Game-theoretic extensions of logistic regression

Guilherme Dean Pelegrina[1*], Miguel Couceiro[2,4], Cristiano Torezzan[3], Leonardo Tomazeli Duarte[3]

[1*]Engineering School, Mackenzie Presbyterian University, São Paulo, São Paulo, Brazil.
[2]INESC-ID, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, Portugal.
[3]School of Applied Sciences, University of Campinas, Limeira, São Paulo, Brazil.
[4]LORIA, CNRS, University of Lorraine, Nancy, France.

*Corresponding author(s). E-mail(s): guilherme.pelegrina@mackenzie.br; Contributing authors: miguel.couceiro@inesc-id.pt; torezzan@unicamp.br; ltduarte@unicamp.br;

Abstract

Logistic regression (LR) is a widely used and well-established tool for classification tasks, valued for its simplicity and interpretability. However, its reliance on linear separators transformed by a sigmoid function limits its flexibility, particularly when dealing with non-linear decision boundaries. To address this limitation, Tehrani *et al.* proposed an extension of LR where the linear separator is replaced by a piecewise-linear known as Choquet integral. While effective, this approach, known as "Choquistic regression", faces the curse of dimensionality, as its parameter count grows exponentially with the number of input features. To mitigate this issue, simplified variants of the Choquet integral, such as the 2-additive Choquet integral, have been proposed. These variants restrict interactions to at most two features, significantly reducing complexity while delivering competitive performance compared to unrestricted models. However, the monotonicity constraints imposed by the Choquistic approach remain a limitation, reducing its applicability to real-world scenarios. In this paper, we propose a novel formulation of logistic regression that integrates the Choquet integral, with parameters defined through a cooperative game framework. Additionally, we explore game-based aggregation functions such as the multilinear model, which also captures feature interactions effectively. To overcome the curse of dimensionality, we utilize 2-additive games, which significantly reduce the number of parameters while preserving competitive accuracy. This reduction in complexity is empirically validated through a comparative study of restricted and unrestricted logistic regression variants on a COVID-19 dataset. Furthermore, the results highlight the interpretability advantages of these models, demonstrating their practical value in real-world applications.

Keywords: Logistic regression, Choquet integral, Cooperative game, Multilinear model, COVID-19

1 Introduction

In recent years, machine learning models became increasingly present in decision support systems and contributed to the automated decisions in various real-world situations (Sarker, 2021). Simple techniques, such as logistic regression (Kleinbaum & Klein, 2002), are particularly appealing for classification tasks due to their fast training process and inherently interpretable outputs. In fact, interpretability has emerged as a critical aspect of Trustworthy Artificial Intelligence (Kaur et al., 2022), emphasizing the importance of understanding how features influence decisions. For instance, when a bank denies a client's credit application, it is important to identify whether financial or personal factors — or a combination of both — underpinned the decision. Similarly, in disease detection, both physicians and patients benefit from interpreting how specific symptoms contribute to the diagnostic outcome.

While simple models are advantageous in terms of interpretability and ease of implementation, many real-world scenarios present a level of complexity that necessitates the use of sophisticated models. Applications in supply chain management (Ni et al., 2020), credit card fraud detection (Bin Sulaiman et al., 2022), and healthcare system management (Li et al., 2021) exemplify situations where complex models, such as Deep Neural Networks (Goodfellow et al., 2016) and Extreme Gradient Boosting (XGBoost) (Chen & Guestrin, 2016), are often required. These models typically achieve superior performance but are frequently regarded as "black boxes" (Lipton, 2018) due to their lack of interpretability.

To bridge this gap, model-agnostic approaches have been developed to estimate feature contributions to a given decision (Molnar, 2021). These methods have been also applied to mitigate bias (Alves et al., 2023; Bhargava et al., 2020). A notable technique employs the game-theoretic concept of the *Shapley value* (Shapley, 1953). In summary, the idea is to model the machine learning task as a cooperative game in which attributes (players) work together to achieve a specific goal, such as optimizing overall classifier performance or explaining a local prediction. The Shapley value quantifies each attribute's marginal contribution to this goal, accounting for attribute interactions and ensuring a fair distribution of the model's "worth". As a result, the Shapley value has become a widely recognized tool for interpreting complex machine learning models.

Despite its utility, the computational inefficiency of Shapley value calculations in model-agnostics approaches poses a significant challenge. The computational cost increases exponentially with the number of attributes, making it impractical for high-dimensional datasets. This limitation highlights the need for models that are easy to train while effectively capturing feature interactions. Such models could offer a practical solution by achieving an optimal balance between computational efficiency, interpretability, and performance.

In this context, logistic regression (Kleinbaum & Klein, 2002) remains a baseline of interpretable machine learning. By extending the principles of linear regression to model the probability of specific events, this model combines input variables through a linear function, which is then passed through a sigmoid function to produce probabilistic outputs. This straightforward design ensures low implementation complexity and simplifies parameter estimation, making logistic regression a practical and widely used choice for classification tasks. It demonstrates how simplicity and interpretability can coexist, even amidst the growing demand for complex decision-making solutions. However, the accuracy of such model is limited by the linear aggregator embedded within the sigmoid function.

To overcome this limitation and enhance model flexibility, Tehrani et al. (2011) proposed integrating the Choquet integral (Choquet, 1954) as the aggregation function in place of the linear component. The parameters of the Choquet integral, known as capacity coefficients, enable the modeling of interactions between features. While the Choquet integral is a nonlinear function, it can be formulated to remain linear in its parameters, allowing the cost function to closely resemble that of traditional logistic regression. This modification significantly improves the model's capacity to capture feature interactions, enhancing its flexibility without compromising simplicity.

However, the model proposed by Tehrani et al. (2011), known as *Choquistic Regression*, utilizes the Choquet integral parameterized by capacity coefficients, which must adhere to monotonicity constraints and satisfy two boundary conditions. These constraints ensure consistency with the theoretical framework of capacities. However, as noted in the literature (Grabisch, 2016; Pelegrina et al., 2023a), the Choquet integral can alternatively be defined using a cooperative game framework instead of capacities. This reformulation removes the requirement for monotonicity constraints and reduces the boundary conditions to a single requirement. As a result, the parameter learning process is simplified, while the model retains its ability to effectively capture feature interactions. This adaptability underscores the potential of cooperative game theory to enhance the practicality and scalability of Choquistic Regression, particularly in addressing complex decision-making challenges.

In this context, this paper presents a novel formulation of logistic regression that incorporates the Choquet integral[1], with parameters defined using a cooperative game framework. Moreover, we exploit another game-base aggregation function, called multilinear model, which has also the ability to model attribute's interaction. Furthermore, we explore the use of 2-additive games, which significantly reduce the number of parameters to be estimated. This approach yields a more generalized logistic regression model compared to the classical version with a linear aggregator, offering increased flexibility while eliminating the parameter constraints present in the model proposed

[1]It is worth highlighting that a preliminary study has already been conducted in the conference paper (Pelegrina et al., 2023b).

by Tehrani et al. (2011). The 2-additive model can be interpreted as a multilinear framework, where the coefficients quantify the significance of pairwise feature interactions. This approach can be particularly beneficial in applications involving binary features, as interactions between two attributes are represented by their binary multiplication, with the corresponding coefficient modeling scenarios where both features are simultaneously true. For example, in medical diagnosis, this framework effectively captures the combined effect of two co-occurring symptoms, enhancing the model's interpretability and predictive capability. We exemplify the application of our proposal in a COVID-19 detection task, as well as in several other datasets frequently used in the literature.

