

Episodes of Exuberance in Housing Markets: In Search of the Smoking Gun

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Abstract In this paper, we examine changes in the time series properties of three widely used housing market indicators (real house prices, price-to-income ratios, and price-to-rent ratios) for a large set of countries to detect episodes of explosive dynamics. Dating such episodes of exuberance in housing markets provides a timeline as

The International House Price Database can be accessed online at <http://www.dallasfed.org/institute/houseprice/index.cfm>. An earlier version of the paper circulated under the title “Monitoring Housing Markets for Periods of Exuberance. An Application of the Phillips et al. (2012, 2013) GSADF Test on the Dallas Fed International House Price Database.”

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well as empirical content to the narrative connecting housing exuberance to the global 2008–09 recession. For our empirical analysis, we employ two recursive univariate unit root tests recently developed by Phillips and Yu (International Economic Review 52(1):201–226, 2011) and Phillips et al. (2015). We also propose a novel extension of the test developed by Phillips et al. (2015) to a panel setting in order to exploit the large cross-sectional dimension of our international dataset. Statistically significant periods of exuberance are found in most countries. Moreover, we find strong evidence of the emergence of an unprecedented period of exuberance in the early 2000s that eventually collapsed around 2006–07, preceding the 2008–09 global recession. We examine whether macro and financial variables help to predict (in-sample) episodes of exuberance in housing markets. Long-term interest rates, credit growth and global economic conditions are found to be among the best predictors. We conclude that global factors (partly) explain the synchronization of exuberance episodes that we detect in the data in the 2000s.

Keywords House prices · Mildly explosive time series · Sup ADF test · Generalized sup ADF test · Panel GSADF · Probit model

JEL Classification: C22 · G12 · R30 · R31

Introduction

The latest boom and bust in international housing markets has generated a renewed interest in the dynamics of house prices. A view shared by many academics and policymakers is that the latest boom period in housing was associated with house prices departing from their intrinsic values, which led to a misallocation of resources, distorted investment patterns, and eventually precipitated the 2008–09 global recession. On this basis, in the aftermath of the crisis, international organizations and central bankers have become increasingly concerned about developments in housing markets across the world (e.g., the IMF established the Global Housing Watch and the Federal Reserve Bank of Dallas created the International House Price Database). Central banks have also become more aware of the role of domestic housing markets in financial stability and the real economy (see, e.g., the 2014 U.K. stress testing exercise of the Bank of England).

In this paper we analyze changes in the time series properties of house prices over the last four decades in an attempt to shed light on three research questions: Firstly, when did house price run-ups turn into episodes of exuberance (displaying explosive dynamics) in domestic housing markets, secondly, was there synchronization across countries that led to global exuberance and, lastly, which were the fundamental factors that aided in the emergence of housing exuberance? By addressing these research questions, we aim to improve our understanding of housing markets in a way that facilitates the development of better monitoring mechanisms and the design of more effective and proactive policies.

An appealing feature of our empirical analysis is the use of panel data from the International House Price Database of the Federal Reserve Bank of Dallas (Mack and

Martínez-García 2011) to detect and date-stamp explosive autoregressive behavior in housing markets. This dataset has a large panel dimension (22 countries), which allows us to have a unique, international perspective on the evolution of housing markets.¹

With respect to our first research question, we employ two univariate econometric tests based on the Augmented Dickey-Fuller (*ADF*) test—namely, the Supremum *ADF* (*SADF*) and the Generalized Supremum *ADF* (*GSADF*)—recently developed by Phillips and Yu (2011, 2015). We examine with these tests whether real house prices and standard long-run anchors of the housing market, such as price-to-income and price-to-rent ratios, exhibit periods of mildly explosive behavior.² The *SADF* and *GSADF* are right-tailed unit root tests that perform recursive *ADF* regressions to detect explosiveness. Their recursive nature makes the *SADF* and *GSADF* tests attractive in our context for two reasons. First, the tests enable us to identify in which (if any) periods a series displayed explosive behavior and, thus, date the emergence of exuberance in domestic housing markets. Second, these tests exhibit higher power in the presence of boom and bust episodes than standard integration/cointegration tests and tests that allow for a single (permanent) change in persistence.³

A timeline of events emerges from the corresponding dating procedure which suggests that the latest boom-bust cycle was unusually widespread across countries and evolved in three phases: A first phase of origination is connected with the U.S. experience during the second half of the 1990s; a second phase of propagation is characterized by widespread and synchronized episodes of exuberance across very different housing markets during the first half of the 2000s; and a final phase where the episode of exuberance burst within a short period of time and was followed by the severe contraction in economic activity of the 2008–09 global recession. In light of the substantial differences across domestic housing markets and the non-tradability of housing, the pattern of near-simultaneous explosive behavior in the first half of the 2000s is a particularly interesting finding.

However, the *SADF* and *GSADF* are univariate testing procedures and, thus, only allow us to draw conclusions at the country level. To address our second research question, we propose a novel extension of the *GSADF* procedure to a panel setting inspired by the work of Im et al. (2003), which exploits the large cross-sectional

¹We complement the Dallas Fed dataset with housing rents from the OECD for 16 of the 22 countries for which there is consistent data over the same sample period (Girouard et al. 2006).

²Mildly explosive behavior is modeled by an autoregressive process with a root that exceeds unity, but remains within the vicinity of one. This represents a small departure from martingale behavior, but is consistent with the submartingale property often used in the rational bubbles literature (see Section “The Univariate *SADF* and *GSADF* Procedures” for further details). Phillips and Magdalinos (2007a, b) and Magdalinos (2012) provide a large sample asymptotic theory for this class of processes that enables econometric inference in this case, unlike for purely explosive processes.

³The *SADF* and *GSADF* tests better detect mildly explosive behavior in time series data than standard methods such as unit root/cointegration tests (Diba and Grossman 1988), but also variance bound tests (LeRoy 1981; Shiller 1981), specification tests (West 1987), and Chow and CUSUM-type tests (Homm and Breitung 2012). A few studies have implemented these techniques in the context of housing markets (Phillips and Yu 2011; Yiu et al. 2013), but only on domestic and not across international markets.

dimension of the Dallas Fed's International House Price Database in order to draw inferences for the global housing market. The extension is easy to compute and, like the univariate methods, provides a date-stamping strategy. The results for our panel *GSADF* test provide a very clear picture of global exuberance in housing markets in the period preceding the 2008–09 global recession between 2000 and 2006.

