Discrete random walks with memory: Models and applications

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Outline

1. Prepare mathematical model
2. Describe its properties
3. Apply it on data

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Random walk

Definition

A man starts from a point O and walks l yards in a straight line; he then turns through any angle whatever and walks another l yards in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $r + \delta r$ from his starting point, O.

[Karl Pearson: The problem of the random walk. (1905)]

Where is the "Drunken sailor"?

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Let $\{X_k\}_{k=1}^{\infty}$ be a sequence of independent, identically distributed discrete random variables. For each positive integer n, let S_n denote the sum $X_1 + X_2 + \cdots + X_n$, with $S_0 = 0$. The sequence $\{S_n\}_{n=1}^{\infty}$ is called a random walk. If the common range of the X_k's is $\mathbb{R}_m$, then $\{S_n\}$ is a random walk in $\mathbb{R}_m$.

For $X_k \sim B(p = \frac{1}{2})$ it is called the standard random walk.

Random walk properties

- Discrete random process
- $n-$dimensional, on a matrix, graph, finite or infinite set
- Self avoiding, reinforced
- Brownian motion, polymer creation, games simulation, sports simulation

Random walk with memory

- Based on standard random walk (Bernoulli distribution with $p = 0.5$, discrete time).
- Constant total step size:

$$l_i^+ + l_i^- = 2 \; \forall i \in \mathbb{N}.$$

- At the beginning the step sizes are equal ($l_1^+ = l_1^- = 1$) and further for $t > 1$ evolve using a memory parameter $\lambda \in (0, 1)$:

$$X_{t-1} = 1 \rightarrow \begin{cases} l_t^+ = \lambda l_{t-1}^+ \\ l_t^- = 2 - \lambda l_{t-1}^+ \end{cases} \qquad X_{t-1} = -1 \rightarrow \begin{cases} l_t^+ = 2 - \lambda l_{t-1}^- \\ l_t^- = \lambda l_{t-1}^- \end{cases}$$

- Loïc Turban. *On a random walk with memory and its relation with markovian processes.* Journal of Physics A: Mathematical and Theoretical (2010).

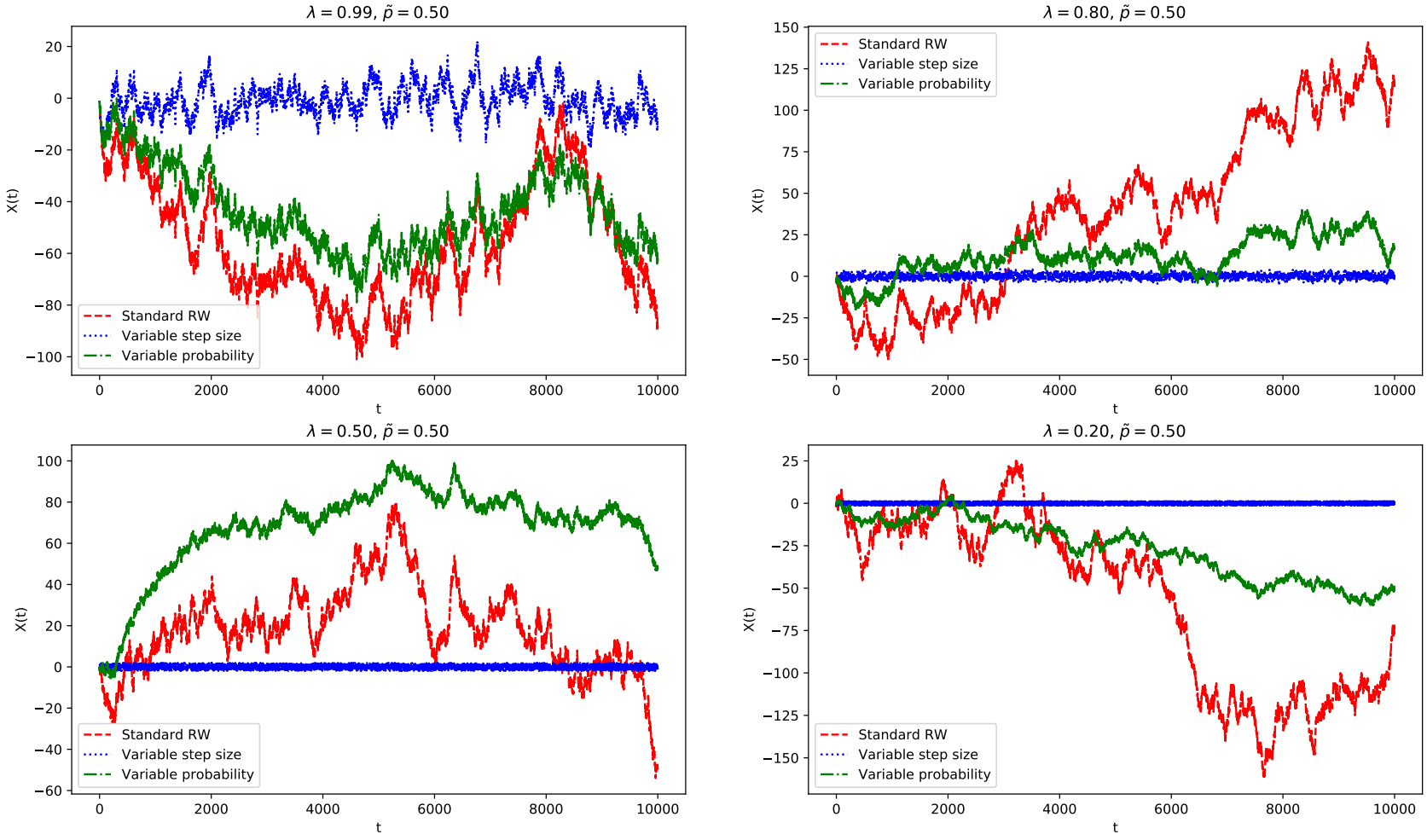
Random walk with varying transition probability

