

Predicting Tennis Outcomes Using Random Walks

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Abstract

Recently introduced random walk model showed its possible use in predicting tennis men Grand Slam tournaments outcomes. The model was then further examined in and its theoretical background was described in more detail. In this paper the model is trained using the most recent data and then applied for in-play betting against a bookmaker. Different betting strategies are tested and the overall performance of the model is evaluated.

1 Introduction

Random walk was first introduced by K. Pearson over 100 years ago [4]. Since then the concept was further elaborated, the random walk was described in much detail and many alternation of the original model were introduced. One of the most recent variations of a random walk is a random walk with varying probabilities, a concept introduced recently by the authors [2]. It is based on a similar concept of a random walk with varying step size introduced by L. Turban [5]. This model seems particularly well suited to model different sporting events, especially tennis matches.

A proper model of a tennis match, and more generally any sporting event, is of a great value. It can serve the athletes and their coaches to improve their strengths and eliminate weaknesses, it can bring new insides into the game and it is especially useful in the sports betting industry. Such model can be used by bookmakers to provide more accurate odds, it can be used by professional bettors to gain an edge against the bookmakers and a robust model can also help to detect fraudsters selling their matches or betting on games where the result was agreed long before the game even started.

The paper is organized as follows. Next chapter briefly introduces the model used for tennis modeling. Section 3 provides general description of the data used, Section 4 describes the model fitting procedure. In Chapter 5 the model is tested against a database of real life bookmaker's odds and its performance is evaluated. Section 6 concludes this paper.

2 Random walk with varying probability

The idea behind the model of a random walk with varying probability is that the probability distribution of the next step depends directly on the result of the previous step in the following manner. Let us have a basic random walk with steps $X \in \{-1, 1\}$ with Bernoulli distribution depending on a parameter p so that $P(x = 1) = p$ and $P(x = -1) = 1 - p$. After each step this probability is altered using a memory coefficient λ and the result of previous step. We can define 2 major types of random walk with varying probability - *success punishing* and *success rewarding*. The first denotes that every time an event occurs, the probability of its repetition decreases, i.e. if step $X_t = 1$ then the probability the next step being 1 again decreases by a parameter λ so that $p_{t+1} = \lambda p_t$. For the *success rewarding* model, the opposite holds and every time an event occurs, the probability of its repetition increases.

The model can be further refined by using two separate parameters, one for each direction, i.e. $\lambda = [\lambda_1, \lambda_2]$. Again, there can be *success punishing* and *success rewarding* model. Thus, together there are four basic model variants. Formal definitions of all four model variants and their detailed description together with the derivation of useful formulas can be found in [3].

2.1 Application for tennis modeling

The random walk model can be applied to model any discrete time random process. Such processes often occur in sports, especially in sports played not for certain amount of time, but for certain amount of points. These sports include for example volleyball, badminton or tennis. Tennis seems as particularly interesting sport in terms of discrete random processes. There are several such processes. Series of matches can be considered a discrete random process, the sets played within a match, games within a set, points within a game or even strokes within a point, they all can be considered discrete random processes. In this paper, the model of a random walk with varying probability was used to model the sets played within a match.

3 Data description

For the purpose of this paper an automatic data gathering tool was created. This tool was continuously scraping data from www.tipssport.cz, the website of the biggest Czech bookmaker, gathering provided odds for the winner of the first set of each match and, if available, also the *in-play* odds for each set played. The tool also stored the results of each particular set played. The tool was developed using the Python programming language with the help of Selenium framework. PostgreSQL database deployed on Digital Ocean infrastructure was used for data storage. For the purpose of this paper, data from February until May 2021 was available.

The data was split into training and testing datasets. Training dataset

contains odds and results of matches played from February till the end of April, May matches represent the testing dataset. In the training dataset, there were 12 372 matches. Complete information, i.e. both odds and result, were available for 3 365 matches (mostly because the bookmaker did not provide *in-play* odds for the matches). As the matches are played as *best-of-three* (or *best-of-five* in case of 2021 ATP Australian Open) there were 8 310 sets played where all information required for the presented model was available. The testing dataset contains 3 076 matches, 1 150 of which with complete information, and there are 2 796 relevant sets.

4 Model training

As mentioned in Section 2, there are 4 basic variants of the model altogether with two (or three in case of $\lambda = [\lambda_1, \lambda_2]$) unknown parameters. Model training thus consists of two steps, best model variant selection and finding the optimal parameter.