The remaining sections of this paper are organized as follows. Section 2 discusses the theoretical aspects employed in our approach. In Section 3, we present the proposed game-based formulation for logistic regression, as well as a discussion on model interpretability. Numerical experiments and the obtained results are addressed in Section 4. Finally, we present the concluding remarks and future perspectives in Section 5.

2 Theoretical background

Throughout this paper, we will mainly focus on binary classification problems. We thus consider $\mathbf{X} \in \mathbb{R}^{n \times m}$ as a set of n samples described by m attributes, and $\mathbf{y} \in \{0,1\}^n$ as the associated vector of labels. We represent each sample by $\mathbf{x}_i = [x_{i,1}, x_{i,2}, \dots, x_{i,m}]$ and the associated label y_i, for $i = 1, \dots, n$. In this setting, the goal is to learn a function $g\colon \mathbf{X} \to [0,1]$ such that $g(\mathbf{x}_i) = \hat{y}_i = P(y_i = 1)$ indicates the probability that $\mathbf{x}_i$ (and the associated y_i) belongs to class 1. We then expect $\hat{y}_i$ as close as possible from y_i for all $i = 1, \dots, n$.

In this paper, we propose a game-theoretic reformulation for logistic regression in order to model the relation between inputs and vector of labels. This section presents the theoretical aspects associated with our proposal.

2.1 Logistic regression

The logistic regression is a simple and useful tool for classification problems (Kleinbaum & Klein, 2002). Instead of modeling a linear relation between $\mathbf{X}$ and $\mathbf{y}$ (which is the case of linear regression), the purpose in logistic regression is to assign probabilities the $P(y_i = 1)$ to each sample i by means of the sigmoid function. Mathematically, one has the following:

$$\hat{y}_i = P(y_i = 1) = \sigma(f(\mathbf{x}_i)) = \frac{1}{1 + \exp\left(-f(\mathbf{x}_i)\right)}, \tag{1}$$

where $f(\mathbf{x}_i) = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_m x_{i,m}$, with $\beta_0, \beta_1, \ldots, \beta_m$ being the model's parameters. Note that the greater $f(\mathbf{x}_i)$, the greater the chance of sample $\mathbf{x}_i$ being classified as 1.

In order to adjust the logistic regression parameters, one frequently uses the cross-entropy as a dissimilarity measure. Clearly, in the case of binary classification, if $\hat{y}_i = P(y_i = 1)$ represents the probability of sample $\mathbf{x}_i$ be classified as class 1, $P(y_i =$

$0) = 1 - P(y_i = 1) = 1 - \hat{y}_i$ represents the probability of sample $\mathbf{x}_i$ be classified as class 0. This observation enables the definition of the cross-entropy-based dissimilarity measure as a loss function for the adjustment of the model's parameters, which, in mathematical terms, is given by:

$$\mathcal{L}_{CE}(\hat{y}_i, y_i) = -\left[y_i \log \hat{y}_i + (1-y)\log\left(1-\hat{y}_i\right)\right], \tag{2}$$

which, ideally, achieves the minimum when $\hat{y}_i = y_i$. Therefore, the optimization problem addressed in logistic regression to learn the relation between inputs and the vector of labels can be formalized as follows

$$\min_{\beta_0,\beta_1,\ldots,\beta_k} \sum_{i=1}^{n} -\left[y_i \log \hat{y}_i + (1-y)\log\left(1-\hat{y}_i\right)\right], \tag{3}$$

where $\hat{y}_i$ is the sigmoid function defined in Equation (1).

An advantage of using the logistic regression in classification problems is that the learned model parameters can be used for interpretability and explainability purposes. To illustrate, consider the following "odds" measure:

$$odds = \frac{P(y_i = 1)}{P(y_i = 0)} = \frac{P(y_i = 1)}{1 - P(y_i = 1)} = \exp\left(\beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \ldots + \beta_r x_{i,m}\right).$$

In order to assess the impact a unit increase in the i-th component $x_{i,j}$, we might thus investigate the following ratio:

$$\begin{aligned}\frac{odds_{x_{i,j}+1}}{odds_{x_{i,j}}} &= \frac{\exp\left(\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_j(x_{i,j}+1) + \ldots + \beta_r x_{i,m}\right)}{\exp\left(\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_j x_{i,j} + \ldots + \beta_r x_{i,m}\right)} \\ &= \exp(\beta_j(x_{i,j}+1) - \beta_j x_{i,j}) = \exp(\beta_j).\end{aligned}$$

The interpretation is as follows. An increase of one unit in $x_{i,j}$ changes the odds by a factor of $\exp(\beta_j)$. For example, if the initial odds are equal to 2 (meaning the probability of the sample being classified as class 1 is twice the probability of being classified as class 0) and $\beta_j = 0.92$, an increase of one unit in x_j will multiply the odds by approximately 2.5, changing it to 5.

2.2 Capacity-based aggregation functions

In the next subsections, we define the aggregation functions adopted (after some considerations) in our formulation. Moreover, we also discuss the concept of k-additive games and how the aggregation functions can be defined in the case of binary features.

2.2.1 Choquet integral and Shapley indices

The Choquet integral (Choquet, 1954) consists in a nonlinear aggregation function that models interactions among attributes. It has been widely used in multi-criteria decision making (MCDM) problems (Grabisch, 1996; Grabisch & Labreuche, 2010).

Let $M = \{1, \ldots, m\}$ be a set of m attributes or criteria in MCDM. The Choquet integral is defined by

$$f_{CI}(\mu, \mathbf{x}_i) = \sum_{i=1}^{m} (x_{i,(j)} - x_{i,(j-1)}) \mu(\{(j), \ldots, (m)\}), \tag{4}$$

where $(\cdot)$ indicates a permutation of indices j such that $x_{i,(1)} \leq x_{i,(2)} \leq \ldots \leq x_{i,(m)}$ (with $x_{i,(0)} = 0$) and $\mu : 2^M \rightarrow \mathbb{R}_+$ is a capacity, *i.e.*, a set function that satisfies the following axioms:

- $A \subseteq B \subseteq M \Rightarrow \mu(A) \leq \mu(B) \leq \mu(M)$ (monotonicity),
- $\mu(\emptyset) = 0$ e $\mu(M) = 1$ (boundedness).

An interesting aspect associated with $\mu(A)$ (the capacity coefficients) is that they allow representation in several other domains (Grabisch, 1997a). For instance, there is a linear transformation from the capacity coefficients to the (generalized) Shapley interaction indices, which includes the well known solution concept in cooperative game theory called Shapley value (Shapley, 1953). This transformation is given by

$$I^{\mathcal{S}}(A) = \sum_{B \subseteq M \backslash A} \frac{(m - |B| - |A|)!\, |B|!}{(m - |A| + 1)!} \left(\sum_{B' \subseteq A} (-1)^{|A| - |B'|} \mu(B \cup B') \right), \tag{5}$$

where $|B|$ represents the cardinality of the subset B. From Equation (5), one may obtain the Shapley value associated with attribute j by the following expression:

$$\phi_j^{\mathcal{S}} = \sum_{B \subseteq M \backslash j} \frac{(m - |B| - 1)!\, |B|!}{m!} \left[\mu(B \cup \{j\}) - \mu(B) \right]. \tag{6}$$

Observe that the Shapley value $\phi_j^{\mathcal{S}}$ represents the marginal contribution of attribute j in the aggregation function.