- Based on standard random walk (Bernoulli distribution with $p = 0.5$, discrete time).
- Step size remains constant, transition probability changes
- First step realized according to starting probability p_0 which then for $t > 1$ evolve using a memory parameter $\lambda \in (0, 1)$:

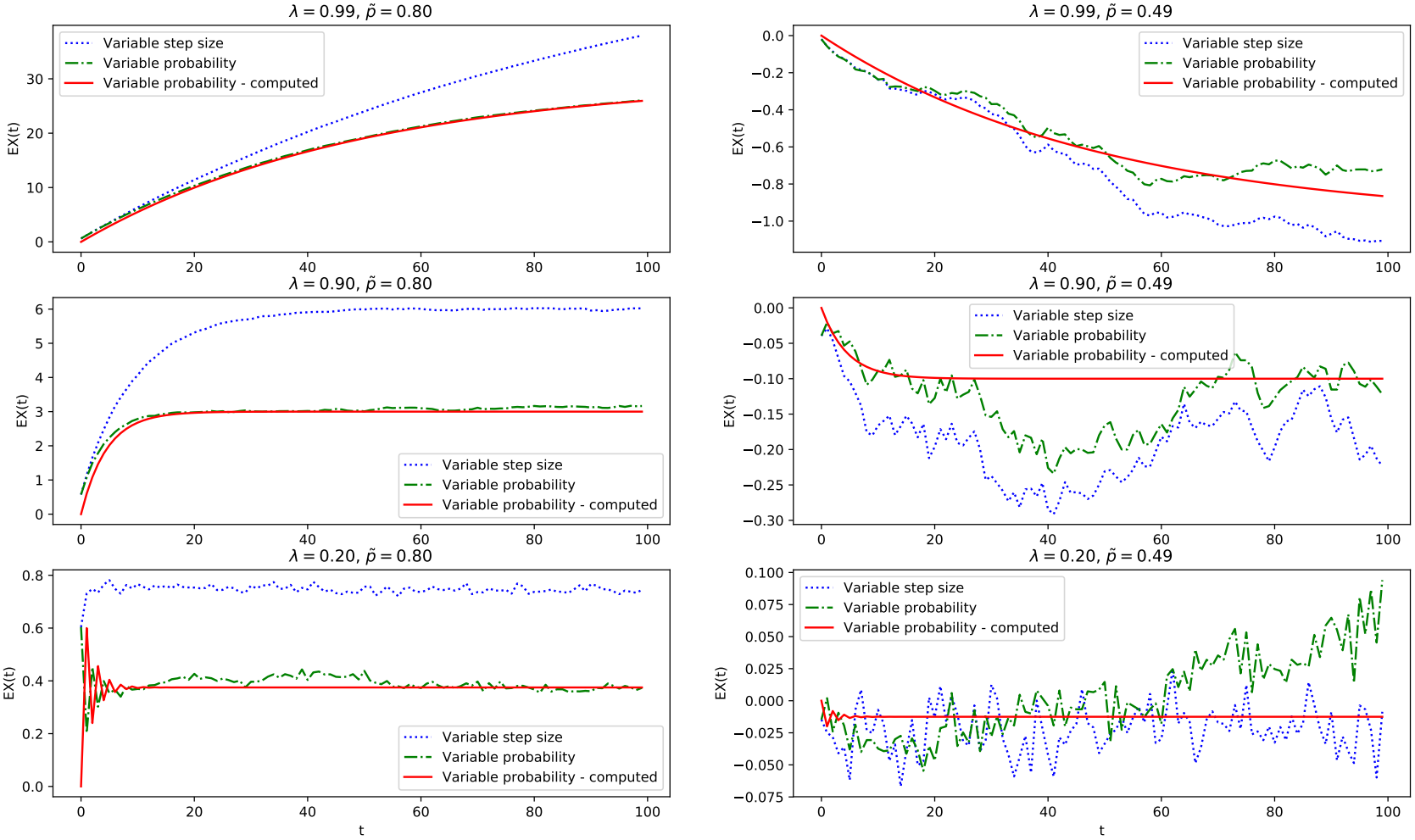
$$X_t = 1 \rightarrow p_t = \lambda p_{t-1}$$

