Non-markov discrete random walks - selected properties

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Abstract

This paper elaborates the newly introduced model of discrete random walks with memory implemented through a memory coefficient λ. It follows on the recent work of the authors and further examines the theoretical properties of the model.

1 Introduction

Sumar vysledku a proc to delat, podobne jako vzdycky.

2 Random walks with memory - overview

Shrnuti dosavadnich vysledku a vypoctenych vzorcu

3 Random walks with memory - properties

Dukazy novych veci, hlavni cast clanku

Proposition 1. *For all $t \geq 2$*

$$Var\, S_t = neco.$$

Proof. From the definition of variance

$$Var\, S_t = ES_t^2 - E^2 S_t$$

and XXXXES_t vzorecXXX follows that to prove the proposition it is enough to prove that

$$ES_t^2 = neco.$$

First of all, let us express $E(S_t^2)$ from the knowledge of past walk development. From the denition of the expected value and the denition of the walk XXXXXX it follows

$$ES_t^2 = E[E(S_t^2|P_{t-1})] = E[E((S_{t-1}+X_t)^2|P_{t-1})] = E(S_{t-1}^2+2(2P_{t-1}-1)S_{t-1}+(2P_{t-1}-1)^2) =$$

$$= E(S_{t-1}^2 + 4P_{t-1}S_{t-1} - 2S_{t-1} + 4P_{t-1}^2 + 4P_{t-1} - 1).$$

The values of ES_t and EP_t are given by XXXXXXX and

$$EP_t^2 = (3\lambda^2 - 2\lambda)^t p_0^2 + \sum_{i=1}^{t} K(i-1)(3\lambda^2 - 2\lambda)^{t-i},$$

with

$$K(t) = k(t) \cdot (-3\lambda^2 + 4\lambda - 1) + (1-\lambda)^2$$

and

$$k(t) = EP_t = (2\lambda - 1)^t p_0 + \frac{1 - (2\lambda - 1)^t}{2},$$

wich follows form the proof of Proposition 2.5 in CITE-AMISTAT. The only unknown element (clen rovnice - jak se to rekne anglicky?) is the mixed one. Let us again express it from the knowledge of past steps as

$$E(P_tS_t) = E[E(P_tS_t|P_{t-1})] = E[E((\lambda P_{t-1} + \frac{1}{2}(1-\lambda)(1-X_t))(S_{t-1}+X_t)|P_{t-1})] =$$

$$= E[(\lambda P_{t-1} + \frac{1}{2}(1-\lambda)(1-(2P_{t-1}-1)))(S_{t-1} + (2P_{t-1}-1))] =$$

$$= E[(\lambda P_{t-1} + (1-\lambda)(1-P_{t-1}))(S_{t-1} + 2P_{t-1} - 1)] =$$

$$= E[(\lambda P_{t-1} + 1 - P_{t-1} - \lambda + \lambda P_{t-1})(S_{t-1} + 2P_{t-1} - 1)] =$$

$$= E[(\lambda P_{t-1}S_{t-1} + S_{t-1} - P_{t-1}S_{t-1} - \lambda S_{t-1} + \lambda P_{t-1}S_{t-1} + 2\lambda P_{t-1}^2 + 2P_{t-1} - 2P_{t-1}^2 - 2\lambda P_{t-1} + 2\lambda P_{t-1}^2 -$$

$$-\lambda P_{t-1} - 1 + P_{t-1} + \lambda - \lambda P_{t-1})] =$$

$$= E((2\lambda-1)P_{t-1}S_{t-1} + (1-\lambda)S_{t-1} + 2(2\lambda-1)P_{t-1}^2 + (3-4\lambda)P_{t-1} - 1 + \lambda).$$

Together we get

$$ES_t^2 = E(S_{t-1}^2 + 4((2\lambda-1)P_{t-2}S_{t-2} + (1-\lambda)S_{t-2} + 2(2\lambda-1)P_{t-2}^2 + (3-4\lambda)P_{t-2} - 1 + \lambda) -$$

$$-2S_{t-1} + 4P_{t-1}^2 + 4P_{t-1} - 1) =$$

$$= E(S_{t-1}^2 + 4((2\lambda-1)P_{t-2}S_{t-2} + (1-\lambda)S_{t-2} + 2(2\lambda-1)P_{t-2}^2 + (3-4\lambda)P_{t-2})) - 2S_{t-1} + 4P_{t-1}^2 + 4P_{t-1}) - 5 + 4\lambda.$$

□

4 Conclusion