Fundamentos de Programação

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Summary

- Recursive functions
  - How it works.
  - The program stack.
  - The rules for termination
- Examples
  - Operations on a list
  - Towers of Hanoi
  - A sorting algorithm (kind of quicksort)

A crazy idea

- What does this function do?

  `sumsq([1, 2, 3])`   `#-> 14`

  - Does it work on an empty list?
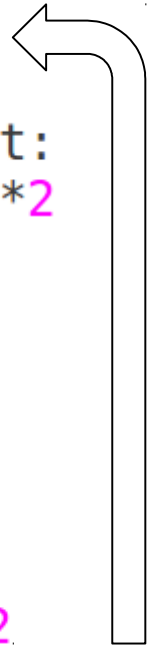 - Can you write it with a generator expression? (Homework!)

```python
def sumsq(lst):
    s = 0
    for x in lst:
        s += x**2
    return s
```

- Check out this weird version!

  - It squares first element;
  - Calls `sumsq` on the rest;
  - And adds.
  - For empty lst, just return zero.

```python
def sumsq2(lst):
    s = 0
    if len(lst) > 0:
        sq0 = lst[0]**2
        s = sq0 + sumsq(lst[1:])
    return s
```

- It is *equivalent* to `sumsq` in every case, but still calls the original `sumsq`. Not very useful.

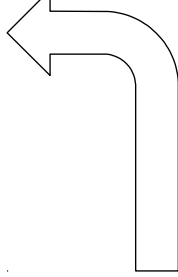- But if `sumsq2 <=> sumsq`, why not **call itself**?

Recursive functions

- This is what would result.

```python
def sumsqR(lst):
    s = 0
    if len(lst) > 0:
        sq0 = lst[0]**2
        s = sq0 + sumsqR(lst[1:])
    return s
```

- This is a **recursive function**: a function that calls itself.

- Notice that there is no loop instruction, but code gets executed several times, anyway.

- How does it work?

How recursion works

- What happens when we call `sumsqR([1, 2, 3])` ?
    - Watch the execution in PythonTutor.

```
sumsqR([1, 2, 3])
|    sumsqR([2, 3])
|    |    sumsqR([3])
|    |    |    sumsqR([])
|    |    |    L>0
|    |    L>9    ( = 3**2 + 0 )
|    L>13    ( = 2**2 + 9 )
L>14    ( = 1**2 + 13 )
```

- Notice that at one point, there are 4 frames in memory.

    - 4 variables named `lst`, 4 named `s`, 3 named `sq0`, but all distinct!

- The frames are stored in the **program stack**.

Example: Towers of Hanoi

- The Towers of Hanoi puzzle (Édouard Lucas, 1883)
- Move tower from A to C, using B temporarily.
  - Move only one disk at a time;
  - No disk may be put on top of a smaller disk.

By Trixx, CC BY-SA 3.0
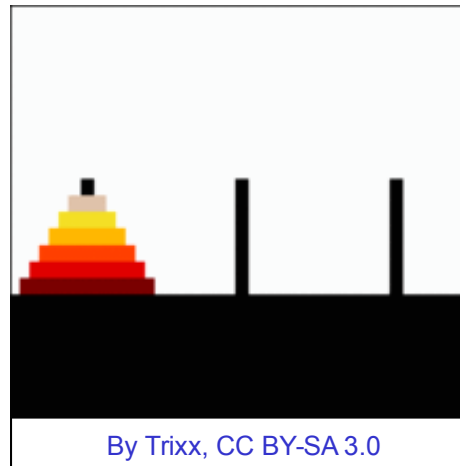
- Now solve it in 4 lines of code!

Example: quicksort

- The quicksort algorithm (C.A.R. Hoare) goes like this:

  1. Pick one of the values in the list (generally the first) and store in T.

  2. Put values smaller than T into a list L1, the others into a list L2.

  3. Sort L1 and L2 (using same algorithm, by the way)

  4. Result is L1 + [T] + L2.

- Of course, there's a few more details (the base case).

```python
def qsorted(lst):
    if len(lst)<=1:        # no need to sort
        return lst[:]      # just return a copy
    T = lst[0]
    L1 = [x for x in lst[1:] if x<T]
    L2 = [x for x in lst[1:] if x>=T]
    return qsorted(L1) + [T] + qsorted(L2)
```

- This is simple to understand and quite efficient!

The actual quicksort modifies the list in-place, and is slightly harder.