$$X_t = -1 \rightarrow p_t = 1 - \lambda(1 - p_{t-1})$$

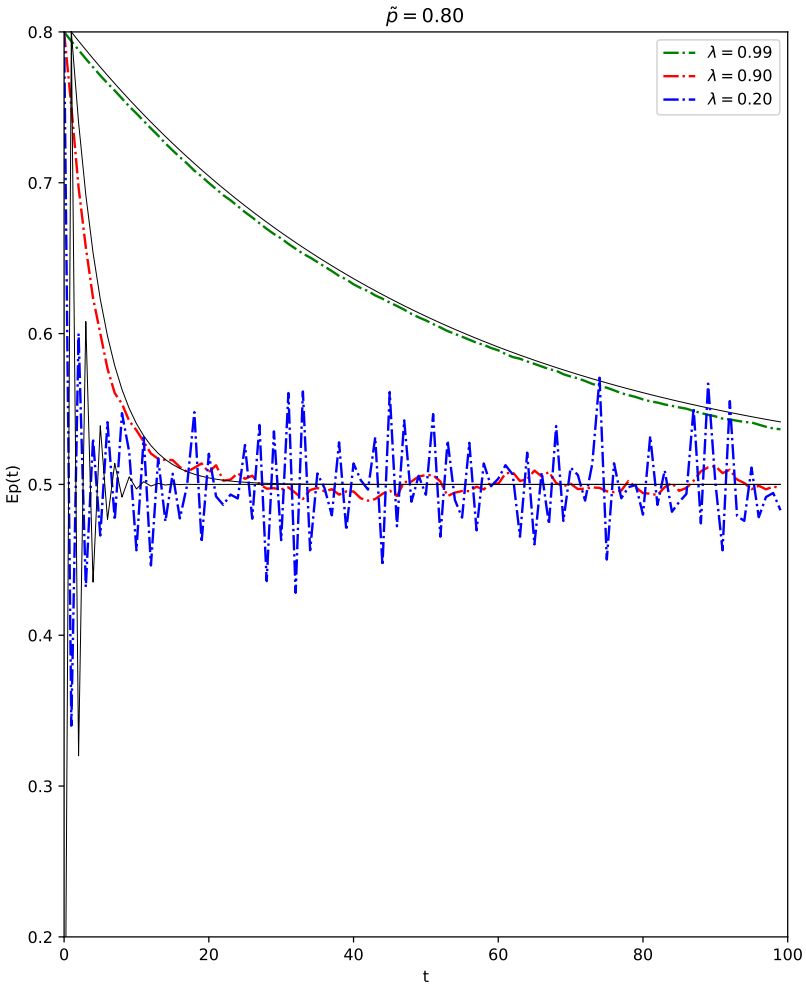
Example - RW evolution

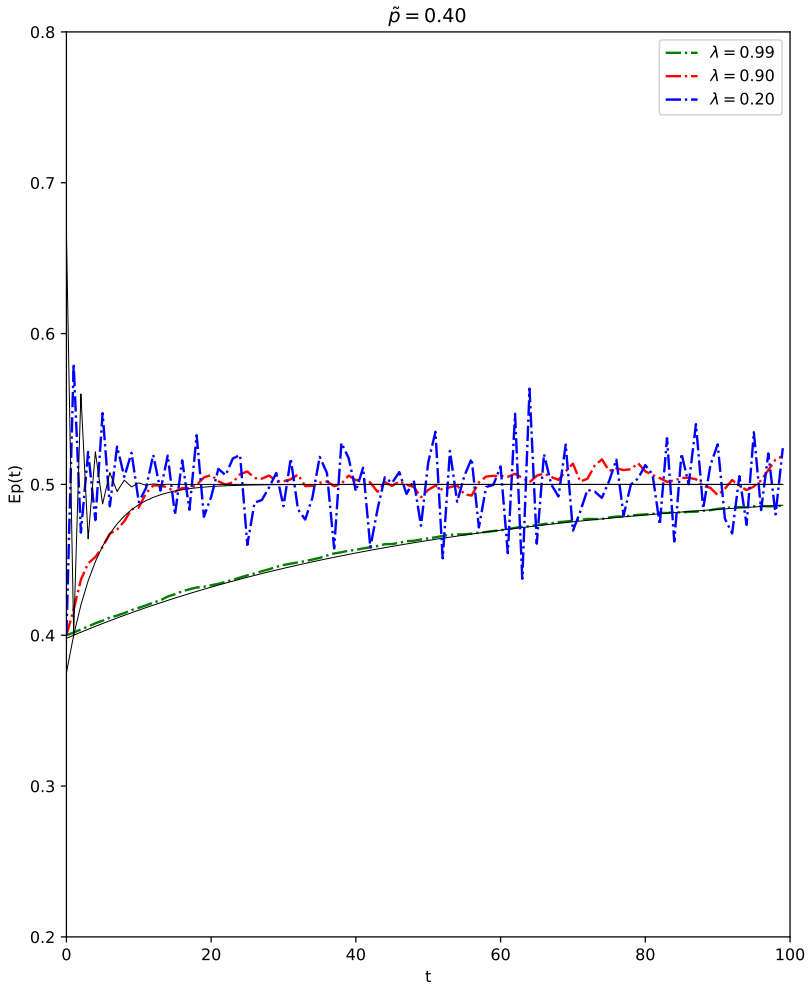


Example - Expected position of the walker

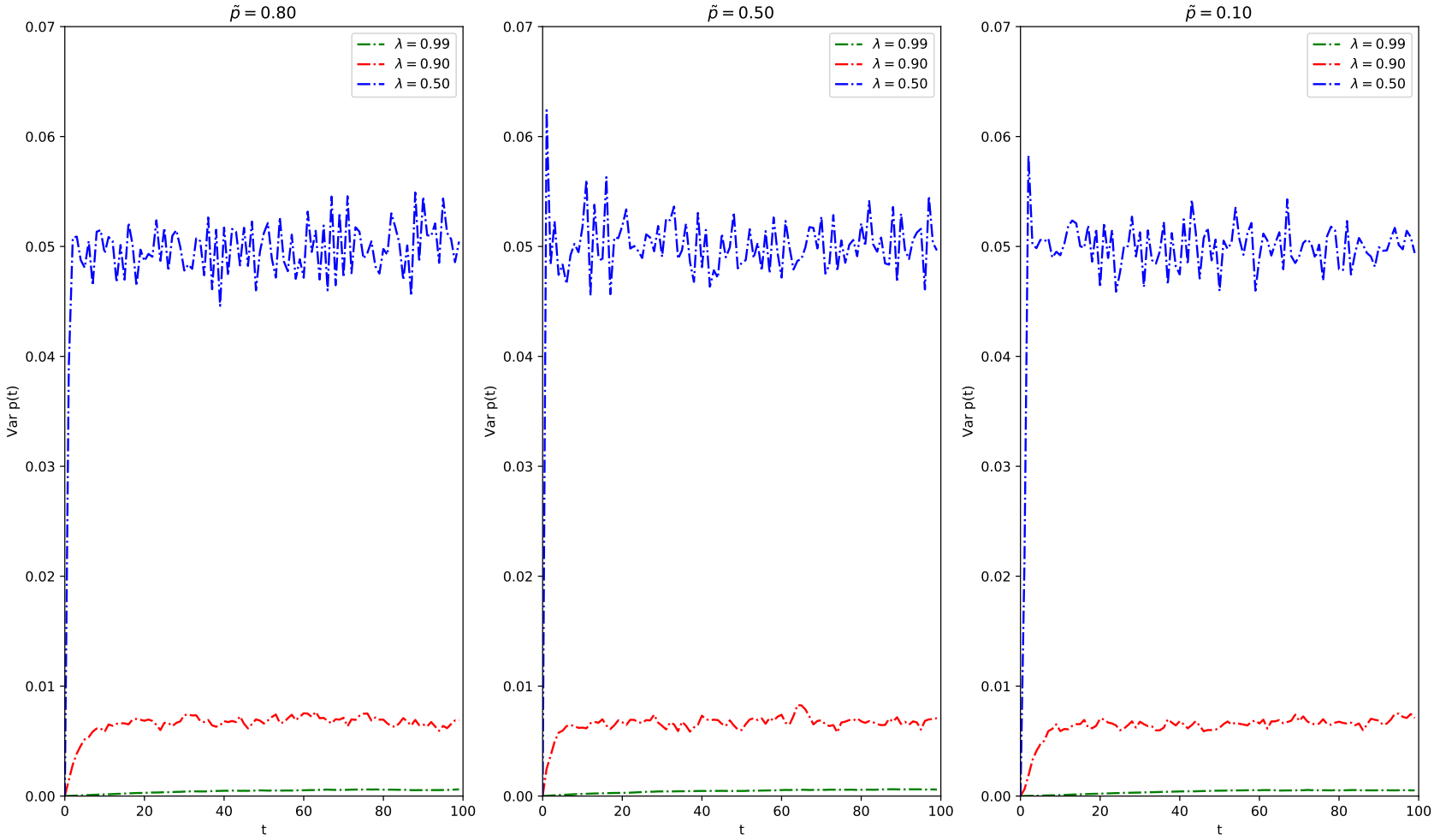


Example - Expected transition probability

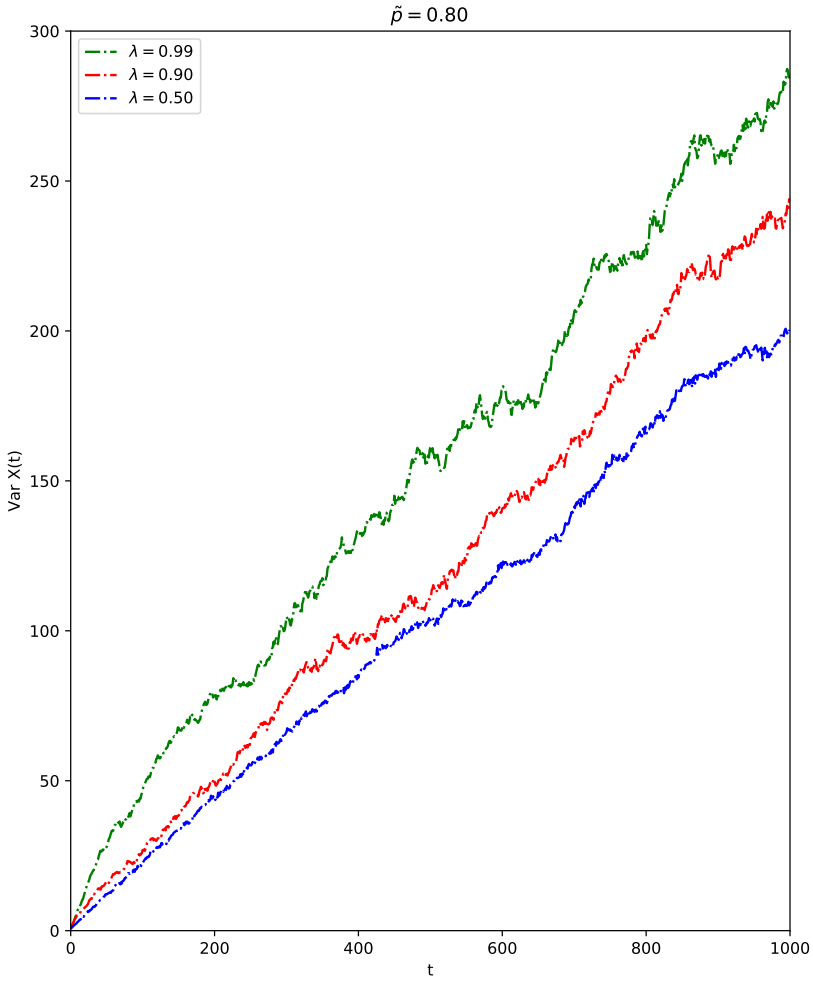


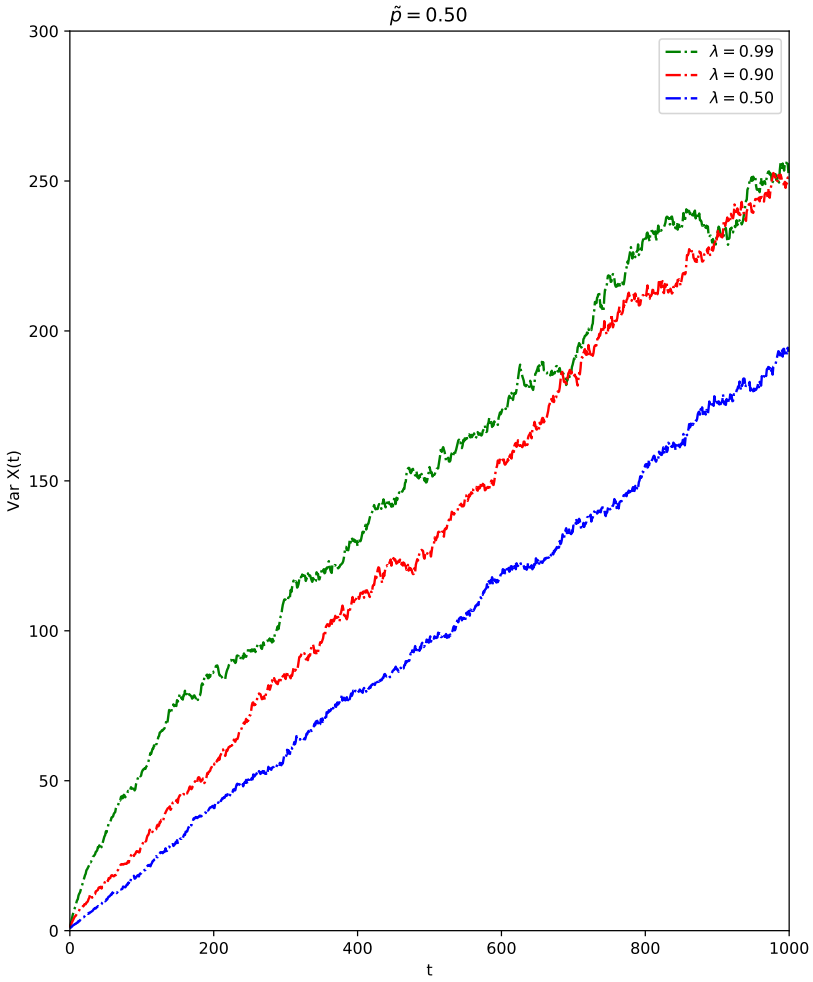


Example - Transition probability variance



Example - Walker's position variance





Alternative definitions

- "Success rewarded"

$$X_{t-1} = 1 \rightarrow p_t = 1 - \lambda(1 - p_{t-1})$$

$$X_{t-1} = 0 \rightarrow p_t = \lambda p_{t-1}$$