Having identified periods of explosive behavior at both the domestic and the international levels, we turn to our third research question, which deals with the factors that contribute to the increase in the likelihood of exuberance emerging in housing. A widely used explanation for the emergence of boom and bust episodes is the existence of housing bubbles (Case and Shiller 2003; Mayer 2011; Shiller 2015). In a rational expectations framework, rational bubbles can arise simply due to market participants' expectations of future price increases (Flood and Hodrick 1990). This type of bubble process follows explosive paths, which makes asset prices diverge from their fundamental values and induces exuberance in housing markets. On the basis of this rationale, several authors have employed integration and cointegration tests to examine the presence of house price bubbles (Hott and Monnin 2008; Mikhed and Zemcik 2009b; André et al. 2014).

As we show in the following section, non-stationary dynamics in house prices may also arise from causes other than bubbles, such as explosive dynamics in economic fundamentals and time-varying discount rates. For this reason, although we recognize that bubbles can cause explosive behavior in house prices, we refrain from using the term bubbles in the interpretation of our findings.

Capozza et al. (2004), for example, find that the time series properties of house prices in 62 United States metro areas depend on a set of economic variables that proxy for information costs, supply costs and expectations.⁴ Agnello and Schuknecht (2011) and Rousová and Van den Noord (2011), among other recent studies, look at fundamental predictors of housing booms and busts as well as turning points in the housing cycle of OECD countries. According to their findings, domestic credit and interest rates are among the most important predictors of booms and busts, while macro aggregates (like unemployment) can be helpful in predicting peaks and troughs. In related research, Chen (2009) and Nyberg (2013) examine which macroeconomic and financial variables impact the future likelihood of a bull or bear market. By analyzing data for 18 OECD countries, they find evidence that asset price booms, characterized by rapid price appreciation above estimated trends, generally are preceded by similarly rapid expansions in monetary aggregates and inflation. They also find that financial variables are useful predictors, especially for bear markets.

We contribute to this literature by assessing whether the probability of a housing market being in a state of exuberance depends on a set of macroeconomic and financial variables that have been reported to be important drivers of house prices. Our results provide evidence in favor of in-sample predictability with the best predictors

⁴The existing empirical evidence points out that house prices may temporarily deviate from fundamentals (e.g., the time series and cross-section evidence in Clayton 1996; Hwang and Quigley 2006; Mikhed and Zemcik 2009a; Capozza et al. 2004; Adams and Fuss 2010) and the importance of the bank lending channel in housing (e.g., Mian and Sufi 2009; Pavlov and Wachter 2011; Berkovec et al. 2012).

being long-run interest rates, private credit growth, demand-side factors (such as per capita real disposable income growth) and global economic conditions. These results highlight the important ripple effects of declining long-term interest rates in the synchronization of housing market exuberance and also conform with the anecdotal evidence that during the boom years prior to the 2008–09 global recession, international investors—reaping the benefits of strong global growth—fueled house price run-ups across many different countries.

The remainder of the paper proceeds as follows: Section “[House Prices, Fundamentals and Rational Bubbles](#)” outlines the standard asset-pricing model of housing and illustrates how explosive behavior in house prices may arise. Section “[Testing for Episodes of Explosive Behavior](#)” describes the *SADF* and *GSADF* testing procedures, as well as the extension of the latter to a panel setting. Section “[Empirical Evidence on International House Prices](#)” presents and discusses the empirical results for the univariate and panel unit root tests as well as the panel probit on the likelihood of an episode of exuberance occurring. Special attention is paid to the U.S., the U.K. and Spain due to their economic size and significance, and because these countries exemplify the distinct patterns observed during the three main phases of the timeline of events that we describe in the paper. Finally, Section “[Concluding Remarks](#)” provides our assessment of the evidence.

House Prices, Fundamentals and Rational Bubbles

A conventional framework for the study of explosive behavior in housing markets is provided by the standard asset-pricing model with risk neutral agents (see, e.g., Clayton 1996; Hiebert and Sydow 2011).⁵ In this framework, the price of housing can be derived from the following no-arbitrage condition,

$$\underbrace{\rho}_{\text{constant risk-free rate}} = \underbrace{\mathbb{E}_t(R_{t+1})}_{\text{expected return on housing}}, \quad (1)$$

where $\rho > 0$ denotes the (for the time being constant) discount rate—the expected net return on an alternative investment opportunity—while \mathbb{E}_t is the expectations operator based on all information available up to time t , and R_{t+1} is the return on housing at time $t + 1$ defined as

$$R_{t+1} \equiv \frac{P_{t+1} + F_{t+1}}{P_t} - 1, \quad (2)$$

where P_t denotes the house price, and F_t is the stream of payoffs (pecuniary or otherwise) derived from housing.

We refer to F_t as the economic fundamentals of the housing market, and work out our analysis with two related specifications. One general specification of F_t includes the payoff stream, X_t , which is given by the observed economic rents of housing,

⁵The asset-pricing approach to housing builds on the extensive rational bubbles literature. See, e.g., the seminal work of Blanchard (1979) and Blanchard and Watson (1982).

including housing services, and another component, U_t , that is either unobserved or reflects mismeasurement of the housing rents, i.e.,

$$F_t = X_t + U_t. \quad (3)$$

The other specification of F_t relates the payoff stream of housing rents, X_t , to macroeconomic fundamentals through a demand equation for rental housing but retains an unobserved (or mismeasurement) term, U_t . Under a set of constraints on preferences, we can derive a linear expenditure system where the demand for rental housing linearly relates housing rents, X_t , to macroeconomic fundamentals such as disposable income, Y_t , i.e.,

$$F_t = \theta_F + \delta Y_t + U_t. \quad (4)$$

The [Appendix](#) provides details on the derivation of this relationship, which captures the affordability determinants of housing.⁶

Using the definition of the return on housing, R_t , in Eq. 2 and rearranging the no-arbitrage condition in Eq. 1 yields the following expression for house prices,⁷

$$P_t = \frac{1}{1 + \rho} \mathbb{E}_t [P_{t+1} + F_{t+1}], \quad (5)$$

which indicates that the price today must be equal to the discounted present value of the expected fundamentals plus the resale price of housing tomorrow. Recursive substitution of this asset-pricing equation yields the standard present-value model for the price of housing (see, e.g., Clayton 1996).

By recursively substituting T periods forward, Eq. 5 can be rewritten as

$$P_t = \mathbb{E}_t \left[\sum_{j=1}^T \left(\frac{1}{1 + \rho} \right)^j F_{t+j} \right] + \mathbb{E}_t \left[\left(\frac{1}{1 + \rho} \right)^T P_{t+T} \right]. \quad (6)$$

According to the above expression, the house price at time t is a function of the expected discounted flow of all future payoffs up to time T plus the discounted resale value of the house at time T . Letting T go to infinity and imposing the transversality $\lim_{T \rightarrow \infty} \mathbb{E}_t \left[\left(\frac{1}{1 + \rho} \right)^T P_{t+T} \right] = 0$ to rule out non-fundamental behavior (bubbles), the unique solution to the expectational difference equation in Eq. 5 yields

$$P_t^* = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \left(\frac{1}{1 + \rho} \right)^j F_{t+j} \right], \quad (7)$$

⁶We implicitly use this demand equation for rental housing to relate house prices to personal disposable income. In doing so, however, the definition of fundamentals has to be augmented with a particular specification of the rental housing demand.

