Suc. de Fib.:

Secc. de núm.: $0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$

$$\text{RR.} \begin{cases} F_n = F_{n-1} + F_{n-2}, & n \geq 2 \\ F_0 = 0 \\ F_1 = 1 \end{cases}$$

Cap. 10

- 10.1 → 1er O.H.
  - EC
  - 4 Problemas.
- 10.2 → 2do O.H.
  - Problemas.
- 10.3 → 1er, 2do O. No homo, $f(n)$
  - $\frac{1}{n}$ No
  - n
  - 2^n
  - 2^{-n} No.
  - $\cos n$
  - $\log n$ No.

MCI → Resolver.

Sol. General

$$F_n = g(n) = 5 + 2^n$$

$$F_{1000} = 5 + 2^{1000}$$

Series:

$$\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} 2^n$$

→ número, es convergente.

→ no es convergente.

$$S = \sum_{n=0}^{10} 2^n = 1+2+2^2+2^3+\cdots+2^{10} = \text{número} . \not{p}$$

Sol. General de una R.R

$$\left\} \ b_0 = 6 \right.$$

$$b_n = 5+2^n$$

$b_0 = 6$

$b_1 = 7$

$b_2 = 9$

$n=3$ $b_3 = b_2 + 2^2 = 9+4 = 13$; $b_3 = 5+2^3$

$$b_{n+1} = 5+2^{n+1} = 5+2.2^n = \underbrace{5+2^n} + \underbrace{2^n} = b_n + 2^n$$

$$b_{n+1} = b_n + 2^n$$

Sol. general

$b_n = 5 + 2^n$

R.R. $\begin{cases} \underbrace{b_n = b_{n-1} + 2^{n-1}}, & n \geq 1 \\ b_0 = 6 \end{cases}$

$\underbrace{b_3} = 5 + 2^3 = 5 + 8 = 13$

$b_0 = 5 + 2^0 = 6$ /

$b_1 = 5 + 2^1 = 7$ /

$b_2 = 5 + 2^2 = 9$ /

$b_3 = 5 + 2^3 = 13$

$\vdots$

$b_0 = 6$

$b_1 = b_0 + 2^0 = 6 + 1 = 7$

$n=2$: $b_2 = 7 + 2^1 = 9$

$b_3 = b_2 + 2^2 = 9 + 4 = 13$

$\vdots$

Sol. general

$\rightarrow C_n = n^2 \quad , n \geq 0$

$$RR \begin{cases} C_n = ? = C_{n-1} + 2(n-1) + 1 & , n \geq 1 \\ C_0 = 0 \end{cases}$$

$C_0 = 0$

$c_1 = 1$

$c_2 = 2^2 = 4$

$c_3 = 3^2 = 9$

$\vdots$

$$C_{n+1} = (n+1)^2 = n^2 + 2n + 1 = C_n + 2n + 1$$

$$C_{n+1} = C_n + (2n + 1)$$

$2x = x + x$

$b_n = 5 + 2^n$

$$b_{n+1} = 5 + 2^{n+1} = \ldots b_n = b_{n-} = \ldots\ldots =$$

$$= 5 + 2^n \cdot 2 = 5 + 2^n + 2^n = b_n + 2^n$$

$$a_n = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \sum_{i=1}^{n} i^3$$

$$a_{1000} = \sum_{i=1}^{1000} i^3 = 1^3 + 2^3 + \cdots + 1000^3 = \cdots$$

$$a_{10000} = 1^3 + 2^3 + 3^3 + \cdots + 10.000^3 = \cdots$$

$n = 1000$

$n = 10000$

RR:

$$a_{n-1} = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + (n-1)^3 = \sum_{i=1}^{n-1} i^3$$

RR. $\begin{cases} a_n - a_{n-1} = n^3, & n \geq 2 \\ a_1 = 1 \end{cases} \longrightarrow a_n \overset{?}{=} 2n^3 + 3n^2 + 6n - 10, \quad n \geq 1$

$$a_{1000} = 2.1000^3 + 3.1000^2 + 6.1000 - 1 \quad \beta$$

$$a_{10.000} =$$

Problema 2.

