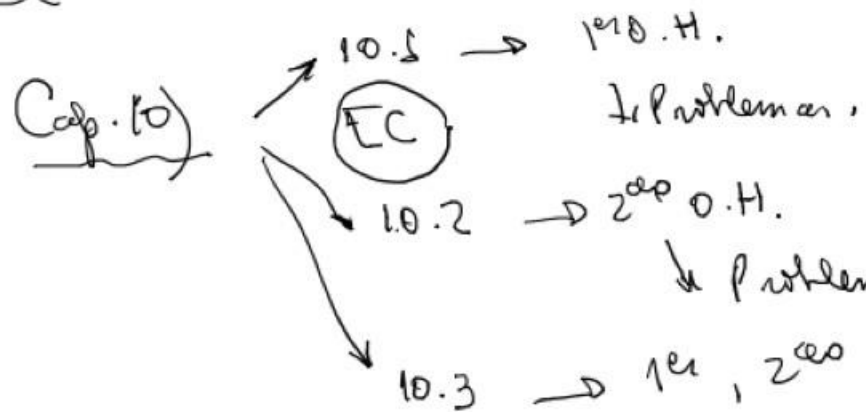


Serie de Fib. :

Serie de num. : 0, 1, 1, 2, 3, 5, 8, 13, 21, ...



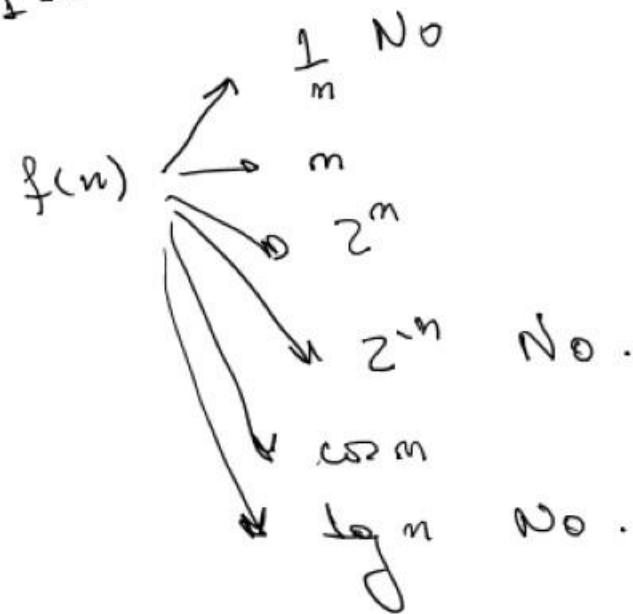
R.R. $\left\{ \begin{array}{l} F_m = F_{m-1} + F_{m-2}, \quad m \geq 2 \\ F_0 = 0 \\ F_1 = 1 \end{array} \right.$

HCI \rightarrow Resolver.

Sol. General

$$F_m = g(m) = 5 + 2^m$$

$$F_{1000} = 5 + 2^{1000}$$



Serie:

$$\sum_{m=0}^{\infty} a_m = \sum_{m=0}^{\infty} 2^m$$

\nearrow mel'ment, es convergente.
 \searrow no ts convergente.

10

$$S = \sum_{m=0}^{\infty} 2^m = 1 + 2 + 2^2 + 2^3 + \dots + 2^{10} = \text{mel'ment. } p$$

Sol. General de l'enc R.R $\left\{ \begin{array}{l} b_0 = 6 \end{array} \right.$

$$b_m = 5 + 2^m$$

$$b_0 = 6$$

$$b_1 = 7$$

$$b_2 = 9$$

$$b_{m+1} = b_m + 2^m$$

$$n=3 : b_3 = b_2 + 2^2 = 9 + 4 = 13 ; b_3 = 5 + 2^3$$

$$b_{m+1} = 5 + 2^{m+1} = 5 + 2 \cdot 2^m = \underbrace{5 + 2^m}_{b_m} + \underbrace{2^m}_{2^m} = b_m + 2^m$$

Sol. general

$$b_m = 5 + 2^n$$

$$b_3 = 5 + 2^3 = 5 + 8 = 13$$

$$b_0 = 5 + 2^0 = 6,$$

$$b_1 = 5 + 2^1 = 7,$$

$$b_2 = 5 + 2^2 = 9,$$

$$b_3 = 5 + 2^3 = 13$$

⋮

R.R.