⁷Log-linear approximations are also commonly used but may be less relevant with nonstationary data where sample means do not converge to population constants (Campbell and Shiller 1988; Campbell et al. 1997). Further discussion on these approximations can be found in Lee and Phillips (2011). In this paper, we work with levels. Using logs does not qualitatively alter the results.

where P_t^* is referred to as the fundamental price of housing due to the fact that it is a function solely of economic fundamentals, F_t , and the discount rate, ρ .⁸ In this case, the housing price corresponds to its fundamental-based price—i.e., $P_t = P_t^*$.

The prediction that house prices are driven solely by economic fundamentals depends crucially on the transversality condition. In the absence of this condition, there exist infinite forward solutions to the difference equation for the price of housing which are of the form (see, e.g., Sargent 1987; Diba and Grossman 1988; LeRoy 2004),

$$P_t = P_t^* + B_t, \quad (8)$$

where P_t^* is the fundamental-based price of housing determined in Eq. 7 and B_t is a non-fundamental, bubble term that satisfies the submartingale property

$$\mathbb{E}_t(B_{t+1}) = (1 + \rho)B_t. \quad (9)$$

Since the discount factor is positive ($\rho > 0$), the bubble term, B_t , is expected to be explosive. Making the discount factor, ρ_t , either stationary or integrated of order 1 is not going to alter the implications of the submartingale in Eq. 9.

A rational bubble occurs when expectations of future price increases rather than fundamentals drive current house prices up (Case and Shiller 2003). Hence, the emergence of such a bubble creates exuberance in the housing market. Intuitively, in the presence of a bubble, buyers are willing to pay prices increasingly higher than the fundamental-based price, P_t^* , because they expect to be compensated through future price increases at a rate that equals the discount rate, $\rho > 0$.

The fact that the bubble term is explosive has important implications for empirical studies. If economic fundamentals follow either a stationary or an integrated process of order 1, which is a typical assumption in the literature, then a reason why house prices can display explosive dynamics is because rational bubbles exist. Given that, one can test for rational bubbles by simply applying right-tailed unit root tests to house prices in order to detect and date-stamp periods of (mildly) explosive behavior in the time series. However, the results of such tests must be interpreted with caution since they are only indicative, not conclusive. As we show next, factors other than bubbles can also give rise to explosive dynamics in house prices.

Explosive Fundamentals Following Campbell and Shiller (1987), we combine Eqs. 7 and 8 to obtain the following expression for house prices,

$$P_t = \frac{1}{\rho}F_t + \left(\frac{1+\rho}{\rho}\right)\mathbb{E}_t\left[\sum_{j=1}^{\infty}\left(\frac{1}{1+\rho}\right)^j\Delta F_{t+j}\right] + B_t, \quad (10)$$

where Δ is the difference operator. A plausible assumption is that the economic rents on housing, F_t , follow a general autoregressive process of order 1,

$$F_t = \phi F_{t-1} + \epsilon_t, \epsilon_t \sim WN(0, \sigma_\epsilon^2), \quad (11)$$

⁸For the standard dividend discount model in which the payoff stream $\{X_t\}_{t=1}^{\infty}$ grows at a constant rate, see Gordon and Shapiro (1956). Blanchard and Watson (1982) and Campbell et al. (1997) examine more general processes for $\{X_t\}_{t=1}^{\infty}$.

where ϵ_t is a white noise process. The stochastic process in Eq. 11 is stationary for $|\phi| < 1$, integrated of order 1 for $\phi = 1$, and explosive for $\phi > 1$. In the absence of bubbles (i.e., if $B_t = 0$ for all t), Eq. 8 implies that the house price equates its fundamental-based price. Hence, Eqs. 10 and 11 imply that⁹

$$P_t = P_t^* = \left(1 + (1 - \phi) \left(\frac{1 + \rho}{1 + \rho - \phi}\right)\right) \frac{1}{\rho} F_t. \quad (12)$$

It follows from Eq. 12 that house prices, P_t , can display explosive dynamics, even if there is no bubble, due to explosive dynamics in fundamentals—i.e., $\phi > 1$. In this case, exuberance in the housing market is inherited from fundamental factors which might not be directly observable.

It is worth noting that, irrespective of the value of ϕ , the ratio of house prices to fundamentals,

$$\frac{P_t}{F_t} = \left(1 + (1 - \phi) \left(\frac{1 + \rho}{1 + \rho - \phi}\right)\right) \frac{1}{\rho}, \quad (13)$$

is non-explosive in the absence of bubbles. On the contrary, in the presence of bubbles, house prices increase driven by expectations faster than fundamentals, causing their ratio to explode. This implies that right-tailed unit root tests applied to price-to-fundamental ratios are more informative about rational bubbles than tests applied to house prices alone. For this reason, we also examine the price-to-rent and the price-to-income ratios in our empirical analysis.¹⁰

It should be noted that working with price-to-fundamental ratios does not make the results of right-tailed unit root tests conclusive for the presence of a bubble. A reason for this is because we do not generally observe all fundamentals, so rather than applying our tests to the unobservable price-to-fundamental ratio, $\frac{P_t}{F_t}$, we rely on the observable price-to-rent $\left(\frac{P_t}{X_t}\right)$ and price-to-income $\left(\frac{P_t}{Y_t}\right)$ ratios.¹¹ Hence, even if there is evidence of explosive behavior in such observable ratios, we cannot truly rule out the possibility that explosiveness is inherited from the unobserved component of fundamentals, U_t —a deficiency that plagues virtually all empirical studies.

Time-Varying Discount Rates Another factor that can cause exuberance in the housing market is time variation in the discount rate, ρ . As with economic fundamentals, the trajectory of the discount rate can have an important effect on the

⁹For a discussion of a more general solution with log-linear approximation methods, see Engsted et al. (2012).

¹⁰The price-to-income ratio in particular provides a metric of house prices relative to the ability of households to pay (Himmelberg et al. 2005; Girouard et al. 2006) and, thus, it incorporates one of the key determinants of the demand for housing.

