

A model of random walk with varying transition probabilities

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Outline

1. Prepare mathematical model

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2. Describe its properties

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2. Describe its properties
3. Apply it on data

Random walk

Definition

A man starts from a point O and walks l yards in a straight line; he then turns through any angle whatever and walks another l yards in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $r + \delta r$ from his starting point, O .

[Karl Pearson: The problem of the random walk. (1905)]

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Where is the “Drunken sailor”?

Random walk

Definition

Let $\{X_k\}_{k=1}^{\infty}$ be a sequence of discrete random variables. For each positive integer n , let S_n denote the sum $X_1 + X_2 + \cdots + X_n$, with $S_0 = 0$. The sequence $\{S_n\}_{n=1}^{\infty}$ is called a random walk. If the common range of the X_k 's is \mathbb{R}_m , then $\{S_n\}$ is a random walk in \mathbb{R}_m .

If for $\forall k; X_k \sim B(p = \frac{1}{2})$, the walk is called the standard random walk.

Random walk properties

- ▶ Discrete random process
- ▶ n —dimensional, on a matrix, graph, finite or infinite set
- ▶ Self avoiding, reinforced
- ▶ Brownian motion, polymer creation, games simulation, sports simulation

Random walks with varying transition probabilities

Nejprve popis, co to vlastne dela, rozdil mezi 2 variantami

Formal definition

formalni definice walku, obe varianty

Example - RW steps

obrazek

ruzna λ , p_0 , oddelena barevne

jedno “pozorovani” a k tomu expected value a variance

Walk step properties

$\text{Exp } X_t, \text{Var } X_t$

Example - RW probabilities

Stejný obrazek jako u steps, ale s probabilities

Walk probabilities properties

stejne jako u steps

Example - RW position

dtto

Walk position properties

dtto

Success rewarding model

Stejne obrazky a slidy jako pro

Overview

Prehledova tabulka se vseimi vzorci pro porovnani

Two-parameter model

Popis a definice 2lambda modelu

Example - two-parameter model

Jeden obrazek na SP, jeden na SR

Real life applications

Zminit studii na US Open sazeni

Summary

novy zajimavy model
odvozene teoreticke vlastnosti
ukazka na realnych datech