$$\begin{cases} b_m = b_{m-1} + 2^{m-1}, & m \geq 1 \\ b_0 = 6 \end{cases}$$

$$b_0 = 6$$

$$b_1 = b_0 + 2^0 = 6 + 1 = 7$$

$$m=2: b_2 = 7 + 2^1 = 9$$

$$b_3 = b_2 + 2^2 = 9 + 4 = 13$$

⋮

Sol. General

$$\rightarrow C_n = n^2, n \geq 0$$

$$RR \begin{cases} C_n = ? = C_{n-1} + 2(n-1) + 1, n \geq 1 \\ C_0 = 0 \end{cases}$$

$$C_0 = 0$$

$$C_1 = 1$$

$$C_2 = 2^2 = 4$$

$$C_3 = 3^2 = 9$$

$$C_{n+1} = (n+1)^2 = n^2 + 2n + 1 = C_n + 2n + 1$$

$$C_{n+1} = C_n + (2(n) + 1)$$

$$2x = x + x$$

$$b_n = 5 + 2^n$$

$$\begin{aligned} \rightarrow b_{n+1} &= 5 + 2^{n+1} = \dots = b_n = b_{n-1} = \dots = \\ &= 5 + 2^n \cdot 2 = \boxed{5 + 2^n} + 2^n = b_n + 2^n \end{aligned}$$

$$a_n = 1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \sum_{i=1}^n i^3$$

$$n = 1000$$

$$n = 10000$$

RR:

$$a_{n-1} = 1^3 + 2^3 + 3^3 + 4^3 + \dots + (n-1)^3 = \sum_{i=1}^{n-1} i^3$$

$$a_{1000} = \sum_{i=1}^{1000} i^3 = 1^3 + 2^3 + \dots + 1000^3 = \dots$$

$$a_{10000} = 1^3 + 2^3 + 3^3 + \dots + 10000^3 = \dots$$

RR: $\begin{cases} a_n - a_{n-1} = n^3, n \geq 2 \\ a_1 = 1 \end{cases}$

$$a_n = 2n^3 + 3n^2 + 6n - 1, n \geq 1$$

$$a_{1000} = 2 \cdot 1000^3 + 3 \cdot 1000^2 + 6 \cdot 1000 - 1 \quad \beta$$

$$a_{10000} =$$

Problema 80

UN NÚMERO PAR DE CEROS

$$\Sigma^1 = \{0, \underbrace{1, 2, \dots, 9}\}$$

$$m = 1 :$$

$$a_1 = 9$$

$$u = 2:$$

$$a_2 = \frac{8.8}{9.9} + 1 = 82$$

$$a_3 = 9^3 + \dots = \dots$$

$$a_3 = 9 \cdot a_2 + 1 \cdot a_1$$

$G_2: \text{---} \text{---} |$

$$a_3: \quad \underline{\underline{\quad \quad}} \mid \quad$$

↓

$$\begin{array}{cc|c} a_2 & 1 & 1 \\ \downarrow & 0 & 0 \\ 82 & 2 & 2 \\ & 3 & 1 \\ & 7 & 8 \end{array} \quad \begin{array}{c} 7 \\ 7 \\ 7 \\ 7 \\ 7 \end{array}$$

$$\mathbb{R}^+ \rightarrow A = (1, 2)$$

$$1, 01$$

$$1, 001$$

$$1, 0001$$

$$\mathbb{Z}^+ \rightarrow A = \{2, 4, 6, 8, 10, \dots\} \subset \mathbb{Z}^+, \text{ primer elemento } = 2$$

Demostrar que para cualquier $n \in \mathbb{Z}^+$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$: $P(n) \rightarrow$ Conjetura Matemática.

$$P(m): 1^2 + 2^2 + 3^2 + \dots + m^2 \stackrel{?}{=} \frac{m(m+1)(2m+1)}{6}, \quad m \geq 1$$

Inducción matemática:

$P(n=1)$: $\sum_{i=1}^1 i^2 = 1^2 = 1$ $\swarrow \searrow$ $\therefore P(n=1)$ es Verdadera.

$$\frac{1 \cdot (1+1) \cdot (2 \cdot 1 + 1)}{6} = \frac{1 \cdot 2 \cdot 3}{6} = 1$$

H.I.) $P(m=k)$: $\left[\sum_{i=1}^k i^2 = \frac{k(k+1)(2k+1)}{6} \right]$

\downarrow
Tesis) $P(m=k+1)$: $\sum_{i=1}^{k+1} i^2 \stackrel{?}{=} \frac{(k+1)(k+2) \cdot (2k+3)}{6}$ β

$$D- \sum_{i=1}^{k+1} i^2 = \underbrace{\sum_{i=1}^k i^2}_{i=1,2,3,\dots,k} + \underbrace{(k+1)^2}_{i=k+1}$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

H.I.