¹¹We note that, apart from income and rent, there are other fundamental drivers of housing prices, such as the cost of foregone interest, the cost of property taxes and maintenance costs (see, e.g., the discussion in Himmelberg et al. 2005). Lack of consistent and comparable data across countries for fundamental factors like this remains a limitation for applied research in housing.

characteristics of the fundamental-based price of housing. For simplicity, we reconsider the asset pricing model of housing in Eq. 5 but with a time-varying discount rate, i.e.,

$$P_t = \frac{1}{1 + \rho_t} \mathbb{E}_t [P_{t+1} + F_{t+1}], \quad (14)$$

and set $\phi = 1$ so that the fundamentals process in Eq. 11 follows a random walk process, i.e.,

$$F_t = F_{t-1} + \epsilon_t, \epsilon_t \sim WN(0, \sigma_\epsilon^2). \quad (15)$$

We are interested in a scenario where there is a gradual and anticipated decline in interest rates over a certain period of time. This change is described by¹²

$$1 + \rho_{t+s} = \begin{cases} 1 + \rho', & \text{for } 0 \leq s \leq k, \\ (1 + \rho_{t+s+1})g, & \text{for } k+1 \leq s < k', \\ 1 + \rho, & \text{for } s \geq k', \end{cases} \quad (16)$$

where $0 < k < k' < \infty$ defines the time window of decline for the discount rate, and $g \geq 1$ determines the gross rate of decline. This interest rate specification collapses to the constant discount rate case whenever $g = 1$, and implies $\rho' > \rho$ whenever $g > 1$. The time variation in Eq. 16 captures the idea that declining rates may have been an important factor in the run-up of house prices leading to the 2008–09 global recession (in line with Bernanke 2005's saving glut theory).

Imposing the transversality condition to rule out non-fundamental bubbles, the unique solution to the present-value model for house prices is given by

$$P_t = P_t^* = \theta_{t-1} F_t, \quad (17)$$

where θ_{t-1} obeys the following difference equation,

$$(1 + \rho_t) \theta_{t-1} = (1 + \theta_t). \quad (18)$$

Combining the solution in Eq. 17 with the specification of fundamentals in Eq. 15, we derive the following process for house prices,

$$P_t = \frac{\theta_{t-1}}{\theta_{t-2}} P_{t-1} + \varepsilon_t, \varepsilon_t \sim WN(0, \theta_{t-1}^2 \sigma_\epsilon^2). \quad (19)$$

Equation 19 shows the potential impact of time variation in the discount rate on the persistence and volatility of house prices.¹³

In the constant discount rate case where $g = 1$, $\theta_t = \frac{1}{\rho}$ and house prices inherit the unit root of fundamentals. Furthermore, house price volatility is that of the

¹²The recursive representation of the discount rate is equivalent to the following alternative characterization,

$$1 + \rho_{t+s} = \begin{cases} (1 + \rho) g^{k'-k}, & \text{for } 0 \leq s \leq k, \\ (1 + \rho) g^{k'-s}, & \text{for } k+1 \leq s < k', \\ 1 + \rho, & \text{for } s \geq k'. \end{cases}$$

¹³For a discussion on the characteristics of the volatility process in house prices with data from the International House Price Database see Mack and Martínez-García (2012). These authors provide empirical evidence of an increase in house price volatility that is consistent with the stylized implications of declining discount rates laid out here.

fundamentals scaled by a constant related to the discount rate. The solution in the general case where $g > 1$ can be characterized by backward induction as follows: For $s \geq k'$, the solution corresponds to the case where the low discount rate remains constant with $\theta_{t+s} = \frac{1}{\rho}$. By taking $\theta_{t+k'} = \frac{1}{\rho}$ as given and using the specification of the discount rate in Eq. 16 and the difference equation in Eq. 18, we can recover $\theta_{t+k'-1}$. In turn, we can similarly use $\theta_{t+k'-1}$ to back out $\theta_{t+k'-2}$ and so on until we recover the entire trajectory back to time t .¹⁴

Figure 1 illustrates the impact of time variation in the discount rate, ρ_t , given in Eq. 16 on the time series features of the house price series, P_t , in Eq. 19 with a simple numerical example.¹⁵ Our findings suggest that declines in interest rates can rationalize (at least qualitatively) run-ups in house prices, the explosiveness in the time series and increased volatility without having to appeal to non-fundamental explanations (rational bubbles).

In short, the above analysis highlights the importance of taking into consideration all three factors (rational bubbles, time variation in the discount rate and explosive fundamentals) when explaining changes in the time series properties of house prices.

Testing for Episodes of Explosive Behavior

This section describes the econometric methods that we employ to test for explosive behavior and provides technical details for their estimation. The first part of the section deals with univariate tests—the Sup ADF (*SADF*) of Phillips and Yu (2011) and the Generalized SADF (*GSADF*) of Phillips et al. (2012, 2015)—and the second part with our proposed extension to a multivariate setting.

The Univariate *SADF* and *GSADF* Procedures

Consider the following ADF regression equation,

$$\Delta y_t = a_{r_1, r_2} + \beta_{r_1, r_2} y_{t-1} + \sum_{j=1}^k \psi_{r_1, r_2}^j \Delta y_{t-j} + \epsilon_t, \epsilon_t \stackrel{iid}{\sim} N(0, \sigma_{r_1, r_2}^2), \quad (20)$$

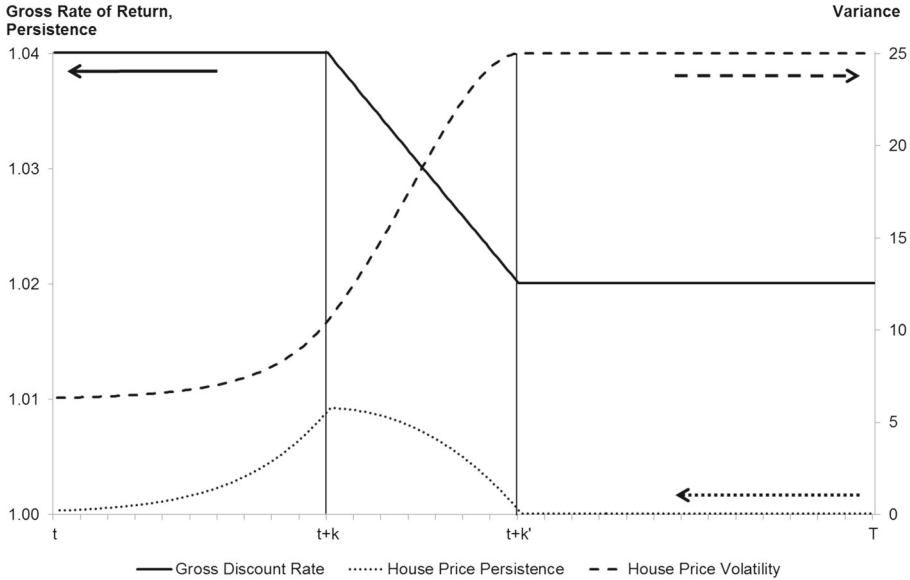
where y_t denotes a generic time series (using the notation of the previous section, y_t can be either P_t , $\frac{P_t}{X_t}$ or $\frac{P_t}{Y_t}$), Δy_{t-j} for $j = 1, \dots, k$ are the differenced lags of the time series, and ϵ_t is the error term. Moreover, r_1 and r_2 denote fractions of the total sample size that specify the starting and ending points of a subsample period, k is the maximum number of lags included in the specification, and a_{r_1, r_2} , β_{r_1, r_2} and ψ_{r_1, r_2}^j with $j = 1, \dots, k$ are regression coefficients.