Similarly to the Shapley value, one may also obtain from Equation (5) the interaction effect between pairs of attributes. The Shapley interaction index between attributes j, j' is given by

$$I_{j,j'}^{\mathcal{S}} = \sum_{B \subseteq M \backslash \{j,j'\}} \frac{(m - |B| - 2)!\, |B|!}{(m - 1)!} \left[\mu(B \cup \{j, j'\}) - \mu(B \cup \{j\}) - \mu(B \cup \{j'\}) + \mu(B) \right] \tag{7}$$

and can be interpreted as follows:

- If $I_{j,j'}^{\mathcal{S}} < 0$, we model a negative interaction (or redundant effect) between attributes j, j'.
- If $I_{j,j'}^{\mathcal{S}} > 0$, we model a positive interaction (complementary effect) between attributes j, j'.

- If $I^{\mathcal{S}}_{j,j'} = 0$, no interaction is modeled between attributes j, j' and they act independently.

As the transformation from $\mu(A)$ to the Shapley indices is linear, we can easily recover the capacity coefficients from $I^{\mathcal{S}}(A)$, $A \subseteq M$. This is achieved by the following equation:

$$\mu(A) = \sum_{B \subseteq M} \gamma^{|B|}_{|A \cap B|} I^{\mathcal{S}}(B), \tag{8}$$

where $\gamma^{|B|}_{|A \cap B|}$ is defined by

$$\gamma^{p'}_{p} = \sum_{q=0}^{p} \binom{p}{q} \alpha_{p'-q}, \tag{9}$$

with

$$\alpha_p = -\sum_{p'=0}^{p-1} \frac{\alpha_{p'}}{p - p' + 1} \binom{p}{p'} \tag{10}$$

being the Bernoulli numbers and $\alpha_0 = 1$.

It is worth remarking that the Choquet integral assumes commensurateness between attributes. Indeed, as the evaluations are ordered in the Choquet integral calculation, all attributes should be comparable. In order to deal with this issue, the attributes evaluations are generally normalized so they can be on the same scale.

2.2.2 Multilinear model and Banzhaf indices

Besides the Choquet integral, another capacity-based aggregation function used in MCDM problems is the multilinear model (Owen, 1972; Pelegrina et al., 2020). It is defined as follows:

$$f_{ML}(\mu, \mathbf{x}_i) = \sum_{A \subseteq M} \mu(A) \prod_{j \in A} x_{i,j} \prod_{j \in \overline{A}} (1 - x_{i,j}), \tag{11}$$

where $x_{i,j} \in [0, 1]$ and $\overline{A}$ is the complement set of A.

Similarly as in the Choquet integral, the multilinear model parameters can be transformed to another representation with interpretations for singletons and pairs of attributes. Associated with the multilinear model, one has the Banzhaf interaction indices (Roubens, 1996), which is based on the Banzhaf value (Banzhaf, 1965). The Banzhaf interaction index $I^{\mathcal{B}}(A)$ is defined as follows:

$$I^{\mathcal{B}}(A) = \frac{1}{2^{|M|-|A|}} \sum_{B \subseteq C \setminus A} \sum_{B' \subseteq A} (-1)^{|A|-|B'|} \mu(B' \cup B). \tag{12}$$

By taking a singleton, one obtains the Banzhaf power index, defined by

$$\phi^{\mathcal{B}}_{j} = \frac{1}{2^{|M|-1}} \sum_{B \subseteq C \setminus \{j\}} \left[\mu(B \cup \{j\}) - \mu(B)\right], \tag{13}$$

which can be interpreted as the marginal contribution of attribute j in the aggregation function. In the case of a pair of attributes j, j', one obtains

$$I^{\mathcal{B}}_{j,j'} = \frac{1}{2^{|M|-2}} \sum_{B \subseteq M \setminus \{j,j'\}} \left[\mu(B \cup \{j,j'\}) - \mu(B \cup \{j\}) - \mu(B \cup \{j'\}) + \mu(B) \right]. \quad (14)$$

The interpretation for the interaction effects are the same as for the Shapley interaction indices.

Given the set of the Banzhaf interaction indices, the capacity $\mu(A)$ may be retrieved by the following linear expression:

$$\mu(A) = \sum_{B \subseteq M} \left(\frac{1}{2} \right)^{|B|} (-1)^{|B \setminus A|} I^{\mathcal{B}}(B). \quad (15)$$

By comparing the Choquet integral and the multilinear model, commensurateness is not needed in the latter aggregation function (as we do not compare attributes values). However, the attribute values should be in the range $[0, 1]$.

2.2.3 The concept of k-additive capacities

Note that the parameters $\mu(A)$ (as well as $I^{\mathcal{S}}(A)$ or $I^{\mathcal{B}}(A)$) in both Choquet integral and multilinear model are defined for each coalition $A \subseteq M$. Therefore, the number of parameters, 2^m, increases exponentially with the number of attributes. In scenarios with a large number of attributes, learning this huge number of parameters may be unpractical. Therefore, aiming at reducing the number of parameters while keeping flexibility to model inter attributes relations, one frequently use k-additive capacities (Grabisch, 1997b). A capacity μ is k-additive if $I^{\mathcal{S}}(A) = 0$ (or $I^{\mathcal{B}}(A) = 0$) for all $A \subseteq M$ such that $|A| > k$. For instance, when assuming a 2-additive capacity, the Choquet integral and the multilinear model can be defined as

$$f_{CI}(\mu, \mathbf{x}_i) = \sum_{j=1}^{m} x_{i,j} \left(\phi^{\mathcal{S}}_j - \frac{1}{2} \sum_{\substack{j'=1, \\ j' \neq j}}^{m} I^{\mathcal{S}}_{j,j'} \right) + \sum_{\substack{j,j'=1, \\ j' \neq j}}^{m} (x_{i,j} \wedge x_{i,j'}) \, I^{\mathcal{S}}_{j,j'}, \quad (16)$$

where $\wedge$ indicates the minimum operator, and

$$f_{ML}(\mu, \mathbf{x}_i) = \sum_{j=1}^{m} x_{i,j} \left(\phi^{\mathcal{B}}_j - \frac{1}{2} \sum_{\substack{j'=1, \\ j' \neq j}}^{m} I^{\mathcal{B}}_{j,j'} \right) + \sum_{\substack{j,j'=1, \\ j' \neq j}}^{m} x_{i,j} x_{i,j'} I^{\mathcal{B}}_{j,j'}. \quad (17)$$

respectively (see (Pelegrina et al., 2020) for further details)[2]. In this case, note that one reduces the number of parameters from 2^m to $m(m+1)/2$. In other words, instead

[2]The ordinal counterpart of k-additivity is known as k-maxitivity (Brabant & Couceiro, 2018) and it has been extensively studied for the Sugeno integral (Couceiro & Marichal, 2010; Grabisch et al., 2009) in the

of a number of parameters that increases exponentially with the number of attributes, it now increases polynomially. This significantly reduces the computational cost in the learning procedure. Although the model reduces interactions for coalitions of cardinality greater than 2, it is known from the literature that the aggregation function still flexible to model inter attributes relation (Grabisch et al., 2006; Grabisch et al., 2002; Pelegrina et al., 2020).