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1) [k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

C.A

$$2k^2 + k + 6k + 6 = 2k^2 + 7k + 6 = 2(k+2)(k+\frac{3}{2}) = (k+2)(2k+3)$$

$$2k^2 + 7k + 6 = 0$$

$$\frac{-7 \pm \sqrt{49 - 48}}{4}$$

$$\frac{-6}{4} = -\frac{3}{2}$$

$$-2$$

1. Demuestre lo siguiente mediante inducción matemática.

a) $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = (n)(2n-1)(2n+1)/3$

b) $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + n(n+2) = (n)(n+1)(2n+7)/6$

c) $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$

d) $\sum_{i=1}^n 2^{i-1} = \sum_{i=0}^{n-1} 2^i = 2^n - 1$

e) $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4} = \left(\sum_{i=1}^n i \right)^2$

f) $1^3 + 3^3 + 5^3 + \dots + (2n-1)^3 = n^2(2n^2-1)$

2. Establezca lo siguiente mediante inducción matemática.

a) $\sum_{i=1}^n i(2^i) = 2 + (n-1)2^{n+1}$

(b) $\sum_{i=1}^n 2(3^{i-1}) = 3^n - 1$
 $\forall n \in \mathbb{N}$

c) $\sum_{i=1}^n (i)(i!) = (n+1)! - 1$

$P(n): \sum_{i=1}^n 2 \cdot (3^{i-1}) = 3^n - 1$

$n=1: \quad \text{L.D.} : 3^1 - 1 = 2$

$\text{L.I.} : \sum_{i=1}^1 2 \cdot 3^{i-1} = 2 \cdot 3^{1-1} = 2$

H.I. : $P(n=k) : \sum_{i=1}^k 2 \cdot 3^{i-1} = 3^k - 1$

\Downarrow
 $P(n=k+1) : \sum_{i=1}^{k+1} 2 \cdot 3^{i-1} \stackrel{?}{=} 3^{k+1} - 1$

Demonstración:

$$\sum_{i=1}^{k+1} 2 \cdot 3^{i-1} = \underbrace{\sum_{i=1}^k 2 \cdot 3^{i-1}}_{1, 2, \dots, k} + \underbrace{2 \cdot 3^k}_{i=k+1} = 1 \cdot 3^k - 1 + 2 \cdot 3^k = 3 \cdot 3^k - 1 = 3^{k+1} - 1 \quad \checkmark$$

$$(a_1 + a_2 + a_3 + a_4) = \underbrace{(a_1 + a_2 + a_3)}_{1, 2, 3} + a_4$$

$$2 + 2 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 + \dots + 2 \cdot 3^{m-1} = 3^m - 1$$

$$\underbrace{2 + 2 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 + \dots + 2 \cdot 3^{100}} = \underbrace{3^{101} - 1}$$

$$1x + 2x = 3x$$

IM

RR.

$\sum_{i=1}^m 2 \cdot 3^{i-1} = 3^m - 1$

$= m^2 + 6m$

$m = 101$

$a_{m+1} =$

- Los números armónicos: $H_1 = 1$; y $\forall n \in \mathbb{Z}^+$: $H_{n+1} = H_n + \left(\frac{1}{n+1}\right)$
- Factorial de un número: $0! = 1$; y $\forall n \in \mathbb{N}_0$: $(n+1)! = (n+1) \cdot n!$

Los **números de Fibonacci** pueden definirse recursivamente de la siguiente manera:

$$\begin{cases} F_0 = 0; F_1 = 1 \\ F_n = F_{n-1} + F_{n-2}, \forall n \in \mathbb{Z}^+, \forall n \geq 2 \end{cases}$$

✓ 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
 F_0, F_1, F_2, F_3

Una sucesión estrechamente relacionada con los números de Fibonacci es la de los **números de Lucas**, la cual se define:

$$\begin{cases} L_0 = 2, L_1 = 1 \\ L_n = L_{n-1} + L_{n-2}, \forall n \in \mathbb{Z}^+, \forall n \geq 2 \end{cases}$$

2, 1, 3, 4, 7, 11, ...
 $\downarrow \quad \downarrow \quad \downarrow$
 $L_0 \quad L_1 \quad L_2$

$$H_1 = 1$$

$$H_2 = H_1 + \frac{1}{2} = 1 + \frac{1}{2}$$

$$H_3 = H_2 + \frac{1}{3} = 1 + \frac{1}{2} + \frac{1}{3}$$

$$H_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$H_5 = H_4 + \frac{1}{5} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \checkmark$$

TORRE DE HANOI.

