Advanced Methods in Applied Statistics: Assignment 2

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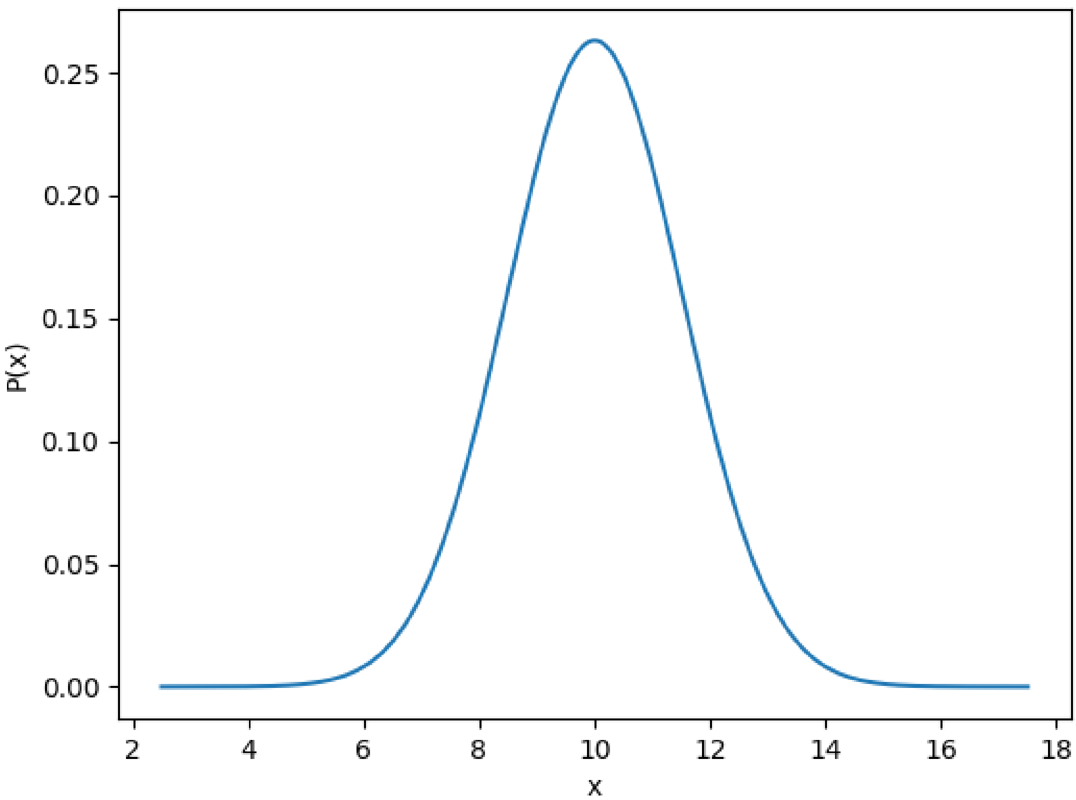
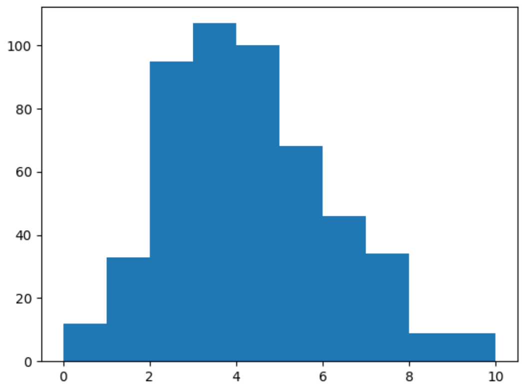
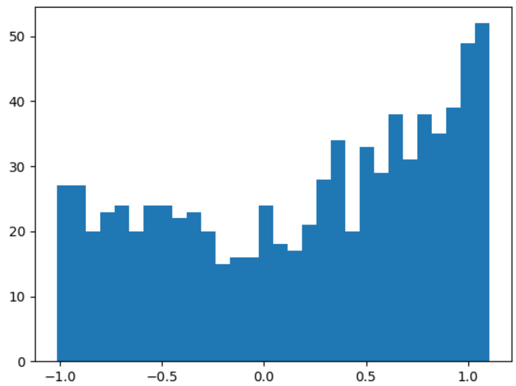