The emergence of explosive behavior in house prices defines a period of exuberance and is indicated by a shift from a random walk—under the assumption that fundamentals are $I(1)$ —to mildly explosive behavior. Therefore, we are interested in

¹⁴We can also show that the persistence term $\frac{\theta_{t-1}}{\theta_{t-2}}$ in the house price equation is bounded below by g and above by $g^{k'-k}$ over the period from t up to $t + k'$.

¹⁵For the numerical example, we set $\rho = 0.02$, $g = 1.0002774397$, $k' - k = 70$ and $\sigma_\epsilon^2 = 0.01$.

Model Simulation Results



SOURCE: Authors' calculations.

Fig. 1 Real house prices: the dynamic effects of an anticipated decline in the discount rate

testing the null hypothesis of a unit root in y_t , $H_0 : \beta_{r_1, r_2} = 0$, against the alternative of mildly explosive behavior, $H_1 : \beta_{r_1, r_2} > 0$. Let

$$ADF_{r_1}^{r_2} = \frac{\hat{\beta}_{r_1, r_2}}{\text{s.e.}(\hat{\beta}_{r_1, r_2})} \quad (21)$$

denote the test statistic corresponding to this null hypothesis. Setting $r_1 = 0$ and $r_2 = 1$ yields the standard ADF test statistic, ADF_0^1 . The limit distribution of ADF_0^1 is given by

$$\frac{\int_0^1 W dW}{\left(\int_0^1 W^2\right)^{\frac{1}{2}}}, \quad (22)$$

where W is a Wiener process. The ADF test compares the ADF_0^1 statistic with the right-tailed critical value from its limit distribution. When the test statistic exceeds the corresponding critical value, the unit root hypothesis is rejected in favor of the alternative of explosive behavior.

Although widely employed, the standard ADF test has extremely low power in detecting episodes of explosive behavior when these episodes end with a large drop in prices, i.e., in the presence of boom-bust dynamics. For example, it is well established that nonlinear dynamics, such as those displayed by episodes of explosiveness

that are periodically collapsing, frequently lead to finding spurious stationarity even though the process is inherently explosive (see, e.g., Evans 1991).¹⁶

In order to deal with the effect of a collapse in house prices on the test's performance, Phillips and Yu (2011) proposed a recursive procedure based on the estimation of the *ADF* regression in Eq. 20 on subsamples of the data. Normalizing the end of the original sample to $T = 1$, the authors propose estimating (20) using a forward expanding sample with the end of the sample period, r_2 , increasing from r_0 (the minimum window size for the fixed initial window) to 1 (the last available observation). For this procedure, the beginning of the sample is held constant at $r_1 = 0$, and the expanding window size of the regression (over the normalized sample) is denoted by $r_w = r_2 - r_1$. Then, while the starting point of the estimation is kept fixed at $r_1 = 0$, the *ADF* regression is recursively estimated, while incrementing the window size, $r_2 \in [r_0, 1]$, by adding one additional observation at a time. Each estimation yields an *ADF* statistic denoted as $ADF_0^{r_2}$.

The Phillips and Yu (2011) test statistic, *SADF*, is defined as the supremum value of the $ADF_0^{r_2}$ sequence expressed as follows:

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2}. \quad (23)$$

Under the null hypothesis of a random walk, the limit distribution of the *SADF* statistic is given by

$$\sup_{r_2 \in [r_0, 1]} \frac{\int_0^{r_2} W dW}{\left(\int_0^{r_2} W^2\right)^{\frac{1}{2}}}. \quad (24)$$

Similar to the standard *ADF* test, when the *SADF* statistic exceeds the right-tailed critical value from its limit distribution, the unit root hypothesis is rejected in favor of explosive behavior.

The *SADF* test performs well when there is a single boom-bust episode within the sample. Simulation experiments in Homm and Breitung (2012) show that the *SADF* test outperforms alternative testing methods—such as the modified versions proposed by Bhargava (1986), Buseti and Taylor (2004), Kim (2000) and Kim et al. (2002)—in the presence of a single change in the persistence from a random walk to an explosive process.¹⁷

More recently, Phillips et al. (2015) derive a new unit root test, *GSADF*, that covers a larger number of subsamples than the *SADF* by allowing both the ending point, r_2 , and the starting point, r_1 , to change. This extra flexibility on the estimation window

¹⁶Evans (1991) shows using simulation methods that standard unit root and cointegration tests cannot reject the null of no explosive behavior when such periodically collapsing episodes are present in the data. Price increases during the boom followed by a decline during the correction phase make it look like a mean-reverting (stationary) process. Intuitively, this is the reason why many non-recursive unit root tests wrongly suggest that processes that incorporate periodically collapsing boom-bust episodes are stationary—as indicated by Evans (1991).

¹⁷These approaches are used to test a permanent change in persistence from a random walk to an explosive process. As a consequence, they perform well only in cases where the series becomes explosive but never bursts in-sample.

results in substantial power gains in comparison to the *SADF*. Furthermore, it makes the *GSADF* test consistent with multiple boom-bust episodes within a given time series—while the *SADF* test is consistent only with a single episode.

The *GSADF* statistic is defined as

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \quad (25)$$

Under the null hypothesis, the limit distribution of the *GSADF* statistic is

$$\sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r) dr [W(r_2) - W(r_1)]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r_2} W(r)^2 dr - \left[\int_{r_1}^{r_2} W(r) dr \right]^2 \right\}^{1/2}} \right\}, \quad (26)$$

where the window size of each estimation is $r_w = r_2 - r_1$. Again, rejection of the unit root hypothesis in favor of explosive behavior requires that the test statistic exceeds the right-tailed critical value from its limit distribution given by Eq. 26.

