-1)$
$M_3(3,-4,0)$



Atenção que o triângulo não é equilátero!

$\cdot\ \overrightarrow{M_2M_3} = M_3 - M_2$
$= (3,-4,0)-(5,3,-1)$
$= (-2; -7; +1)$

$r: (x,y,z) = (2,1,3) + k(-2,-7,1),\ k \in \mathbb{R}$

Repitas 24

24) A: $x+2y+2z=1$, $d=-1$
B: $x+\alpha y+2z=\beta$, $d=-\beta$

Pontos de B

$B\} (-\alpha y - 2z + \beta ; y ; z)$ $\quad \vec{n}=(1,2,2)$

$d(B, A) = 3$

$\Leftrightarrow \frac{|-\alpha y - \cancel{2z} + \beta + 2y + \cancel{2z} - 1|}{\sqrt{1+4+4}} = 3$

$\Leftrightarrow \frac{|-\alpha y + 2y + \beta - 1|}{\sqrt{9}} = 3$, $\sqrt{9}=3$

$|x| = \begin{cases} x & x \\ -x \end{cases}$

$\Leftrightarrow |(-\alpha+2)y + \beta - 1| = 9$

$\Leftrightarrow \begin{cases} (-\alpha+2)y + \beta - 1 = 9 \\ -((-\alpha+2)y + \beta - 1) = 9 \end{cases} \Leftrightarrow \begin{cases} (-\alpha+2)y + \beta = 10 \\ -((-\alpha y + 2y) + \beta - 1) = 9 \end{cases}$

$\Leftrightarrow \begin{cases} -\alpha y + 2y + \beta = 10 \\ \alpha y - 2y - \beta + 1 = 9 \end{cases}$