Example: quicksort

KWICK SÖRT

idea-instructions.com/quick-sort/
v1.0, CC by-nc-sa 4.0   IDEA
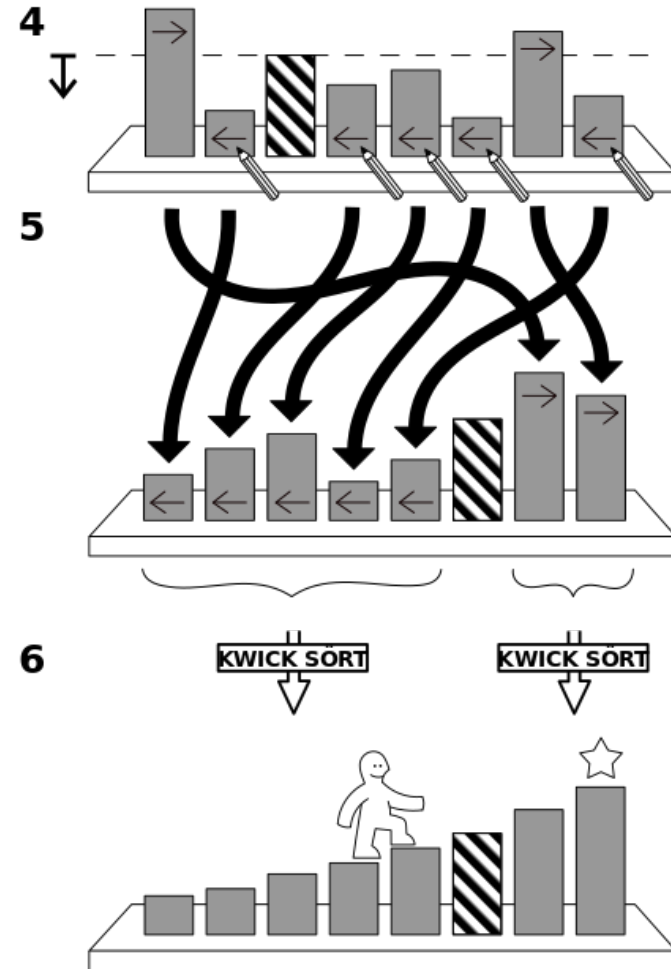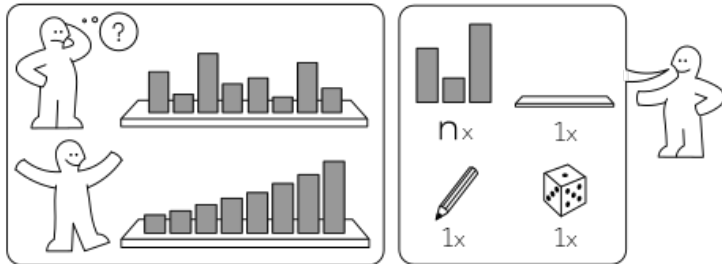
The rules of recursion termination

To guarantee that a recursive function **terminates**, it must obey some **rules**!

1. There must be cases that can be solved *without* recursive calls. These are called the **base cases**.

   - In `sumsqR`, the base case is `len(lst)==0`. In <u>that case</u>, return 0.

2. In the other cases, the *context passed* to recursive calls **must always differ** from the *context received*.*

   - In `sumsqR`, the argument `lst[1:] != lst` (always)

3. In successive recursive calls, the context must **converge** towards the base cases.

   - In `sumsqR`, the `lst` is shortened each time, until it's empty.

* The *context* is the set of arguments (and global values) that have an impact on the base case / recursive case selection.

Recursion vs repetition

- Any problem that can be solved by repetition may be solved by recursion, and vice-versa.

- For certain complex problems, recursive solutions are usually more concise and easier to understand.

- Recursive implementations may incur some time and memory cost because of functions calls and stack usage.

- If the problem has a simple iterative solution, that is usually the most efficient, too.

Writing recursive functions

- To develop a recursive function to solve some problem, there are some tricks that help.

    1. Start by defining the **arguments** you need, what they **mean**, and the **result** you **expect**, as *rigorously* as possible.

    2. Now, **assume** the function will work. Describe how the solution to a problem can be obtained from the solutions of **smaller** versions of the problem. This will be the recursive part of the algorithm.

    3. Finally, determine what are the **base cases**: which conditions have a trivial solution? This will be the non-recursive part of the algorithm. (Hint: usually, base cases are conditions outside the *domain* of the recursive call.)

- While in step 2, you may realize that you need extra arguments. Just add them and go back to step 1.