The Date-Stamping Strategy If the null of a unit root in y_t is rejected, then the *SADF* and *GSADF* procedures can be used to obtain a chronology of exuberance in the housing market. The identification of periods where house prices (or price-to-fundamental ratios) displayed mildly explosive behavior is particularly important since it is a necessary condition for shedding light on the factors and developments that led to the 2008–09 global recession. We focus on the date-stamping strategy associated with the *GSADF* procedure due to its favorable power properties and its consistency in the presence of multiple boom-bust episodes.

Phillips et al. (2012, 2015) recommend a dating strategy under the *GSADF* approach based on the Backward *SADF* statistic, i.e.,¹⁸

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} SADF_{r_1}^{r_2}. \quad (27)$$

The origination date of the period of exuberance is defined as the first observation for which the *BSADF* statistic exceeds its critical value,

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) > scu_{[r_2 T]}^\alpha \right\}, \quad (28)$$

and the termination date as the first observation after \hat{r}_e for which the *BSADF* falls below its critical value,

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < scu_{[r_2 T]}^\alpha \right\}, \quad (29)$$

where $scu_{[r_2 T]}^\alpha$ is the $100(1 - \alpha)\%$ critical value of the *SADF* based on $[r_2 T]$ observations and α is the chosen significance level. The consistency of the above

¹⁸The Backward *SADF* (*BSADF*) statistic relates to the *GSADF* statistic as follows,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} \{ BSADF_{r_2}(r_0) \}.$$

dating strategy in the presence of one or two explosive periods that are periodically collapsing is established in Phillips et al. (2015).

Technical Details The computation of the *SADF*, *GSADF* and *BSADF* test statistics necessitates the selection of the minimum window size, r_0 , and the autoregressive lag length, k . Regarding the minimum window size, this has to be large enough to allow initial estimation, but it should not be too large to avoid missing short episodes of exuberance. We follow Phillips et al. (2012) and set the minimum size equal to 36 observations. This minimum window size is also close to the size suggested by the rule of thumb of Phillips et al. (2015), $r_0 = 0.1 + 1.8/\sqrt{T}$.¹⁹

With respect to the autoregressive lag length, k , we evaluate our results primarily for two cases, k equal to 1 and 4.²⁰ Our findings do not appear very sensitive to the lag length specification and, for this reason, we only report results for $k = 4$. More sophisticated lag length selection procedures based on information criteria (such as the Modified Information Criteria of Ng and Perron 2001) and sequential hypothesis testing (Ng and Perron 1995) could, in principle, be applied but with a higher computational cost. Moreover, Phillips et al. (2012) show that such procedures can result in severe size distortions, and a reduction in power of both the *SADF* and *GSADF* tests. That is, they frequently lead to rejecting the null hypothesis when in fact the time series follows a unit root process, and not rejecting the null when the time series is explosive.

The implementation of the unit root tests also requires the limit distributions of the *SADF*, *GSADF* and *BSADF* test statistics. These distributions are non-standard and depend on the minimum window size. Hence, critical values have to be obtained through Monte Carlo simulations. We obtain finite sample critical values by generating 2000 replications of a driftless random walk process with $N(0, 1)$ errors.²¹

Finally, the researcher may choose to neglect very short periods of exuberance by setting a minimum duration period. Phillips et al. (2015) recommend a minimum duration of $\log(T)/T$. Since all of our house price time series have 154 quarterly observations, the minimum duration that we adhere to in our empirical evaluation corresponds to 5 quarters.

The Panel *GSADF* Procedure

The International House Price Database of the Federal Reserve Bank of Dallas has a large cross-sectional dimension of which we use data for 22 countries in our study. The *SADF* and *GSADF* tests described in the previous sub-section, however, can only be applied country by country and, hence, cannot exploit the panel nature of our

¹⁹Exploring alternative minimum window sizes can be computationally demanding since for each r_0 new critical values must be computed.

²⁰The choice of a fixed lag length is appealing because it allows us to employ a recursive least squares approach, which substantially reduces the computational cost of estimation.

²¹Using asymptotic critical values doesn't qualitatively change our results. Asymptotic values are provided in Phillips et al. (2015).

dataset. To the best of our knowledge, there is no sequential right-tailed unit root test for panel data in the literature. Inspired by the work of Im et al. (2003), we propose an extension of the *GSADF* test procedure to heterogeneous panels.²²

Consider the panel version of the *ADF* regression equation in Eq. 20,

$$\Delta y_{i,t} = a_{i,r_1,r_2} + \beta_{i,r_1,r_2} y_{i,t-1} + \sum_{j=1}^k \psi_{i,r_1,r_2}^j \Delta y_{i,t-j} + \epsilon_{i,t}, \quad (30)$$

where $i = 1, \dots, N$, denotes the country index, and the remaining variables are defined as in the previous sub-section. We are interested in testing the null hypothesis of a unit root, $H_0 : \beta_{i,r_1,r_2} = 0$ for all N countries, against the alternative of explosive behavior in a subset of countries, $H_1 : \beta_{i,r_1,r_2} > 0$ for some i . This alternative allows for β_{i,r_1,r_2} to differ across countries and, in that sense, is more general than approaches based on the homogeneous alternative hypothesis.

We propose a panel unit root test computed by taking the average of the individual *BSADF* statistics at each time period. To facilitate the analysis, we adjust the notation for univariate test statistics to include the country index, i ,

$$ADF_{i,r_1}^{r_2} = \frac{\hat{\beta}_{i,r_1,r_2}}{\text{s.e.}(\hat{\beta}_{i,r_1,r_2})}, \quad (31)$$

$$SADF_{i,r_1}^{r_2} = \sup_{r_2 \in [r_1, r_2]} ADF_{i,r_1}^{r_2}, \quad (32)$$

and

$$BSADF_{i,r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} SADF_{i,r_1}^{r_2}. \quad (33)$$

The panel *BSADF* can now be defined as

$$\text{Panel } BSADF_{r_2}(r_0) = \frac{1}{N} \sum_{i=1}^N BSADF_{i,r_2}(r_0). \quad (34)$$

The panel *BSADF* statistic is particularly appealing because it measures the degree of *overall* exuberance in international housing markets. Having defined the panel *BSADF*, the definition of the panel *GSADF* follows naturally. It is simply the supremum of the panel *BSADF*, i.e.,

$$\text{Panel } GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} \text{Panel } BSADF_{r_2}(r_0). \quad (35)$$

The results of Maddala and Wu (1999) and Chang (2004) show that the distribution of panel unit root tests based on mean unit root statistics is not invariant to cross-sectional dependence of the error terms, ϵ_i . In light of the ample evidence of strong financial linkages across countries (see, e.g., Lane and Milesi-Ferretti 2003), the assumption of uncorrelated shocks seems unrealistic even for international housing markets. In order to draw inferences in this context, we adopt a sieve bootstrap approach that is designed specifically to allow for cross-sectional error dependence. Details of the sieve bootstrap can be found in the [Appendix](#).