2.2.4 A special case with binary features

In some applications, the set of attributes is composed by binary data. For instance, this scenario is common in disease detection, as the attributes may describe the presence ($x_{i,j} = 1$) or absence ($x_{i,j} = 0$) of a set of symptoms. In this context, both the Choquet integral and the multilinear model boil down to very simple aggregation functions. Assume that $\mathbf{x}_{i,\mathbf{1}_A}$ represents a sample such that $x_{i,j} = 1$ if $j \in A$ and 0 otherwise. It is easy to see that, when all attributes are either 0 or 1, the Choquet integral and the multilinear model can be respectively defined by

$$f_{CI}(\mu, \mathbf{x}_{i,\mathbf{1}_A}) = \mu(A) = \sum_{B \subseteq M} \gamma_{|A \cap B|}^{|B|} I^{\mathcal{S}}(B)$$

and

$$f_{ML}(\mu, \mathbf{x}_{i,\mathbf{1}_A}) = \mu(A) = \sum_{B \subseteq M} \left(\frac{1}{2}\right)^{|B|} (-1)^{|B \setminus A|} I^{\mathcal{B}}(B).$$

Observe that, if one assumes a 2-additive capacity, the Choquet integral and the multilinear model are respectively defined by

$$f_{CI}(\mu, \mathbf{x}_{i,\mathbf{1}_A}) = \sum_{j \in A}^{m} \left(\phi_j^{\mathcal{S}} - \frac{1}{2} \sum_{\substack{j'=1, \\ j' \neq j}}^{m} I_{j,j'}^{\mathcal{S}} \right) + \sum_{\{j,j'\} \subseteq A} I_{j,j'}^{\mathcal{S}} \tag{18}$$

and

$$f_{ML}(\mu, \mathbf{x}_{i,\mathbf{1}_A}) = \sum_{j \in A}^{m} \left(\phi_j^{\mathcal{B}} - \frac{1}{2} \sum_{\substack{j'=1, \\ j' \neq j}}^{m} I_{j,j'}^{\mathcal{B}} \right) + \sum_{\{j,j'\} \subseteq A} I_{j,j'}^{\mathcal{B}} \tag{19}$$

2.3 Choquistic regression

The use of the Choquet integral in the logistic regression formulation has already been exploited by Tehrani et al. (2011). The idea was to replace the linear function within the sigmoid function (Equation (1)) by the (capacity-based) Choquet integral. The obtained probability becomes the following:

$$P(y_i = 1) = \frac{1}{1 + \exp\left(-\rho\left(f_{CI}(\mu, \mathbf{x}_i) - \beta_0\right)\right)}, \tag{20}$$

framework of ordinal aggregation, monotonic classification and qualitative preference learning (Brabant et al., 2020; Couceiro et al., 2019).

where ρ and β_0 are constants. An important aspect that should be considered in this reformulation is the "direction" of attribute maximization. Indeed, the monotonicity constraints imply that all attributes are in the same direction of maximization. In other words, if an increase in the value of a specific attribute leads to the sample belonging to class 0, this attribute should be normalized. This normalization ensures that, for all attributes, the higher the attribute value, the higher the likelihood of the sample being classified as belonging to class 1.

Another aspect pointed out by the authors is the presence of several constraints when learning the Choquistic regression parameters. Different from the classical logistic regression model whose optimization problem is unconstrained, in the Choquistic regression proposed by Tehrani et al. (2011) there are several constraints, as the estimated parameters must satisfy both boundedness ($\mu(\emptyset) = 0$ and $\mu(M) = 1$) and monotonicity ($A \subseteq D \subseteq M$, $\mu(A) \leq \mu(D) \leq \mu(M)$) conditions.

This imposes a difficulty when solving such an optimization problem. In order to deal with it, both the k-additive and the heuristic approaches have been used to reduce the number of parameters to be estimated and to be able to tackle the constrained optimization problem (Tehrani, 2021).

3 Proposed framework

In this section, we present reformulation of logistic regression rooted in game theoretic approaches. Firstly, we present the proposed models as well as the optimization problems that we deal with. Thereafter, we provide a discussion on model interpretability.

3.1 Game-theoretic approaches for logistic regression

As mentioned in Grabisch (2016), the Choquet integral can be defined by means of a game υ instead of a capacity μ. Moreover, a recent work (Pelegrina et al., 2023a) has adopted such a formulation to deal with the task of machine learning interpretability. What is particularly interesting in this case is that, by adopting the game υ, the constraints boil down to $\upsilon(\emptyset) = 0$, as the remaining ones are no longer necessary. Indeed, in a game-theoretic reformulation, the value function does not need to be monotone with respect to the inclusion of player coalitions and, hence, the satisfaction of monotonicity constraints is no longer required. Furthermore, for a game the upper bound condition $\mu(M) = 1$ is not necessary since we do not impose any restriction on the value function $\upsilon(M)$. Therefore, the remaining condition $\upsilon(\emptyset) = 0$ can easily be incorporated into the Choquet integral without imposing an additional constraint.

Once we modify the definition of the Choquet integral by adopting game-theoretic ideas, the optimization problem that we deal with in the proposed game-based Choquistic regression is given by

$$\min_{\beta_0, \upsilon(A), A\subseteq M} \sum_{i=1}^{n} -\left[y_i \log \frac{1}{1+\exp\left(-\gamma\left(f_{CI}(\upsilon,\mathbf{x}_i)-\beta_0\right)\right)} \right.$$
$$\left. + (1-y)\log\left(1 - \frac{1}{1+\exp\left(-\gamma\left(f_{CI}(\upsilon,\mathbf{x}_i)-\beta_0\right)\right)}\right)\right], \tag{21}$$

where

$$f_{CI}(\upsilon,\mathbf{x}_i) = \sum_{i=1}^{m}(x_{i,(j)} - x_{i,(j-1)})\upsilon(\{(j),\ldots,(m)\}), \tag{22}$$

where the game $\upsilon : 2^M \to \mathbb{R}$ is such that $\upsilon(\emptyset) = 0$. Surely, instead of $f_{CI}(\upsilon,\mathbf{x}_i)$ one might also consider the 2-additive game-based Choquet integral, given by

$$f_{CI}(\upsilon,\mathbf{x}_i) = \sum_{j=1}^{m} x_{i,j}\left(\phi_j^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{S}}\right) + \sum_{\substack{j,j'=1,\\ j'\neq j}}^{m} \left(x_{i,j}\wedge x_{i,j'}\right) I_{j,j'}^{\mathcal{S}}, \tag{23}$$

where all $\phi_j^{\mathcal{S}}$ and $I_{j,j'}^{\mathcal{S}}$, $j,j' = 1,\ldots,m$, can be calculated from the linear transformations presented in Equations (6) and (7), respectively, by replacing the capacity μ by the game υ.