4.1 Starting probability selection

The first unknown parameter of the model is the starting probability p_0 . In tennis modeling, coefficient $p_0 = p_A$ means the probability that Player A will win the first set. This probability (or its estimate) is implicitly given by the odds $o = [o_A, o_B]$ provided by bookmaker. Roughly, $p_A \approx \frac{1}{o_A}$. This estimate can be further refined. First, for typical odds there holds that $\frac{1}{o_A} + \frac{1}{o_B} > 1$ (i.e., typically the bookmaker provides *subfair* odds with bookmaker's winning margin). First step is thus the normalization of odds. It turns out that the simple normalization, i.e. $p_A = n_1(o) = \frac{\frac{1}{o_A}}{\frac{1}{o_A} + \frac{1}{o_B}}$ does not provide reasonable results. In fact, bookmakers margin is usually unevenly distributed towards the outsider. Therefore, an alternative normalization function can be used

$$p_A = n_2(o) = \frac{1}{o_A} + \frac{o_A o_B - o_A - o_B}{o_B(o_A + o_B)}. \quad (1)$$

This can be further improved by introducing a third normalization function $\mu(o, t)$, with a real parameter t linearly extrapolating n_1 and n_2 such that $\mu(o, 0) = n_2(o)$ and $\mu(o, 2) = n_1(o)$.

To obtain reasonable estimation of the model parameter p_0 from provided odds, the first step is to estimate parameter t . This was done using the maximal likelihood estimate method and the training set. Bookmaker's odds for the first set and the actual result of the first set were used. The likelihood function is defined as

$$L = \prod_{i=1}^N (x_i \mu(o_i, t) + (1 - x_i)(1 - \mu(o_i, t))),$$

where N is the number of matches in the training dataset, o_i is the bookmaker's odds for the first set in the i -th match and x_i is the result of the first set in the

i -th match, $x_i = 1$ if Player A won the set, $x_i = 0$ otherwise. For computational reasons the *log-likelihood* $L_l = \log(L)$ was used, i.e. the function

$$L_l = \sum_{i=1}^N \ln(x_i \mu(o_i, t) + (1 - x_i)(1 - \mu(o_i, t))).$$

Numerical methods, namely the function `minimize_scalar` from the Python SciPy package, were used to obtain optimal value of parameter $t = 4.03e - 06$. We can thus consider $t = 0$ and use function n_2 for normalization.

4.2 Model and memory parameter selection

For each match with data for at least 2 sets available (i.e. with at least 1 prediction opportunity), the first set winning probability p_0 was derived using the procedure and parameter from previous paragraph. Then, for each of the four model variants the optimal value of coefficient λ was found again using the maximum likelihood estimate

$$L = \sum_{i=1}^N \sum_{j=1}^{N_i} x_{ij} p_{ij} + (1 - x_{ij})(1 - p_{ij}),$$

where N denotes the number of matches in testing dataset, N_i the number of predictable sets in i -th match, x_{ij} is the result of the j -th set in the i -th match, $x_{ij} = 1$ if Player A won the set, $x_{ij} = 0$ and p_{ij} is the probability of Player A winning the set computed using the currently considered model variant.

Finally, the Akaike Information Criterion $AIC = 2k - 2\ln(L)$, which considers the number of parameters to correctly the best model, was used. Here k is the number of model parameters and L is the maximal likelihood.

Based on the available training data the *single lambda success rewarding* model variant was selected, with $\lambda = 0.826$ and p_0 computed for each match using the n_2 normalization function defined in (1).

5 Model application for in-play betting

To test the quality of the model, following experiment was performed. For each match in the dataset, first the starting probability was derived using the procedure described in Section 4.1. Then, it was iterated over the sets played, and for each set, the set winning probability was computed using the *success rewarding* model and memory parameter $\lambda = 0.826$ as specified in Section 4.2. This probability was compared to odds provided by the bookmaker and possibly a virtual bet was made.

