Complex physics: Midterm exam

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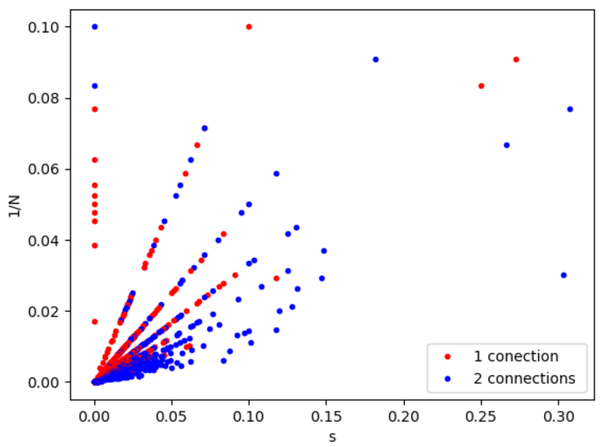
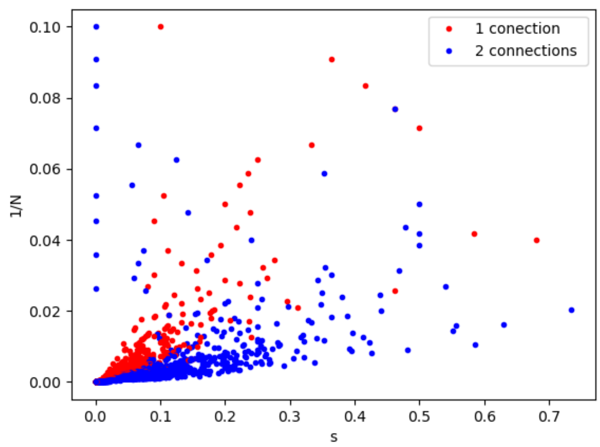
1 Percolation

1.1 and 1.2



(a) p=0.3



(b) p=0.7

We can see that for d=1 or d=2 when N $\rightarrow \infty$ we recover the thermodyamic limit and the system tends to the origin of coordinates so $s \rightarrow 0$. This is independent of p.

What is dependent of p though is the average size of the cluster. It is easy to see in the graphs that the sizes are much lower for d=1.

Additionally, my intuition for d=2 was that when we introduce additional connections such as in this case, more paths are likely to be opened, hence more possibilities of of percolation. Consequently the critical probability p_c shall be lowered. Indeed we can clearly see in both graphs that for same sizes, in general the size of the cluster is bigger for d=2 than for d=1 (the points are further in x axis).This is actually the idea behind renormalization. We "zoom out" the system so the correlation length increases and the system tends to its extreme version.

1.3

If we now introduce random connections we obtain the following results.

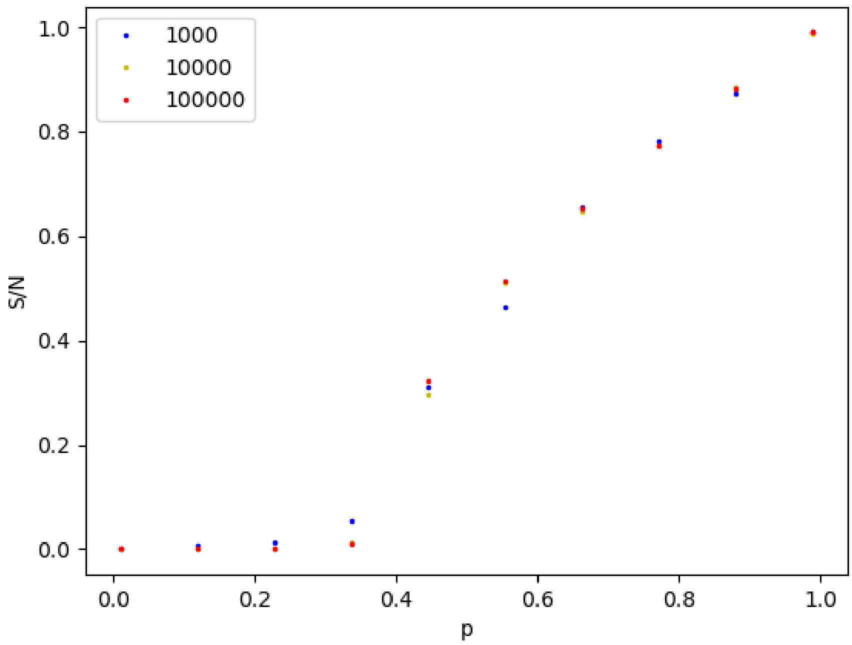


Figure 1: Percolation for random connections

There seems to be a critical probability below which the size of the cluster is almost 0.