Dating episodes of exuberance in the global housing market can be performed by comparing the panel *BSADF* with the sequence of bootstrap critical values. The

²²We are grateful to an anonymous referee for motivating this extension.

origination date is set equal to the first observation that the panel *BSADF* statistic exceeds the $100(1 - \alpha)\%$ critical value obtained from the bootstrap procedure, and the termination date is set equal to the first observation that the panel *BSADF* falls below the $100(1 - \alpha)\%$ critical value.

Empirical Evidence on International House Prices

The sources and methodology used to construct the 22-country panel of house prices and personal disposable income from the International House Price Database are documented in Mack and Martínez-García (2011). The data on real house prices and real personal disposable income per capita is reported quarterly, deflated with the PCE deflator, and covers the period from the first quarter of 1975 to the second quarter of 2013. From this data, we construct an affordability index for housing as the ratio (in percent) of real house prices over real personal disposable income per capita for each country. For 16 out of the 22 countries that we consider, a quarterly index of real housing rents is also available from the OECD (Girouard et al. 2006). We use this rent index to construct the price-to-rent ratio for the 16 countries with available data.

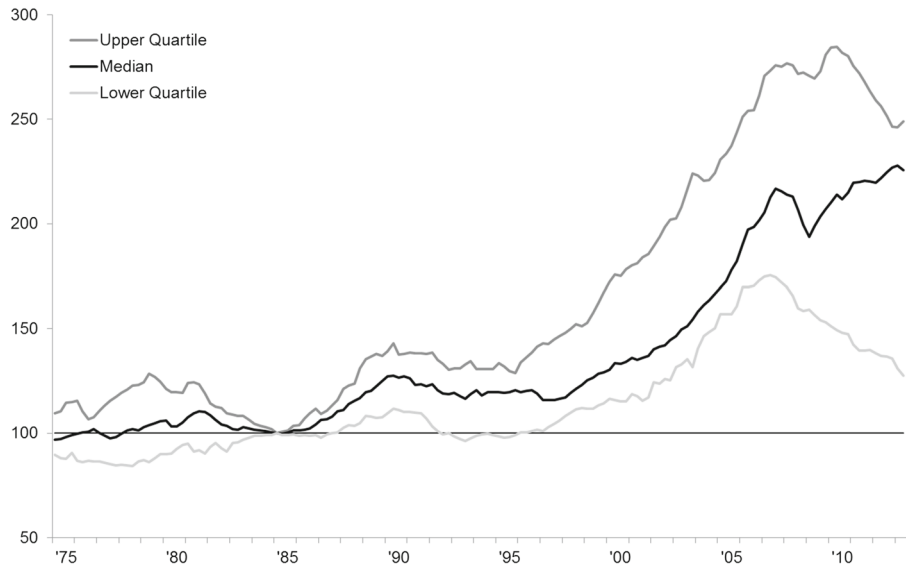
The sample period covered in our dataset includes several recessions, which makes it ideal for contrasting the timeline of the boom and bust in international housing markets prior to the 2008–09 global recession against the experience during previous periods of contraction in economic activity. The median and lower and upper quartile of house prices of all 22 countries are displayed in Fig. 2. We observe from the figure that for the median country, real house prices troughed in the mid-1990s and peaked around 2006. Furthermore, we can see that the run-up in real house prices during this period is widespread—an observation that has fueled the view that the latest boom-bust episode in housing had a part in the 2008–09 global recession. This view is further supported by the fact that the ratios of house prices to their long-run anchors (income and rents) display similar patterns (see Fig. 3). This motivates our first research question of whether there is formal statistical evidence that substantiates the claim that housing markets were exuberant prior to the 2008–09 global recession.

Empirical Findings: A Chronology of Exuberance in International Housing Markets

Table 1 reports results for the *SADF* and *GSADF* tests on real house prices, the price-to-income ratio and the price-to-rent ratio. A comparison of the results of the two methods reveals large differences. Starting with the *GSADF* test, there is strong evidence of exuberance in real house prices with the null hypothesis of a unit root being rejected for all countries but three: Finland, Italy and South Korea. The evidence in favor of mildly explosive behavior remains strong when we look at the price-to-income ratio (the null cannot be rejected only for Finland, Italy, South Korea and Norway) and the price-to-rent ratio (the null cannot be rejected only for Germany, France and Italy). Turning to the *SADF* test, we observe that the number of rejections of the unit root hypothesis is substantially smaller than that for the *GSADF*,

Real House Price Index

1985 Q1=100



SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Fig. 2 Real house prices: cross-country characteristics

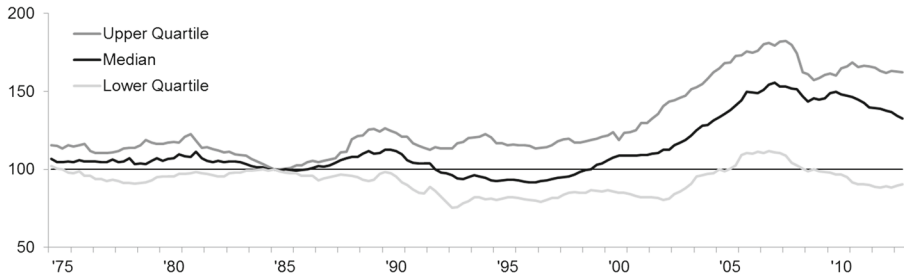
which is in line with the higher power of the latter. In particular, the *SADF* test cannot reject the null for more than half of the countries of our sample (12 out of the 22) when we examine real house prices. When we look at the price-to-income and price-to-rent ratios, the evidence in favor of exuberance becomes even weaker with fewer rejections. Overall, these results indicate that episodes of explosive behavior were widespread across a large number of countries and that more powerful testing procedures than *SADF* are required to detect them.²³

We now turn to the chronology of exuberance. Figures 4 and 5 show the periods during which the series under examination displayed explosive dynamics (i.e., the periods during which the estimated *BSADF* statistics exceed the corresponding 95 % critical values). Focusing on the results for real house prices, three phases can be identified in connection with the latest boom and bust episode in international housing markets.

²³National house price indices aggregate across dwelling types and diverse locations within a country which may impact the performance of the econometric tests described in the previous section. In order to examine the effect of aggregation on the properties of the *SADF* and *GSADF* tests, we have conducted a large simulation experiment based on the S&P/Case-Shiller 10-City Composite Home Price Index and its constituent series. The results of the simulation experiment (which are available upon request from the authors) illustrate that aggregating lowers the power of both the *SADF* and *GSADF* tests. The effect is much larger for the *SADF* test than for the *GSADF* test, which gives us another reason to prefer the latter in our econometric strategy.