Choosing the correct betting strategy is one of the key elements of successful betting. It depends on the underlying model, available bookmaker's odds, bankroll, internal bookmaker's policies and many other parameters. There exists a large number of possible approaches and the detailed description of them

Strategy	Bet	$E(w)$	$E(w p = \frac{1}{a})$	$Var(w)$	$Var(w p = \frac{1}{a})$
Naive	1	$pa - 1$	0	$pa^2(1 - p)$	$a(1 - \frac{1}{a})$
Odds	$\frac{1}{a}$	$p - \frac{1}{a}$	0	$p(1 - p)$	$\frac{1 - \frac{1}{k}}{k}$
Prob.	p	$p(pa - 1)$	0	$p^3a^2(1 - p)$	$\frac{1 - \frac{1}{k}}{k}$
General	u	$u(pa - 1)$	0	$u^2pa^2(1 - p)$	$u^2a(1 - \frac{1}{a})$

Table 1: Theoretical values of wins and variances for different betting strategies. p is the probability of winning, a is the odds provided by bookmaker.

is beyond the scope of this paper. For testing purposes, there were three basic strategies tested. First, the naive betting strategy, where simply 1 unit was bet every time. Then, the probability base strategy, where p (probability of winning) units was bet. Finally the odds based strategy, where $\frac{1}{odds}$ was bet. The strategies differ in the expected wins and their variance. The theoretical properties of the betting strategies can be observed in Table 1 with the special case where $p = \frac{1}{a}$, i.e. in case of fair odds [1]. Besides the different betting amount strategies, it is also important to choose when to bet. The basic strategy is to bet always when $p > \frac{1}{odds}$. It turns out that it is more favorable to bet only when there is a margin present, i.e. when $p > \frac{1}{odds} \cdot m$, where m is a margin parameter, $m > 1$.

For $m = 1.2$, i.e. 20% margin, there are 65 virtual bets made and a significant profit is achieved. One of the parameters to evaluate a quality of a betting strategy is the return of investment (ROI), i.e. $\frac{total\ profit}{total\ bankroll\ needed}$. For this parameter, the naive betting yields $ROI = 148\%$, probability based betting $ROI = 98\%$ and odds based betting $ROI = 111\%$. The development of the portfolio balance can be seen in Figure 1.

6 Conclusion and future work

In this paper, a recently introduced model of a random walk with varying probabilities was used to predict the development of a tennis match. The quality of the predictions was tested against a real life odds provided by a bookmaker and the results confirm big potential of the model for tennis modeling. To better validate the model quality a larger dataset has to be acquired. The model can be further improved, for example by introducing a variable memory coefficient λ or by combining the model with some of the more classical, regression based approaches. This will be subject of a further research.

7 Remarks

The source code containing all functionality mentioned in this article is freely available as open source at GitHub¹. More results can be also obtained from

¹<https://github.com/tomaskourim/mathsport2021>

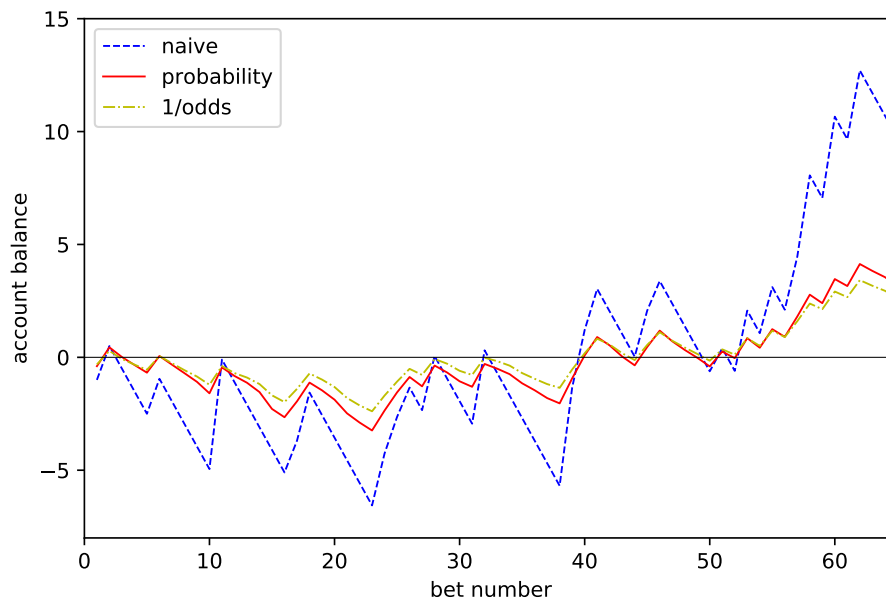


Figure 1: A graph showing the account balance development for different betting strategies.

the same repository. The data used in this paper are available from the author upon request.

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