2 Ising model

2.1

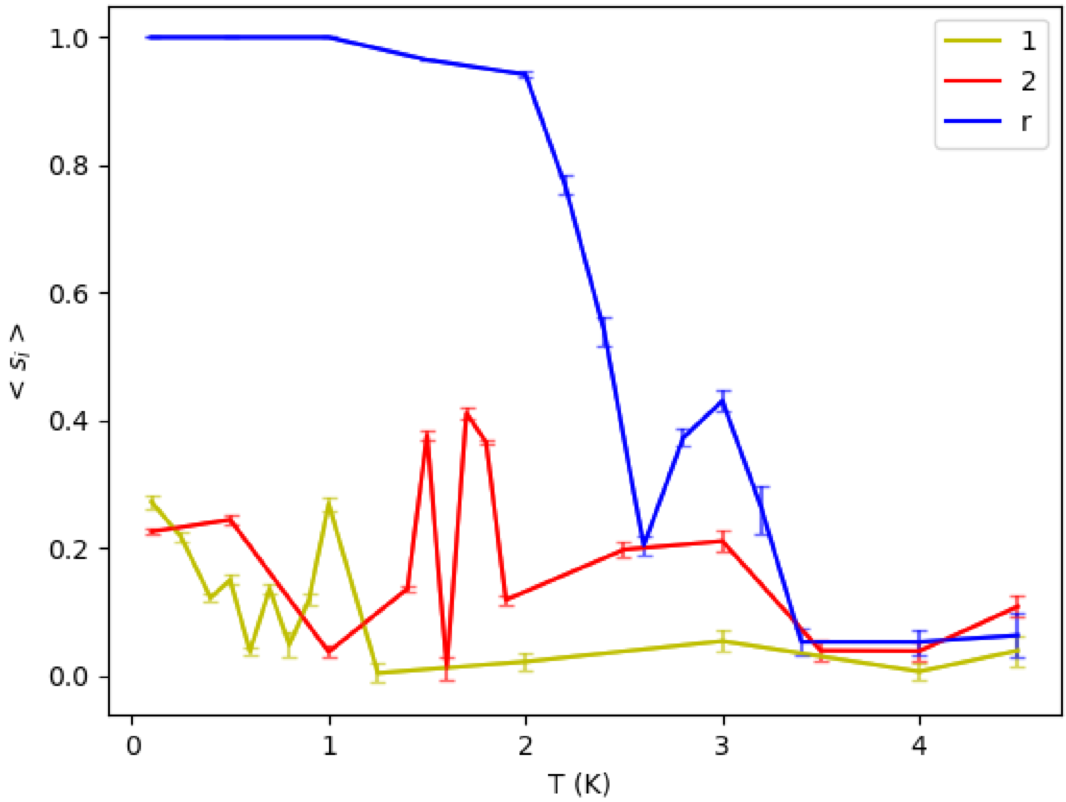


Figure 2: 1D Ising model for 1, 2 and random connections

We don't see a critical temperature for D=1 or D=2. The results fluctuate around the same average (near to zero) independently of T. For random connections we do though.

2.2

For d=2 the correlation is present and we don't see any phase transition (as happens in d=1 case). However, when we introduce random connections and we increase the number of sites we clearly see a phase transition as in the D>1 cases. Indeed, when $N \to \infty$ is increasing, random edges are likely to connect totally uncorrelated nodes as happens in the 2D case. Therefore we recover the latter and a phase transition is possible.

2.3

The mean field ising model solution predicts that $T_c = \frac{Jz}{kb}$. This is quite consistent with the simulation results for the first two cases where $T_c = z = 1$ and $T_c = z = 2$ respectively. For the random connections case the mean field solution doesn't hold. This is because the key assumption in m.f.s is that the correlation between nearest neighbours can be neglected but in the case of random connections we are relating sites that are surrounding so this assumption is no longer correct.

2.4

We first compute $F_{ground} = -J \sum_{<ij>} s_i s_j - k_B T \log \Omega$ We have the same number of bonds than in 1D chain and the number of states is still all up or all down son $F_{ground} = -2JN - k_B T \log 2$ If we now add a domain wall let's take first a look at the energy. The double connection makes that one domain wall affects 2nd nearest neighbours also. Hence, one domain wall flips at minimum three bonds. If we also take into account the fact that the chain is cyclic, additional bond flips would appear (minimum +2 maximum +3). So in total we have $E_{max} = -J \sum_{<ij>} s_i s_j = -2JN - (-4) + 4 = -2J(N-4)$ and $E_{min} = -2J(N-3)$ The degeneracy depends on when we put the walls (N possibilities) and the cyclic effect (N-1 possibilities). It is therefore $\Omega = N(N-1)$

Computing $\Delta F = F_{ground} - F_{wall} = -6J + k_b T(\log(N(N-1)) - \log(2))$ which is higher than 0 for $N \to \infty$. We recover the case of the 1 D chain where there is no phase transition possible for T>0