The same procedure can be used to formulate the game-theoretic variant of the Multilinear logistic regression model, which consists in replacing the linear function within the sigmoid function of Equation (1) by the game-based multilinear model. The optimization problem then becomes the following:

$$\min_{\beta_0, \upsilon(A), A\subseteq M} \sum_{i=1}^{n} -\left[y_i \log \frac{1}{1+\exp\left(-\gamma\left(f_{ML}(\upsilon,\mathbf{x}_i)-\beta_0\right)\right)} \right.$$
$$\left. + (1-y)\log\left(1 - \frac{1}{1+\exp\left(-\gamma\left(f_{ML}(\upsilon,\mathbf{x}_i)-\beta_0\right)\right)}\right)\right], \tag{24}$$

where

$$f_{ML}(\upsilon,\mathbf{x}_i) = \sum_{A\subseteq M}\upsilon(A)\prod_{j\in A} x_{i,j}\prod_{j\in\overline{A}}\left(1-x_{i,j}\right).$$

Also, instead of $f_{ML}(\upsilon,\mathbf{x}_i)$ one may also consider the 2-additive game-based multilinear model, given by

$$f'_{ML}(\upsilon,\mathbf{x}_i) = \sum_{j=1}^{m} x_{i,j}\left(\phi_j^{\mathcal{B}} - \frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{B}}\right) + \sum_{\substack{j,j'=1,\\ j'\neq j}}^{m} x_{i,j}x_{i,j'}I_{j,j'}^{\mathcal{B}}, \tag{25}$$

where all $\phi_j^{\mathcal{B}}$ and $I_{j,j'}^{\mathcal{B}}$, $j, j' = 1, \ldots, m$, can be calculated from the linear transformations presented in Equations (13) and (14), respectively, by replacing the capacity μ by the game υ.

It is worth highlighting the advantages of our proposal in comparison with the existing Choquistic regression model. Removing the constraints from the optimization model by assuming a game instead of a capacity, our proposal becomes simpler to solve when compared to the model proposed by Tehrani et al. (2011). Indeed, as the optimization problems (21) and (24) are unconstrained and the aggregation functions within the sigmoid function are linear in the parameters, solving them is similar to solve the optimization problem (3) for the traditional logistic regression (surely, with more parameters to be learned). This facilitates the learning procedure, as one may exploit the same strategy used for the traditional logistic regression to solve the proposed game-based approaches. Moreover, as the parameters are free to take both positive and negative values, another advantage lies in simplifying the use of the propose models. Indeed, adjusting the direction of attributes maximization is no longer needed.

3.2 Model interpretability

Recall from Section 2.1 that the logistic regression parameters can be interpreted by means of the odds ratio. However, such interpretation in Choquistic or multilinear logistic regression is not straightforward. As in the Choquistic regression commensurateness is to be assumed between attributes, evaluating how an increase of one unit of $x_{i,j}$ changes the odds depends on the comparison between such novel value with the remaining attributes values. Moreover, the normalization procedure frequently adopted to ensure commensurateness also makes this analysis difficult as the scale changes. This is also the case for the multilinear logistic regression, as one needs to re-scale the data in the range $[0, 1]$. Finally, in comparison with the classical logistic regression, game-base aggregation functions have parameters associated with coalitions of attributes. Therefore, in order to evaluate the impact on the odds by increasing $x_{i,j}$, one needs to investigate all parameters associated with attribute j, which includes all coalitions in which such an attribute is included.

Besides this difficulty in interpreting the estimated parameters in any classification problem, this task becomes easier in the case of binary attributes. In this scenario, one may evaluate the impact on the odds when an attribute changes from 0 to 1 (e.g., from the absence of a symptom to the presence of such a symptom). Assume a 2-additive game-based Choquet integral with binary attributes. By changing $x_{i,j^*} = 0$

to $x_{i,j^*}=1$, the odds of $x_{i,\mathbf{1}_{A\cup\{j^*\}}}$ is given by:

$$\begin{aligned}
odds_{x_{i,\mathbf{1}_{A\cup\{j^*\}}}} &= \exp\left(\sum_{j\in\{A\cup\{j^*\}\}}^{m}\left(\phi_j^{\mathcal{S}}-\frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{S}}\right)+\sum_{\{j,j'\}\subseteq\{A\cup\{j^*\}\}} I_{j,j'}^{\mathcal{S}}\right)\\
&= \exp\left(\sum_{j\in A}^{m}\left(\phi_j^{\mathcal{S}}-\frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{S}}\right)+\phi_{j^*}^{\mathcal{S}}-\frac{1}{2}\sum_{\substack{j=1,\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}}\right.\\
&\qquad\qquad\qquad\left.+\sum_{\{j,j'\}\subseteq A} I_{j,j'}^{\mathcal{S}}+\sum_{\substack{j\in A,\\ j\neq j^*}} I_{j^*,j}^{\mathcal{S}}\right).
\end{aligned}$$

As the odds of $x_{i,\mathbf{1}_A}$ is given by

$$odds_{x_{i,\mathbf{1}_A}} = \exp\left(\sum_{j\in A}^{m}\left(\phi_j^{\mathcal{S}}-\frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{S}}\right)+\sum_{\{j,j'\}\subseteq A} I_{j,j'}^{\mathcal{S}}\right),$$

the impact on the odds ration by changing $x_{i,j} = 0$ to $x_{i,j} = 1$ is given as follows:

$$\begin{aligned}
\frac{odds_{x_{i,\mathbf{1}_{A\cup\{j^*\}}}}}{odds_{x_{i,\mathbf{1}_A}}} &= \exp\left(\sum_{j\in A}^{m}\left(\phi_j^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{S}}\right) + \phi_{j^*}^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j=1,\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}} + \sum_{\{j,j'\}\subseteq A} I_{j,j'}^{\mathcal{S}}\right. \\
&\quad \left. + \sum_{\substack{j\in A,\\ j\neq j^*}} I_{j^*,j}^{\mathcal{S}} - \sum_{j\in A}^{m}\left(\phi_j^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j'=1,\\ j'\neq j}}^{m} I_{j,j'}^{\mathcal{S}}\right) + \sum_{\{j,j'\}\subseteq A} I_{j,j'}^{\mathcal{S}}\right) \\
&= \exp\left(\phi_{j^*}^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j=1,\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}} + \sum_{\substack{j\in A,\\ j\neq j^*}} I_{j^*,j}^{\mathcal{S}}\right) \\
&= \exp\left(\phi_{j^*}^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j\in A,\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j\in \bar{A},\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}} + \sum_{\substack{j\in A,\\ j\neq j^*}} I_{j^*,j}^{\mathcal{S}}\right) \\
&= \exp\left(\phi_{j^*}^{\mathcal{S}} + \frac{1}{2}\sum_{\substack{j\in A,\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}} - \frac{1}{2}\sum_{\substack{j\in \bar{A},\\ j\neq j^*}}^{m} I_{j^*,j}^{\mathcal{S}}\right).
\end{aligned}$$

Therefore, the odds changes by a factor which contains (i) the marginal contribution of the included binary attribute j^*, (ii) the (half of) its interaction effects with the others attributes in A and (iii) the negative of (half of) its interaction effects with the others attributes outside A. This same reasoning can be used to derive the interpretability from the game-based 2-additive multilinear model.