UN NÚMERO PAR DE CEROS $\qquad \Sigma = \{0, \underbrace{1, 2, \ldots, 9}\}$

$n=1$: $\qquad a_1 = 9$

$n=2$: $\qquad a_2 = \underset{9 \cdot 9}{\cancel{9} \cdot 8} + 1 = 82$

$a_3 = 9^3 + \cancel{\ldots} = \ldots\ldots$

a_2: _ _ |

a_3: $\underbrace{\_\ \_}$ | _

$a_3 = 9 \cdot a_2 + 1\, a_1$

$=00$

$\underbrace{a_2}$

82

1 1	7
0 0	7
2 2	7
3 1	7
7 8	7

11

00

22

$\mathbb{R}^+ \longrightarrow A = (1, 2)$

1,01

1,001

1,0001

$\mathbb{Z}^+ \longrightarrow A = \{2, 4, 6, 8, 10, \ldots\} \subset \mathbb{Z}^+$, primer elemento $= 2$

Demostrar que para cualquier $n \in \mathbb{Z}^+$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$: $P(n) \to$ Conjetura Matemática.

$$P(n):\ 1^2 + 2^2 + 3^2 + \dots + n^2 \overset{?}{=} \frac{n(n+1)(2n+1)}{6}, \quad n \geq 1$$

Inducción matemática:

$P(n=1)$:

$$\sum_{i=1}^{1} i^2 = 1^2 = 1$$

$$\frac{1.(1+1).(2.1+1)}{6} = \frac{1.2.3}{6} = 1$$

$\therefore$ $P(n=1)$ es Verdadera.

H.I). $P(n=k)$: $\left[\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6} \right]$

$\Downarrow$

Tesis) $P(n=k+1)$: $\sum_{i=1}^{k+1} i^2 \overset{?}{=} \frac{(k+1)(k+2).(2k+3)}{6}$ B

D-/ $\displaystyle\sum_{i=1}^{k+1} i^2 = \underbrace{\sum_{i=1}^{k} i^2}_{i=1,2,3,\dots,k} + \underbrace{(k+1)^2}_{i=k+1} = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

H.I.

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)\left[k(2k+1) + 6(k+1)\right]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\underbrace{\left(1^2 + 2^2 + 3^2 + 4^2 + \dots + (k-1)^2 + k^2\right)}_{k} + \underbrace{(k+1)^2}$$

$$\underbrace{\sum_{i=1}^{k} i^2 + (k+1)^2}$$

C.A

$$2k^2 + k + 6k + 6 = 2k^2 + 7k + 6 = 2(k+2)\left(k + \frac{3}{2}\right) = (k+2)(2k+3)$$

$$2k^2 + 7k + 6 = 0$$

$$\frac{-7 \pm \sqrt{49-48}}{4} \begin{cases} \frac{-6}{4} = -\frac{3}{2} \\ -2 \end{cases}$$

1. Demuestre lo siguiente mediante inducción matemática.

a) $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = (n)(2n-1)(2n+1)/3$

b) $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n+2) = (n)(n+1)(2n+7)/6$

c) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

d) $\sum_{i=1}^{n} 2^{i-1} = \sum_{i=0}^{n-1} 2^i = 2^n - 1$

e) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \left(\sum_{i=1}^{n} i\right)^2$

f) $1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2-1)$

2. Establezca lo siguiente mediante inducción matemática.

a) $\sum_{i=1}^{n} i(2^i) = 2 + (n-1)2^{n+1}$

b) $\sum_{i=1}^{n} 2(3^{i-1}) = 3^n - 1$

$\forall m \in \mathbb{N}$

c) $\sum_{i=1}^{n} (i)(i!) = (n+1)! - 1$

$P(m): \sum_{i=1}^{m} 2.(3^{i-1}) = 3^m - 1$

$m=1$: LD: $3^1 - 1 = 2$

LI: $\sum_{i=1}^{1} 2.3^{i-1} = \underset{\downarrow i=1}{2.3^{1-1}} = 2$ ✓

H.I: $P(m=k): \sum_{i=1}^{k} 2.3^{i-1} = 3^k - 1$

$\Downarrow$

$P(m=k+1): \sum_{i=1}^{k+1} 2.3^{i-1} \overset{?}{=} 3^{k+1} - 1$

Demostración:

$$\sum_{i=1}^{k+1} 2.3^{i-1} = \underbrace{\sum_{i=1}^{k} 2.3^{i-1}}_{1,2,\dots,k} + \underbrace{2.3^{k}}_{i=k+1} = 1.3^{k}-1 + 2.3^{k} = 3.3^{k}-1 = 3^{k+1}-1$$

$$(a_1+a_2+a_3+a_4) = \underbrace{(a_1+a_2+a_3)}_{1,2,3}+a_4$$

$$1x+2x=3x$$

I.M

$$a_n = \sum_{i=1}^{n} 2.3^{i-1} = \underbrace{3^{n}-1}_{¿?}$$

$$= n^2+6n$$

RR.