1



(a) Poisson



(b) Polynomial

2

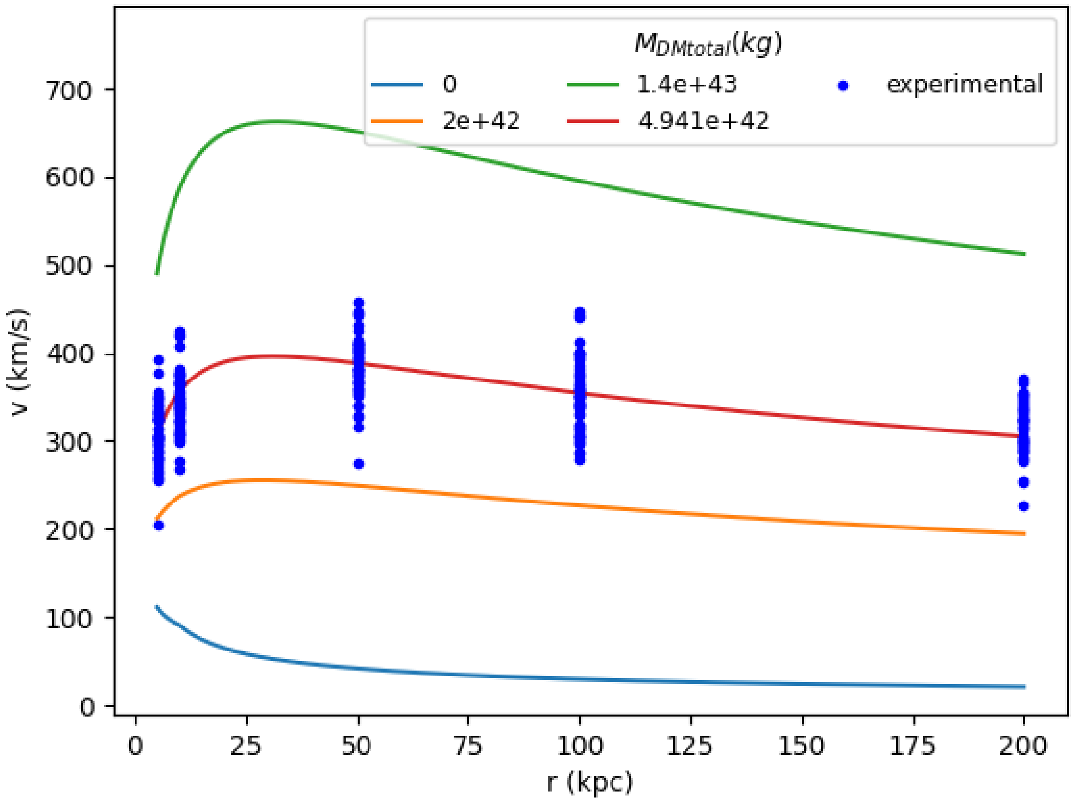


Figure 1: Experimental observations and hipotesis testing with different masses

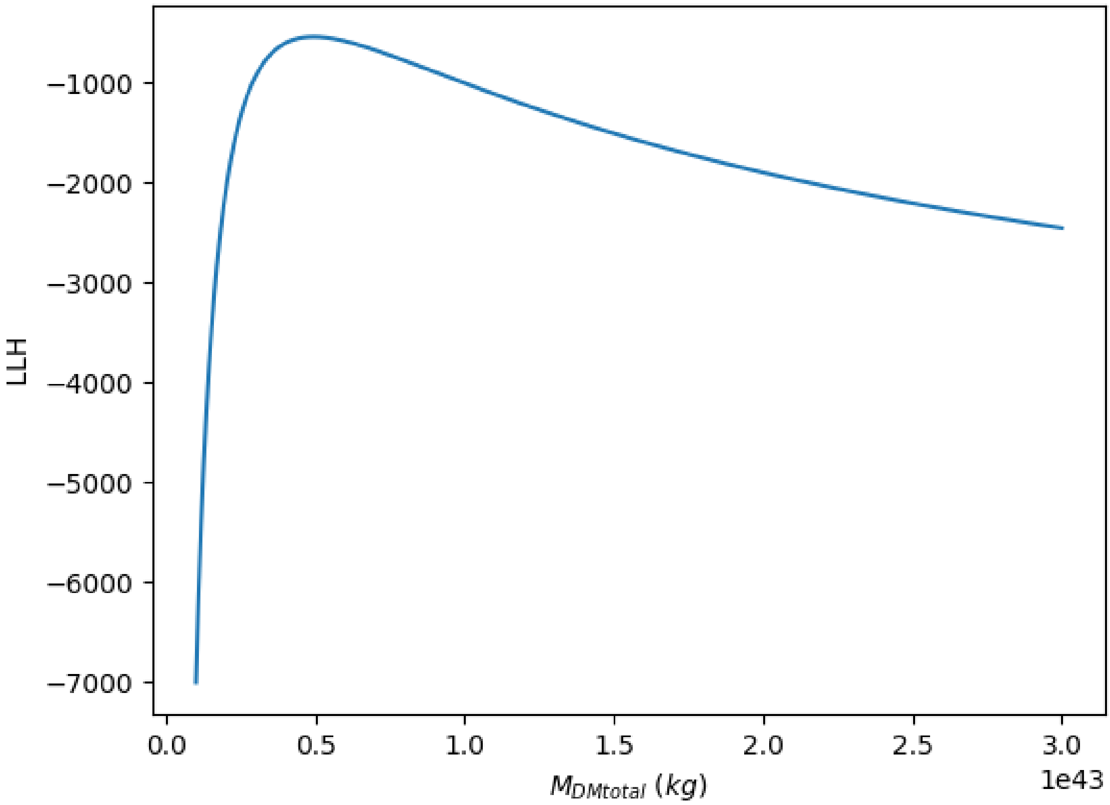


Figure 2: LLH sampling for possible values of $M_{DMtotal}$

I found the minimum of the LLH function to be at $M_{DMtot} = 4.941 \cdot 10^{42}\ kg$, so assuming that indeed the points follow the gaussian distribution described in the assignment this is the estimation that best fits the data. Moreover the product of gaussian distributions also has gaussian shape. Hence we can without making use of any approximation we can define σ as the distance at which LLH has fallen to -0.5 its value. We found $\sigma = 6.5 \cdot 10^{40}$, $\quad 2\sigma = 1.3 \cdot 10^{41}$, $\quad 3\sigma = 19.5 \cdot 10^{40}$.[1]

[1] My minimizers didn't work so I had to do it by visual inspecting the plot

3

3.1

From Bayes theorem:

$$P(A_i|D) = \frac{P(D|A_i)P(A_i)}{P(D)} = \frac{P(D|A_i)P(A_i)}{\sum_j P(D|A_j)P(A_j)}$$

We can hence calculate the probabilities for a product to come for any factory:

Factory	$P(A_i\|D)$
0	0.213740
1	0.183206
2	0.152672
3	0.213740
4	0.236641

Table 1: Probabilities of defective procedence

3.2

I ran a MCMC algorithm that sthocastically found the best configuration of defect rates that satisfied both similarity condition lowest value condition.

previous	new	u(new)
2.0	2.19	0.12
4.0	5.15	0.32
10.0	15.5	1.0
3.5	3.9	0.2
3.1	3.211	0.099

Table 2: Deective changes for the case of 5 factories in %

3.3

The similar algorithm can be used for the case of 14 samples:

previous	new	u(new)
0.020	0.0202	0.0074
0.040	0.05	0.19
0.100	0.10	0.21
0.035	0.06	0.27
0.022	0.0221	0.0090
0.092	0.15	0.60
0.120	0.3	1.2
0.070	0.0700	0.0053
0.110	0.15	0.59
0.020	0.3	1.0
0.070	0.3	1.4
0.060	0.23	0.92
0.099	0.34	1.4
0.082	0.6	2.8

Table 3: Deective changes for the case of 5 factories in fraction