① $\forall n \in \mathbb{N}_0: \sum_{i=0}^n L_i = L_{n+2} - 1 \rightarrow ?_{\text{prop}} \rightarrow \text{Prop. que depende de } n \rightarrow \text{I.M.}$

② $\forall n \in \mathbb{Z}^+: L_n = F_{n-1} + F_{n+1}$

$n=0$:

LD: $L_2 - 1 = 3 - 1 = 2 \quad \left] = \therefore P(n=0) \text{ es } V\right.$

LI: $\sum_{i=0}^0 L_i = L_0 = 2$

H.I: $P(n=k): \sum_{i=0}^k L_i = L_{k+2} - 1$

Probar $P(n=k+1): \sum_{i=0}^{k+1} L_i \stackrel{?}{=} L_{k+3} - 1$

Demstración:

$$\sum_{i=0}^{k+1} L_i = \underbrace{\sum_{i=0}^k L_i}_{\text{H.I}} + L_{k+1} = L_{k+2} - 1 + L_{k+1} = L_{k+3} - 1$$

$L_k, L_{k+1}, L_{k+2}, L_{k+3}$

Diagram showing the relationship between consecutive Fibonacci numbers: $L_k + L_{k+1} = L_{k+2}$ and $L_{k+1} + L_{k+2} = L_{k+3}$.

$$P(n): L_n = F_{n-1} + F_{n+1}, \quad n \in \mathbb{Z}^+ \quad (n \geq 1)$$

$n=1$: LD: $F_0 + F_2 = 0 + 1 = 1$ \leftarrow $P(n=1)$ es cierta.

LI: $L_1 = 1$

$n=2$: LD: $F_1 + F_3 = 1 + 2 = 3$ \leftarrow $P(n=2)$ es cierta.

LI: $L_2 = 3$

H.I.) $\begin{cases} P(n=k-1) \text{ S.I.}: L_{k-1} = F_{k-2} + F_k \\ P(n=k) \text{ S.I.}: L_k = F_{k-1} + F_{k+1} \end{cases}$ ✓

Tesis) $P(n=k+1): L_{k+1} = F_k + F_{k+2}$?

D-/ $L_{k+1} = L_k + L_{k-1} = F_{k-1} + F_{k+1} + \underbrace{L_{k-1}}_{= F_{k-2} + F_k} = F_{k-1} + F_{k+1} + F_{k-2} + F_k = F_k + F_{k+2}$ \leftarrow H.I.