$$2.+2.3+2.3^2+2.3^3+\dots+2.3^{n-1} = 3^{n}-1$$

$$\underbrace{2+2.3+2.3^2+2.3^3+\dots+2.3^{100}} = \underbrace{3^{101}-1}$$

$, n=101$

$a_{n+1} =$

- Los números armónicos: $H_1 = 1$; y $\forall n \in \mathbb{Z}^+$: $H_{n+1} = H_n + \left(\frac{1}{n+1}\right)$
- Factorial de un número: $0! = 1$; y $\forall n \in \mathbb{N}_0$: $(n+1)! = (n+1).n!$

Los ***números de Fibonacci*** pueden definirse recursivamente de l
siguiente manera:

$$\begin{cases} F_0 = 0;\ F_1 = 1 \\ F_n = F_{n-1} + F_{n-2},\ \forall n \in \mathbb{Z}^+,\ \forall n \geq 2 \end{cases}$$

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

F_0 F_1 F_2 F_3

Una sucesión estrechamente relacionada con los números de Fibon
es la de los ***números de Lucas***, la cual se define:

$$\begin{cases} L_0 = 2,\ L_1 = 1 \\ L_n = L_{n-1} + L_{n-2},\ \forall n \in \mathbb{Z}^+,\ \forall n \geq 2 \end{cases}$$

2, 1, 3, 4, 7, 11, ...

L_0 L_1 L_2

$H_1 = 1$

$H_2 = H_1 + \frac{1}{2} = 1 + \frac{1}{2}$

$H_3 = H_2 + \frac{1}{3} = 1 + \frac{1}{2} + \frac{1}{3}$

$H_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$H_5 = H_4 + \frac{1}{5} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

TORRE DE HANOI.

3. $\forall n \in \mathbb{N}_0$: $\sum_{i=0}^{n} L_i = L_{n+2} - 1$ $\longrightarrow$ Prop $\rightarrow$ Prop. que depende de n $\rightarrow$ IM.

4. $\forall n \in \mathbb{Z}^+$: $L_n = F_{n-1} + F_{n+1}$

$\underline{n=0}$:

$$\left.\begin{array}{l} LD: \ L_2 - 1 = 3 - 1 = 2 \\ LI: \ \sum_{i=0}^{0} L_i = L_0 = 2 \end{array}\right] = \quad \therefore P(n=0) \text{ es V}$$

$\underline{H.I}$: $P(n=k)$: $\sum_{i=0}^{k} L_i = L_{k+2} - 1$

$L_k, L_{k+1}, L_{k+2}, L_{k+3}$

+

Probar $P(n=k+1)$: $\sum_{i=0}^{k+1} L_i \overset{?}{=} L_{k+3} - 1$

$\underline{\text{Demostración:}}$

$$\sum_{i=0}^{k+1} L_i = \sum_{i=0}^{k} L_i + L_{k+1} = L_{k+2} - 1 + L_{k+1} = L_{k+3} - 1$$

$P(n): L_n = F_{n-1} + F_{n+1}, \; n \in \mathbb{Z}^+ \; (n \geq 1)$

$\underline{n=1}$: LD: $F_0 + F_2 = 0 + 1 = 1$
LI: $L_1 = 1$
$\Rightarrow P(n=1)$ es cierta.

$\underline{n=2}$: LD: $F_1 + F_3 = 1 + 2 = 3$
LI: $L_2 = 3$
$\Rightarrow P(n=2)$ es cierta.

H.I.) $\begin{cases} P(n=k-1) \; S(k-1): L_{k-1} = F_{k-2} + F_k & \checkmark \\ P(n=k) \; S(k): L_k = F_{k-1} + F_{k+1} & \checkmark \end{cases}$

Tesis) $P(n=k+1): L_{k+1} \overset{?}{=} F_k + F_{k+2}$

D-/ $L_{k+1} = L_k + L_{k-1} = \underbrace{F_{k-1} + F_{k+1}}_{HI.} + \underbrace{L_{k-1}}_{=F_{k-2}+F_k} = F_{k-1} + F_{k+1} + F_{k-2} + F_k = F_k + F_{k+2}$

$n=10$
$n=11$
$n=12$
$n=100$:

1000
1001
1.000.000 → k
1.000.001 → k+1

Detengo!!!

IM, Forma alternativa.