4 Experiments and results

In this section, we present experimental results that demonstrate the effectiveness of the proposed methodology compared to logistic regression variants available in the literature. Specifically, we first provide a comparison between classical logistic regression and the proposed game-theoretic approaches based on freely available benchmark datasets. Subsequently, we conduct experiments focused on COVID-19 screening using symptom-based actual data, emphasizing the interpretability and practical applicability of the models.

4.1 Comparing different logistic regression formulations

As a first experiment, we compare the performance of different logistic regression variants using the scikit-learn implementation of *LogisticRegression* in Python. We considered the following models[3]:

- LR: Classical logistic regression,
- CR: Logistic regression based on the game-based Choquet integral,
- CR^{2add}: Logistic regression based on the 2-additive game-based Choquet integral,
- MLR: Logistic regression based on the game-based multilinear model,
- MLR^{2add}: Logistic regression based on the 2-additive game-based multilinear model,

We also considered two solvers available in the scikit-learn implementation, namely, *newton-cg* and *sag*. Moreover, we evaluated the performance of each classifier in several real datasets, that freely available and briefly described in Table 1.

Table 1 Datasets description.

Dataset	**# of samples**	**# of attributes**
BANK: Banknote authentication[4]	1372	4
BLOOD: Blood transfusion[5]	748	4
MAMMO: Mammographic mass[6]	831	4
RAISIN: Raisin (Kecimen and Besni)[7]	900	7
RICE: Rice (Commeo and Osmancik)[8]	3810	7
DIAB: Diabetes PIMA[9]	768	8
SKIN: Skin segmentation dataset[10]	245057	3

For each dataset, we calculated the overall accuracy over 50 simulations. For each simulation, we split the datasets into training (80%) and testing (20%). We also used the over sampling strategy to rebalance the classes before training. The averaged accuracy as well as the 95% confidence interval (using the t Distribution) for both training and test data are presented in Tables 2 and 3, respectively. For each row, the accuracies in bold indicate the higher results which belong to the same confidence interval.

From Table 2 we can see that the unrestricted models (*i.e.,* without the 2-additivity restriction) had better accuracies on the training data for most datasets. This is expected as these models have exponentially more parameters, and thus presenting greater flexibility and ability to fit the training data.

[3]It is worth mentioning that we did not compare the results with the capacity-based logistic regression as our proposal generalizes it and, therefore, is more flexible to adjust the parameters. Moreover, our proposal as well as the classical logistic regression can use the scikit-learn implementation, which is not the case in for the capacity-based logistic regression.

[4]https://archive.ics.uci.edu/dataset/267/banknote+authentication

[5]https://www.kaggle.com/datasets/whenamancodes/blood-transfusion-dataset

[6]https://archive.ics.uci.edu/dataset/161/mammographic+mass

[7]https://archive.ics.uci.edu/dataset/850/raisin

[8]https://www.kaggle.com/datasets/muratkokludataset/rice-dataset-commeo-and-osmancik

[9]https://www.kaggle.com/datasets/uciml/pima-indians-diabetes-database

[10]https://archive.ics.uci.edu/dataset/229/skin+segmentation

Table 2 Overall accuracy and 95% confidence interval (both in %) in training data.

Dataset	Solver	LR	CR	CR^{2add}	MLR	MLR^{2add}
BANK	*newton-cg*	99.03 [98.97, 99.09]	**100** -	**100** -	**100** -	**100** -
	sag	99.02 [98.97, 99.07]	**100** -	**100** -	**100** -	**100** -
BLOOD	*newton-cg*	**69.94** [69.72, 70.16]	**69.59** [69.36, 69.82]	69.50 [69.27, 69.73]	**70.04** [69.81, 70.27]	**69.68** [69.48, 69.88]
	sag	**69.94** [69.72, 70.06]	69.56 [69.33, 69.79]	69.50 [69.27, 69.73]	**70.13** [69.91, 70.35]	69.65 [69.44, 69.86]
MAMMO	*newton-cg*	80.96 [80.74, 81.18]	**81.68** [81.43, 81.93]	**81.68** [81.45, 81.91]	80.93 80.69, 81.17	80.96 [80.72, 81.20]
	sag	80.96 [80.74, 81.08]	**81.69** [81.45, 81.93]	**81.68** [81.45, 81.91]	80.93 [80.70, 81.16]	80.96 [80.72, 81.20]
RAISIN	*newton-cg*	85.98 [85.77, 86.19]	88.57 [88.41, 88, 73]	88.26 [88.09, 88.43]	**91.16** [91.01, 91.31]	88.67 [88.51, 88.83]
	sag	86.12 [85.92, 86.32]	**88.34** [88.18, 88.50]	**88.02** [87.85, 88.19]	**88.04** [87.89, 88.19]	87.59 [87.43, 87.75]
RICE	*newton-cg*	92.79 [92.73, 92.85]	93.06 [93.00, 93.12]	92.86 [92.80, 92.90]	**93.45** [93.39, 93.51]	93.01 [92.96, 93.06]
	sag	92.81 [92.75, 92.87]	**93.00** [92.94, 93.06]	92.82 [92.76, 92.88]	92.85 [82.80, 92.90]	92.76 [92.70, 92.82]
DIAB	*newton-cg*	74.71 [74.48, 74.94]	82.18 [81.83, 82.53]	78.48 [78.15, 78.81]	**94.45** [94.12, 94.78]	77.49 [77.27, 77.71]
	sag	74.70 [74.47, 74.93]	**81.74** [81.44, 82.04]	78.50 [78.18, 78.82]	**81.98** [81.68, 82.28]	77.48 [77.26, 77.70]
SKIN	*newton-cg*	94.03 [94.02, 94.04]	**96.84** [96.83, 96.85]	**96.84** [96.83, 96, 85]	96.39 [96.38, 96.40]	96.02 [96.00, 96.04]
	sag	94.03 [94.02, 94.04]	**96.84** [96.83, 96.85]	**96.84** [96.83, 96.85]	96.39 [96.38, 96.40]	96.02 [96.00, 96.04]

For the test data the results (see Table 3) were quite varied and less conclusive. Good results were expected for the 2-additive variants of logistic regression models due to the reduction in the number of parameters, which reduces overfitting and thus enables greater generalization. Indeed, by considering the confidence interval, the 2-additive variants as well as the unrestricted models show similar performances. However, by looking at the overall accuracies, in some cases the unrestricted model performed better than the 2-additive ones. It is worth remarking that there is a considerable complexity gain in the latter due to the substantial reduction in the number of parameters to learn.

Moreover, as we will see in next subsection on real-world data, the 2-additive version of the game-theoretic approaches to logistic regression not only provides competitive results compared to logistic regression, but also enhances the interpretation of results.