$n=10$
 $n=11$
 $n=12$
 $n=13$
 $n=14$
 $n=15$
 $n=16$
 $n=17$
 $n=18$
 $n=19$
 $n=20$
 $n=21$
 $n=22$
 $n=23$
 $n=24$
 $n=25$
 $n=26$
 $n=27$
 $n=28$
 $n=29$
 $n=30$
 $n=31$
 $n=32$
 $n=33$
 $n=34$
 $n=35$
 $n=36$
 $n=37$
 $n=38$
 $n=39$
 $n=40$
 $n=41$
 $n=42$
 $n=43$
 $n=44$
 $n=45$
 $n=46$
 $n=47$
 $n=48$
 $n=49$
 $n=50$
 $n=51$
 $n=52$
 $n=53$
 $n=54$
 $n=55$
 $n=56$
 $n=57$
 $n=58$
 $n=59$
 $n=60$
 $n=61$
 $n=62$
 $n=63$
 $n=64$
 $n=65$
 $n=66$
 $n=67$
 $n=68$
 $n=69$
 $n=70$
 $n=71$
 $n=72$
 $n=73$
 $n=74$
 $n=75$
 $n=76$
 $n=77$
 $n=78$
 $n=79$
 $n=80$
 $n=81$
 $n=82$
 $n=83$
 $n=84$
 $n=85$
 $n=86$
 $n=87$
 $n=88$
 $n=89$
 $n=90$
 $n=91$
 $n=92$
 $n=93$
 $n=94$
 $n=95$
 $n=96$
 $n=97$
 $n=98$
 $n=99$
 $n=100$
 $n=101$
 $n=102$
 $n=103$
 $n=104$
 $n=105$
 $n=106$
 $n=107$
 $n=108$
 $n=109$
 $n=110$
 $n=111$
 $n=112$
 $n=113$
 $n=114$
 $n=115$
 $n=116$
 $n=117$
 $n=118$
 $n=119$
 $n=120$
 $n=121$
 $n=122$
 $n=123$
 $n=124$
 $n=125$
 $n=126$
 $n=127$
 $n=128$
 $n=129$
 $n=130$
 $n=131$
 $n=132$
 $n=133$
 $n=134$
 $n=135$
 $n=136$
 $n=137$
 $n=138$
 $n=139$
 $n=140$
 $n=141$
 $n=142$
 $n=143$
 $n=144$
 $n=145$
 $n=146$
 $n=147$
 $n=148$
 $n=149$
 $n=150$
 $n=151$
 $n=152$
 $n=153$
 $n=154$
 $n=155$
 $n=156$
 $n=157$
 $n=158$
 $n=159$
 $n=160$
 $n=161$
 $n=162$
 $n=163$
 $n=164$
 $n=165$
 $n=166$
 $n=167$
 $n=168$
 $n=169$
 $n=170$
 $n=171$
 $n=172$
 $n=173$
 $n=174$
 $n=175$
 $n=176$
 $n=177$
 $n=178$
 $n=179$
 $n=180$
 $n=181$
 $n=182$
 $n=183$
 $n=184$
 $n=185$
 $n=186$
 $n=187$
 $n=188$
 $n=189$
 $n=190$
 $n=191$
 $n=192$
 $n=193$
 $n=194$
 $n=195$
 $n=196$
 $n=197$
 $n=198$
 $n=199$
 $n=200$
 $n=201$
 $n=202$
 $n=203$
 $n=204$
 $n=205$
 $n=206$
 $n=207$
 $n=208$
 $n=209$
 $n=210$
 $n=211$
 $n=212$
 $n=213$
 $n=214$
 $n=215$
 $n=216$
 $n=217$
 $n=218$
 $n=219$
 $n=220$
 $n=221$
 $n=222$
 $n=223$
 $n=224$
 $n=225$
 $n=226$
 $n=227$
 $n=228$
 $n=229$
 $n=230$
 $n=231$
 $n=232$
 $n=233$
 $n=234$
 $n=235$
 $n=236$
 $n=237$
 $n=238$
 $n=239$
 $n=240$
 $n=241$
 $n=242$
 $n=243$
 $n=244$
 $n=245$
 $n=246$
 $n=247$
 $n=248$
 $n=249$
 $n=250$
 $n=251$
 $n=252$
 $n=253$
 $n=254$
 $n=255$
 $n=256$
 $n=257$
 $n=258$
 $n=259$
 $n=260$
 $n=261$
 $n=262$
 $n=263$
 $n=264$
 $n=265$
 $n=266$
 $n=267$
 $n=268$
 $n=269$
 $n=270$
 $n=271$
 $n=272$
 $n=273$
 $n=274$
 $n=275$
 $n=276$
 $n=277$
 $n=278$
 $n=279$
 $n=280$
 $n=281$
 $n=282$
 $n=283$
 $n=284$