- Different coefficients for different events
- Generally n possible steps and m different coefficients λ affecting the transition probabilities
- Possible applications in
  - sports modeling
  - reliability and survival analysis
  - medical research
- Discrete alternative to random processes with memory

Alternative definitions

- "Success rewarded"
- Different coefficients for different events

$$X_{t-1} = 1 \rightarrow p_t = \lambda_1 p_{t-1}$$

$$X_{t-1} = 0 \rightarrow p_t = 1 - \lambda_2(1 - p_{t-1})$$

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$$p_t = f(p_{t-1}, X_{t-1}, \lambda_1, \ldots, \lambda_m)$$

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RWWM Properties

- tabulka 4x3 s vlastnostmi jednotlivych prochazek
- Expected prob
- Expected position
- Expected variance
- Comparision of the previous properties aplied for different model types

Asymptotic behaivor

- Convergence to standard RW??
- predchozi tabulka s hodnotami v nekonecnu

Data generation

- Ways to generate data, different conditions, number of repetitions
- popis jaka data generuju

Fitting on generated data

- Which types are detectable and predictable from the data
- Error rates
- Grafy?

Real life examples

- Kratce popsat co jsem delal do Aten
- Zminit, ze jsem desne vydelal na US Open

Results

- Zajimavy nastroj s possible implementations
- link na github kam neco nahraju

Next steps

- Model implementation
  - λ optimization
  - p_1 optimization
- Model improvement
  - Other versions of random walk with memory
  - Combination with other approaches
- Model testing
  - Model evaluation granularity
  - Performance on a larger dataset
  - Bbetting module for more bookmakers
  - Application of the model to *best-of-three* matches
- Application in other domains

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Thank you.

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