4.2 Application on a Real-World COVID-19 Dataset

The dataset used in this application was provided by the "Dados do Bem" app, a large-scale Brazilian initiative that integrates an app-based symptom tracker with a public testing program (Dantas et al., 2021). Upon registration, all users provided informed consent for the use of de-identified data in non-commercial research. All data were anonymized in compliance with the Brazilian General Data Protection Regulation (LGPD) and are publicly available (Dantas & et al., 2020). Between April 28, 2020, and July 16, 2020, a total of 337,435 individuals reported their symptoms through

Table 3 Overall accuracy and 95% confidence interval (both in %) in test data.

Dataset	Solver	LR	CR	CR^{2add}	MLR	MLR^{2add}
BANK	*newton-cg*	98.84 [98.72, 98.96]	99.84 [99.78, 99.90]	99.97 [99.95, 99.99]	**100** **-**	**100** **-**
	sag	98.88 [98.75, 99.01]	99.85 [99.80, 99.90]	**100** **-**	**100** **-**	**100** **-**
BLOOD	*newton-cg*	**70.17** **[69.42, 72.92]**	**69.01** **[68.22, 69.80]**	**69.00** **[68.21, 69.79]**	**69.81** **[69.13, 70.49]**	**69.89** **[69.19, 70.59]**
	sag	**70.17** **[69.42, 70.92]**	**68.96** **[68.18, 69.74]**	**69.03** **[68.25, 69.81]**	**70.24** **[69.52, 70.96]**	**69.89** **[69.19, 70.59]**
MAMMO	*newton-cg*	**80.53** **[79.68, 81.38]**	**80.24** **[79.39, 81.09]**	**80.58** **[79.68, 81.48]**	**79.76** **[78.87, 80.65]**	**80.26** **[79.42, 81.10]**
	sag	**80.53** **[79.68, 81.38]**	**80.24** **[79.39, 81.09]**	**80.58** **[79.68, 81.48]**	**79.79** **[78.90, 80.68]**	**80.24** **[79.40, 81.08]**
RAISIN	*newton-cg*	85.60 [84.96, 86.24]	**86.01** **[85.41, 86.61]**	**86.82** **[86.24, 87.40]**	85.44 [84.92, 85.96]	**86.98** **[86.46, 87.50]**
	sag	**85.83** **[85.19, 86.48]**	**86.60** **[86.00, 87.20]**	**87.03** **[86.40, 87.66]**	**87.08** **[86.47, 87.69]**	**86.83** **[86.23, 87.43]**
RICE	*newton-cg*	**92.67** **[92.48, 92.86]**	**92.48** **[92.28, 92.68]**	**92.58** **[92.39, 92.77]**	**92.52** **[92.30, 92.74]**	**92.73** **[92.53, 92.93]**
	sag	**92.67** **[92.48, 92.86]**	**92.55** **[92.35, 92.75]**	**92.58** **[92.39, 92.77]**	**92.62** **[92.42, 92.82]**	**92.61** **[92.41, 92.81]**
DIAB	*newton-cg*	74.44 [73.55, 75.33]	**75.36** **[74.61, 76.11]**	**75.10** **[74.26, 75.94]**	**76.38** **[75.53, 77.23]**	74.50 [73.57, 75.43]
	sag	74.43 [73.54, 73.32]	**75.44** **[74.68, 76.20]**	**75.27** **[74.40, 76.14]**	**75.28** **[74.37, 76.19]**	**74.51** **[73.58, 75.44]**
SKIN	*newton-cg*	94.02 [93.99, 94.05]	**96.84** 96.83, 96.85	**96.84** **[96.83, 96.85]**	96.39 [96.37, 96.41]	96.02 [96.00, 96.04]
	sag	94.02 [93.99, 94.05]	**96.84** **[96.83, 96.85]**	**96.84** **[96.83, 96.85]**	96.39 [96.37, 96.41]	96.02 [96.00, 96.04]

the app. Among these, 64,175 users underwent testing; however, 132 had inconclusive test results, and 12,264 did not report any symptoms. These cases were excluded from the analysis. The analysis perfomed in this paper includes only symptomatic patients with conclusive test results (positive or negative), resulting in a total of $N = 51,813$ samples, of which 8,111 (15.6%) tested positive. The antibody test used in the study, the Wondfo COVID-19 IgM/IgG test, had a sensitivity of 86.43% and a specificity of 99.57%.

The objective of this application is to develop a simple and interpretable screening model for COVID-19, designed for practical application in clinical settings. To achieve this, we selected 9 features from the original dataset based on expert consultation and variable selection techniques. These variables are binary indicators for sore throat, runny nose, diarrhea, myalgia, nausea, cough, shortness of breath, fever (> 38°C), and loss of smell. The target variable is a binary outcome indicating whether an individual tested positive or negative for COVID-19.

Our model advances beyond the study of (Dantas et al., 2021) by incorporating the interactions between symptoms, a critical aspect in the field of healthcare. Symptom interactions often play a significant role in diagnosing and understanding the progression of diseases, as the simultaneous presence of certain symptoms can indicate more severe or distinct conditions. By modeling these interactions explicitly, our method enhances both the predictive power and interpretability of the screening model. This improvement is particularly valuable in clinical decision-making, where

Table 4 Overall accuracy and 95% confidence interval (both in %) for COVID-19 dataset.

	Solver	LR	CR^{2add}	MLR^{2add}
Training data	*newton-cg*	68.57 [68.54, 68.60]	**68.92 [68.89, 68.95]**	**68.92 [68.89, 68.95]**
	sag	68.57 [68.54, 68.60]	**68.92 [68.89, 68.95]**	**68.92 [68.89, 68.95]**
Test data	*newton-cg*	68.53 [68.43, 68.63]	**68.84 [68.75, 68.93]**	**68.84 [68.75, 68.93]**
	sag	68.53 [68.43, 68.63]	**68.84 [68.75, 68.93]**	**68.84 [68.75, 68.93]**

understanding the interplay of symptoms can guide more accurate diagnoses and targeted interventions.

4.3 Results

The overall accuracy over 50 simulations as well as the standard deviation for both training (80%) and test (20%) data are presented in Table 4. As in the previous experiments, we also used the over sampling strategy to rebalance the positive and negative cases before training.

Aiming at interpreting the estimated parameters, let us consider the game-based 2-additive Choquistic regression and the *newton-cg* solver. The Shapley values ϕ_j^S for each attribute j are presented in Figure 1. As we can see, the symptoms with higher impact on classifying patients as having COVID-19 are loss of smell, shortness of breath and fever. On the other hand, symptoms as runny nose, sore throat and diarrhea are pointed as important to classify as not having COVID-19. These results are in accordance with the literature on COVID-19 symptoms for the first variant. For instance, as pointed out by Rodrigues et al. (2022), fever, cough and loss of smell were among the most common symptoms in the first variant, while less common were sore throat and diarrhea (among others).

Besides the marginal contributions, one may also investigate how symptoms interacts towards the COVID-19 detection. Figure 2 presents the obtained Shapley interaction indices among symptoms. Note that loss of smell is positively correlated with runny nose and shortness of breath. One may interpret this results as a patient having such symptoms, simultaneously, increases the chance to be classified as having COVID-19. Loss of smell and fever are negatively correlated, which suggests that both symptoms are common in patients with COVID-19. An interesting finding is noted between nausea and both cough and shortness of breath. Although the impact of nausea into the COVID-19 classification is not high, in combination with cough and shortness of breath, this symptoms helps to detect the disease (due to the complementary effect). This was also reported in case studies (Groff et al., 2021).