 $n=285$
 $n=286$
 $n=287$
 $n=288$
 $n=289$
 $n=290$
 $n=291$
 $n=292$
 $n=293$
 $n=294$
 $n=295$
 $n=296$
 $n=297$
 $n=298$
 $n=299$
 $n=300$
 $n=301$
 $n=302$
 $n=303$
 $n=304$
 $n=305$
 $n=306$
 $n=307$
 $n=308$
 $n=309$
 $n=310$
 $n=311$
 $n=312$
 $n=313$
 $n=314$
 $n=315$
 $n=316$
 $n=317$
 $n=318$
 $n=319$
 $n=320$
 $n=321$
 $n=322$
 $n=323$
 $n=324$
 $n=325$
 $n=326$
 $n=327$
 $n=328$
 $n=329$
 $n=330$
 $n=331$
 $n=332$
 $n=333$
 $n=334$
 $n=335$
 $n=336$
 $n=337$
 $n=338$
 $n=339$
 $n=340$
 $n=341$
 $n=342$
 $n=343$
 $n=344$
 $n=345$
 $n=346$
 $n=347$
 $n=348$
 $n=349$
 $n=350$
 $n=351$
 $n=352$
 $n=353$
 $n=354$
 $n=355$
 $n=356$
 $n=357$
 $n=358$
 $n=359$
 $n=360$
 $n=361$
 $n=362$
 $n=363$
 $n=364$
 $n=365$
 $n=366$
 $n=367$
 $n=368$
 $n=369$
 $n=370$
 $n=371$
 $n=372$
 $n=373$
 $n=374$
 $n=375$
 $n=376$
 $n=377$
 $n=378$
 $n=379$
 $n=380$
 $n=381$
 $n=382$
 $n=383$
 $n=384$
 $n=385$
 $n=386$
 $n=387$
 $n=388$
 $n=389$
 $n=390$
 $n=391$
 $n=392$
 $n=393$
 $n=394$
 $n=395$
 $n=396$
 $n=397$
 $n=398$
 $n=399$
 $n=400$
 $n=401$
 $n=402$
 $n=403$
 $n=404$
 $n=405$
 $n=406$
 $n=407$
 $n=408$
 $n=409$
 $n=410$
 $n=411$
 $n=412$
 $n=413$
 $n=414$
 $n=415$
 $n=416$
 $n=417$
 $n=418$
 $n=419$
 $n=420$
 $n=421$
 $n=422$
 $n=423$
 $n=424$
 $n=425$
 $n=426$
 $n=427$
 $n=428$
 $n=429$
 $n=430$
 $n=431$
 $n=432$
 $n=433$
 $n=434$
 $n=435$
 $n=436$
 $n=437$
 $n=438$
 $n=439$
 $n=440$
 $n=441$
 $n=442$
 $n=443$
 $n=444$
 $n=445$
 $n=446$
 $n=447$
 $n=448$
 $n=449$
 $n=450$
 $n=451$
 $n=452$
 $n=453$
 $n=454$
 $n=455$
 $n=456$
 $n=457$
 $n=458$
 $n=459$
 $n=460$
 $n=461$
 $n=462$
 $n=463$
 $n=464$
 $n=465$
 $n=466$
 $n=467$
 $n=468$
 $n=469$
 $n=470$
 $n=471$
 $n=472$
 $n=473$
 $n=474$
 $n=475$
 $n=476$
 $n=477$
 $n=478$
 $n=479$
 $n=480$
 $n=481$
 $n=482$
 $n=483$
 $n=484$
 $n=485$
 $n=486$
 $n=487$
 $n=488$
 $n=489$
 $n=490$
 $n=491$
 $n=492$
 $n=493$
 $n=494$
 $n=495$
 $n=496$
 $n=497$
 $n=498$
 $n=499$
 $n=500$
 $n=501$
 $n=502$
 $n=503$
 $n=504$
 $n=505$
 $n=506$
 $n=507$
 $n=508$
 $n=509$
 $n=510$
 $n=511$
 $n=512$
 $n=513$
 $n=514$
 $n=515$
 $n=516$
 $n=517$
 $n=518$
 $n=519$
 $n=520$
 $n=521$
 $n=522$
 $n=523$
 $n=524$
 $n=525$
 $n=526$
 $n=527$
 $n=528$
 $n=529$
 $n=530$
 $n=531$
 $n=532$
 $n=533$
 $n=534$
 $n=535$
 $n=536$
 $n=537$
 $n=538$
 $n=539$
 $n=540$
 $n=541$
 $n=542$
 $n=543$
 $n=544$
 $n=545$
 $n=546$
 $n=547$
 $n=548$
 $n=549$