As a last analysis, one may also investigate the odds ratio by the inclusion of a symptom. Suppose a patient $\mathbf{x}_{i,\mathbf{1}_A}$ with $A = \{\text{fever}, \text{ myalgia}, \text{ nausea}\}$. The associated probability $P(y_i = 1) \approx 0.5836$, which leads to an odds ratio of

$$odds_{x_{i,\mathbf{1}_A}} = 0.5836/0.4164 \approx 1.40.$$

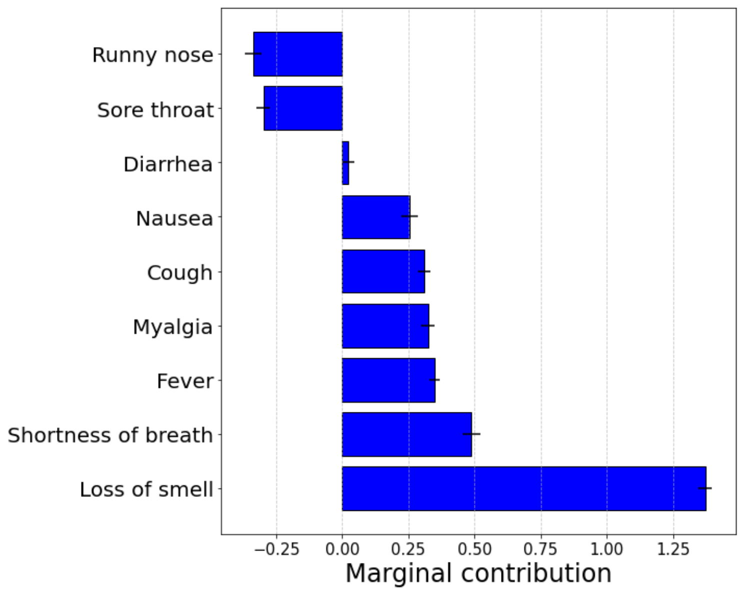

Fig. 1 Marginal contributions (ϕ_j^S) and standard deviations of symptoms in COVID-19 dataset.

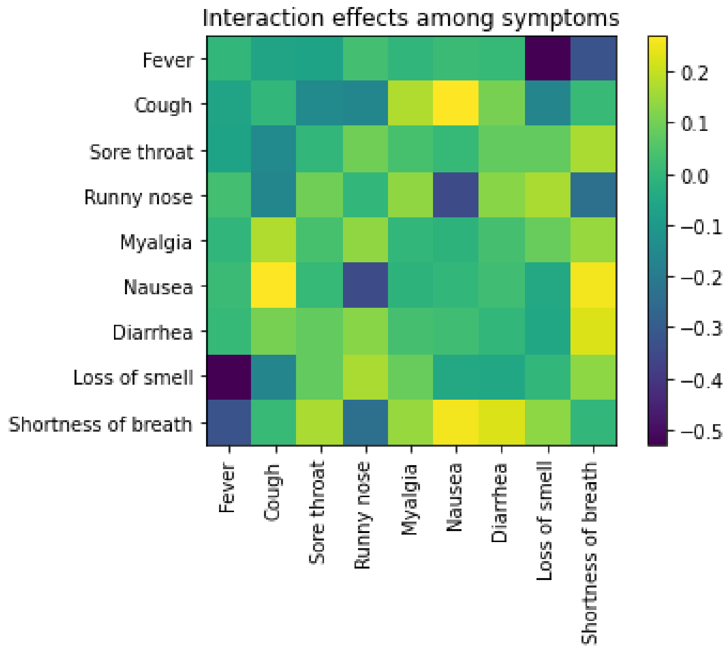

Fig. 2 Interaction effects ($I_{j,j'}^S$) among symptoms in COVID-19 dataset.

If this patient has also loss of smell, the impact on the odds will be the following:

$$
\begin{aligned}
\frac{odds_{x_{i,\mathbf{1}_{A\cup\{\text{loss of smell}\}}}}}{odds_{x_{i,\mathbf{1}_A}}} &= \exp\Bigg(1.330 + \frac{1}{2}\left(-0.524 + 0.043 - 0.101\right) \\
&\qquad - \frac{1}{2}\left(-0.126 + 0.059 + 0.201 - 0.096 + 0.129\right)\Bigg) \\
&= \exp\left(0.955\right) \approx 2.60.
\end{aligned}
$$

This means that the inclusion of loss of smell in this patient increases the odds by a factor of 2.60, i.e., the novel odds will be $1.40 * 2.60 = 3.64$. The chance that this patient has COVID-19 will be approximately 3.64 times the chance that he/she will not have such a disease (instead of 1.4 times if he/she has not loss of smell).

This type of analysis is highly valuable in medical practice, as it quantifies the impact of individual and/or combined symptoms on the likelihood of a diagnosis. Such information helps prioritize patients, improving triage accuracy and optimizing resource allocation. In the context of COVID-19, these analyses support rapid, data-driven decisions, enabling the effective identification of high-risk patients and tailored clinical interventions.

5 Conclusion and Future work

In this paper, we explore several extensions and variants of logistic regression by replacing the linear aggregator in the sigmoid function with alternative aggregation functions and transitioning from a capacity-based framework to one rooted in game theory. This shift allowed us to leverage the advantages of the previously proposed "Choquistist" regression framework while addressing its primary limitations, such as the curse of dimensionality and the challenges of monotonicity constraints. By incorporating game-theoretic concepts, our methodology improves the flexibility and interpretability of logistic regression, making it better suited for real-world applications.

To mitigate the computational complexity associated with the Choquet integral, we introduced simplified variants, such as the 2-additive Choquet integral, which restricts interactions to pairs of features. This approach not only significantly reduces the number of parameters but also retains competitive performance compared to unrestricted models. Additionally, we explored the multilinear model as an alternative aggregation function, further demonstrating the utility of game-based frameworks in capturing feature interactions effectively.

We validated our approach through a series of experiments on both benchmark datasets and real-world data, including a large and practical COVID-19 dataset. These experiments demonstrated that the proposed methodology achieves competitive performance compared to classical logistic regression in terms of accuracy while maintaining a high level of interpretability. The reduced number of attributes, combined with the clear interpretability of feature interactions, enhances the model's suitability for practical applications, particularly in resource-constrained environments where rapid and reliable decision-making tools are essential.

As a future perspective, it is possible to refine the proposed models by incorporating regularization techniques into the optimization process to prevent overfitting, particularly for the 2-additive variants. Tailored regularization strategies for parameters associated with different cardinalities may also be worth exploring, as such approaches could balance model complexity. Another promising direction is the extension of our framework to multiclass classification problems, which broadens its applicability to more complex scenarios. Additionally, integrating feature selection techniques could improve model efficiency, especially for high-dimensional datasets. These enhancements would not only strengthen the proposed methodology but also expand its practical relevance across diverse domains where feature interactions are critical.

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Declarations

The authors have no competing interests to declare that are relevant to the content of this article.

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