 $n=550$
 $n=551$
 $n=552$
 $n=553$
 $n=554$
 $n=555$
 $n=556$
 $n=557$
 $n=558$
 $n=559$
 $n=560$
 $n=561$
 $n=562$
 $n=563$
 $n=564$
 $n=565$
 $n=566$
 $n=567$
 $n=568$
 $n=569$
 $n=570$
 $n=571$
 $n=572$
 $n=573$
 $n=574$
 $n=575$
 $n=576$
 $n=577$
 $n=578$
 $n=579$
 $n=580$
 $n=581$
 $n=582$
 $n=583$
 $n=584$
 $n=585$
 $n=586$
 $n=587$
 $n=588$
 $n=589$
 $n=590$
 $n=591$
 $n=592$
 $n=593$
 $n=594$
 $n=595$
 $n=596$
 $n=597$
 $n=598$
 $n=599$
 $n=600$
 $n=601$
 $n=602$
 $n=603$
 $n=604$
 $n=605$
 $n=606$
 $n=607$
 $n=608$
 $n=609$
 $n=610$
 $n=611$
 $n=612$
 $n=613$
 $n=614$
 $n=615$
 $n=616$
 $n=617$
 $n=618$
 $n=619$
 $n=620$
 $n=621$
 $n=622$
 $n=623$
 $n=624$
 $n=625$
 $n=626$
 $n=627$
 $n=628$
 $n=629$
 $n=630$
 $n=631$
 $n=632$
 $n=633$
 $n=634$
 $n=635$
 $n=636$
 $n=637$
 $n=638$
 $n=639$
 $n=640$
 $n=641$
 $n=642$
 $n=643$
 $n=644$
 $n=645$
 $n=646$
 $n=647$
 $n=648$
 $n=649$
 $n=650$
 $n=651$
 $n=652$
 $n=653$
 $n=654$
 $n=655$
 $n=656$
 $n=657$
 $n=658$
 $n=659$
 $n=660$
 $n=661$
 $n=662$
 $n=663$
 $n=664$
 $n=665$
 $n=666$
 $n=667$
 $n=668$
 $n=669$
 $n=670$
 $n=671$
 $n=672$
 $n=673$
 $n=674$
 $n=675$
 $n=676$
 $n=677$
 $n=678$
 $n=679$
 $n=680$
 $n=681$
 $n=682$
 $n=683$
 $n=684$
 $n=685$
 $n=686$
 $n=687$
 $n=688$
 $n=689$
 $n=690$
 $n=691$
 $n=692$
 $n=693$
 $n=694$
 $n=695$
 $n=696$
 $n=697$
 $n=698$
 $n=699$
 $n=700$
 $n=701$
 $n=702$
 $n=703$
 $n=704$
 $n=705$
 $n=706$
 $n=707$
 $n=708$
 $n=709$
 $n=710$
 $n=711$
 $n=712$
 $n=713$
 $n=714$
 $n=715$
 $n=716$
 $n=717$
 $n=718$
 $n=719$
 $n=720$
 $n=721$
 $n=722$
 $n=723$
 $n=724$
 $n=725$
 $n=726$
 $n=727$
 $n=728$
 $n=729$
 $n=730$
 $n=731$
 $n=732$
 $n=733$
 $n=734$
 $n=735$
 $n=736$
 $n=737$
 $n=738$
 $n=739$
 $n=740$
 $n=741$
 $n=742$
 $n=743$
 $n=744$
 $n=745$
 $n=746$
 $n=747$
 $n=748$
 $n=749$
 $n=750$
 $n=751$
 $n=752$
 $n=753$
 $n=754$
 $n=755$
 $n=756$
 $n=757$
 $n=758$
 $n=759$
 $n=760$
 $n=761$
 $n=762$
 $n=763$
 $n=764$
 $n=765$
 $n=766$
 $n=767$
 $n=768$
 $n=769$
 $n=770$
 $n=771$
 $n=772$
 $n=773$
 $n=774$
 $n=775$
 $n=776$
 $n=777$
 $n=778$
 $n=779$
 $n=780$
 $n=781$
 $n=782$
 $n=783$
 $n=784$
 $n=785$
 $n=786$
 $n=787$
 $n=788$
 $n=789$
 $n=790$
 $n=791$
 $n=792$
 $n=793$
 $n=794$
 $n=795$
 $n=796$
 $n=797$
 $n=798$
 $n=799$
 $n=800$
 $n=801$
 $n=802$
 $n=803$
 $n=804$
 $n=805$
 $n=806$
 $n=807$
 $n=808$
 $n=809$
 $n=810$
 $n=811$
 $n=812$
 $n=813$
 $n=814$
 $n=815$
 $n=816$
 $n=817$
 $n=818$
 $n=819$
 $n=820$
 $n=821$
 $n=822$
 $n=823$
 $n=824$
 $n=825$
 $n=826$
 $n=827$
 $n=828$
 $n=829$
 $n=830$
 $n=831$
 $n=832$
 $n=833$
 $n=834$
 $n=835$
 $n=836$
 $n=837$
 $n=838$
 $n=839$
 $n=840$
 $n=841$
 $n=842$
 $n=843$
 $n=844$
 $n=845$
 $n=846$
 $n=847$
 $n=848$
 $n=849$
 $n=850$
 $n=851$
 $n=852$
 $n=853$
 $n=854$
 $n=855$
 $n=856$
 $n=857$
 $n=858$
 $n=859$
 $n=860$
 $n=861$
 $n=862$
 $n=863$
 $n=864$
 $n=865$
 $n=866$
 $n=867$
 $n=868$
 $n=869$
 $n=870$
 $n=871$
 $n=872$
 $n=873$
 $n=874$
 $n=875$
 $n=876$
 $n=877$
 $n=878$
 $n=879$
 $n=880$
 $n=881$
 $n=882$
 $n=883$
 $n=884$
 $n=885$
 $n=886$
 $n=887$
 $n=888$
 $n=889$
 $n=890$
 $n=891$
 $n=892$
 $n=893$
 $n=894$
 $n=895$
 $n=896$
 $n=897$
 $n=898$
 $n=899$
 $n=900$
 $n=901$
 $n=902$
 $n=903$
 $n=904$
 $n=905$
 $n=906$
 $n=907$
 $n=908$
 $n=909$
 $n=910$
 $n=911$
 $n=912$
 $n=913$
 $n=914$
 $n=915$
 $n=916$
 $n=917$
 $n=918$
 $n=919$
 $n=920$
 $n=921$
 $n=922$
 $n=923$
 $n=924$
 $n=925$
 $n=926$
 $n=927$
 $n=928$
 $n=929$
 $n=930$
 $n=931$
 $n=932$
 $n=933$
 $n=934$
 $n=935$
 $n=936$
 $n=937$
 $n=938$
 $n=939$
 $n=940$
 $n=941$
 $n=942$
 $n=943$
 $n=944$
 $n=945$
 $n=946$
 $n=947$
 $n=948$
 $n=949$
 $n=950$
 $n=951$
 $n=952$
 $n=953$
 $n=954$
 $n=955$
 $n=956$
 $n=957$
 $n=958$
 $n=959$
 $n=960$
 $n=961$
 $n=962$
 $n=963$
 $n=964$
 $n=965$
 $n=966$
 $n=967$
 $n=968$
 $n=969$
 $n=970$
 $n=971$
 $n=972$
 $n=973$
 $n=974$
 $n=975$
 $n=976$
 $n=977$
 $n=978$
 $n=979$
 $n=980$
 $n=981$
 $n=982$
 $n=983$
 $n=984$
 $n=985$
 $n=986$
 $n=987$
 $n=988$
 $n=989$
 $n=990$
 $n=991$
 $n=992$
 $n=993$
 $n=994$
 $n=995$
 $n=996$
 $n=997$
 $n=998$
 $n=999$
 $n=1000$
 $n=1001$
 $n=1002$
 $n=1003$
 $n=1004$
 $n=1005$
 $n=1006$
 $n=1007$
 $n=1008$
 $n=1009$
 $n=1010$
 $n=1011$
 $n=1012$
 $n=1013$
 $n=1014$
 $n=1015$
 $n=1016$
 $n=1017$
 $n=1018$
 $n=1019$
 $n=1020$
 $n=1021$
 $n=1022$
 $n=1023$
 $n=1024$
 $n=1025$
 $n=1026$
 $n=1027$
 $n=1028$
 $n=1029$
 $n=1030$
 $n=1031$
 $n=1032$
 $n=1033$
 $n=1034$
 $n=1035$
 $n=1036$
 $n=1037$
 $n=1038$
 $n=1039$
 $n=1040$
 $n=1041$
 $n=1042$
 $n=1043$
 $n=1044$
 $n=1045$
 $n=1046$
 $n=1047$
 $n=1048$
 $n=1049$
 $n=1050$
 $n=1051$
 $n=1052$
 $n=1053$
 $n=1054$
 $n=1055$
 $n=1056$
 $n=1057$
 $n=1058$
 $n=1059$
 $n=1060$
 $n=1061$
 $n=1062$
 $n=1063$
 $n=1064$
 $n=1065$
 $n=1066$
 $n=1067$
 $n=1068$
 $n=1069$
 $n=1070$
 $n=1071</$