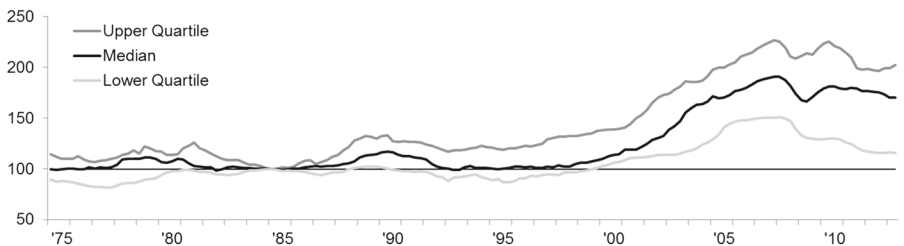
Ratio of House Price to PDI Per Capita

1985 Q1=100



Ratio of House Price to Rent

1985 Q1=100



SOURCE: Federal Reserve Bank of Dallas' International House Price Database; OECD; authors' calculations.

Fig. 3 Price-to-income ratio and price-to-rent ratio: cross-country characteristics

In the first phase, between the mid-1990s and the early 2000s, real house prices became explosive in the U.S. and Ireland. There were also concurrent episodes detected for Norway and Switzerland at the time, but those seem to have evolved and eventually collapsed on their own.

In the second phase of the boom period, i.e., by the early-to-mid 2000s, the number of countries with explosive real house prices shot up to 18 (out of a total of 22). This near-simultaneous exuberance in house prices is particularly interesting given the substantial differences across domestic housing markets and the non-tradability of housing. As evident from Figs. 4 and 5, this phenomenon has no precedent, at least in our sample period. A potential explanation for the first two phases is that the boom in house prices that originated in the U.S. propagated to international housing markets with perceived greater opportunities or lower risks. The pattern of propagation observed in the data and the high synchronization suggest that a common factor may have contributed to house price exuberance spreading across countries. In this regard, ripple effects from the decline in world interest rates experienced during the 2000s and housing bubbles are prime candidates (Case and Shiller 2003).

In the third and final phase, the run-up in house prices started to be perceived as not sustainable, and concerns about real economic activity in both the U.S. and around the world grew. The episode ended with a collapse of house prices around 2006, making apparent the economic implications of the boom and bust for the U.S. and the world.

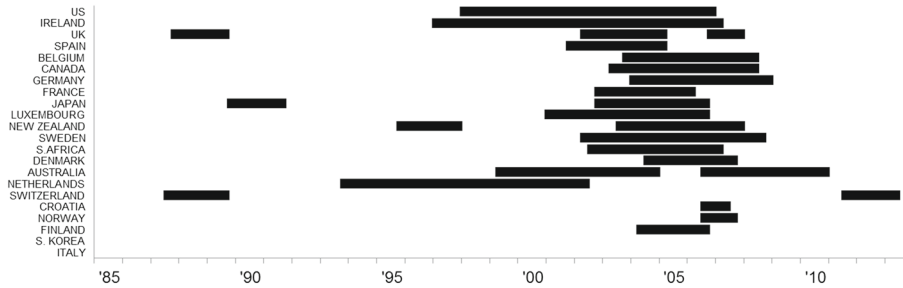
Table 1 Results for the univariate SADF and GSADF tests

Country	Real house prices		Price-to-income ratio		Price-to-rent ratio	
	<i>SADF</i>	<i>GSADF</i>	<i>SADF</i>	<i>GSADF</i>	<i>SADF</i>	<i>GSADF</i>
Panel A: Test statistics						
Australia	2.23***	6.18***	1.08*	2.57***	2.21***	6.40***
Belgium	0.97	2.98***	−0.25	2.92***	0.03	3.41***
Canada	0.32	3.76***	−1.13	2.16**	0.11	3.78***
Switzerland	1.64**	2.3**	1.20*	2.08**	0.47	1.90**
Germany	−0.59	2.10**	0.57	2.55***	0.51	0.85
Denmark	1.31**	2.83***	−0.03	1.76*	0.37	2.98***
Spain	0.39	3.34***	0.01	1.84**	−0.16	2.20**
Finland	0.94	1.45	−0.89	0.96	1.70**	1.70*
France	1.35**	2.21**	−0.03	2.46***	−0.36	1.48
United Kingdom	1.83**	3.34***	1.50**	2.65***	−0.26	3.02***
Ireland	2.59***	3.71***	2.01***	2.19**	2.33***	4.54***
Italy	−1.28	−0.38	−1.52	0.85	−1.93	−0.26
Japan	1.66**	3.76***	0.88	4.63***	1.44**	2.24**
South Korea	−1.11	−0.32	0.49	0.49	—	—
Luxembourg	1.65**	3.89***	−0.27	1.59*	—	—
Netherlands	−0.43	4.13***	−0.17	3.13***	−1.49	3.55***
Norway	0.85	1.75*	0.22	0.31	—	—
New Zealand	1.77**	2.35**	0.43	3.10***	2.82***	3.89***
Sweden	0.18	3.79***	0.23	3.34***	—	—
United States	1.52**	3.81***	−0.78	3.47***	−0.55	3.64***
South Africa	−0.92	3.93***	−1.35	3.44***	—	—
Croatia	0.03	1.64*	0.87	2.23**	—	—
Panel B: Critical Values						
90 %	0.98	1.54	0.98	1.54	0.98	1.54
95 %	1.25	1.80	1.25	1.80	1.25	1.80
99 %	1.89	2.39	1.89	2.39	1.89	2.39

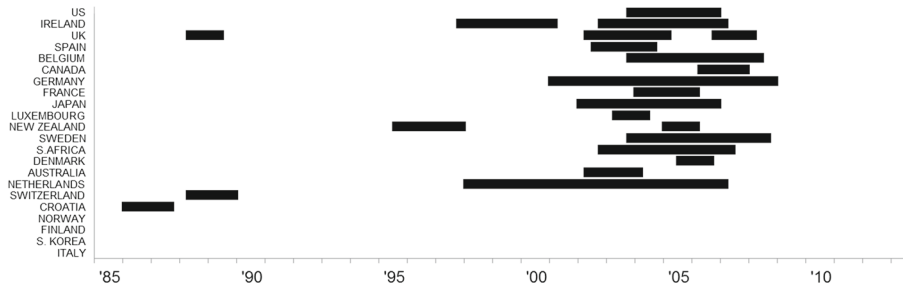
Note: *, ** and *** denote statistical significance at the 10, 5 and 1 percent significance levels respectively. All results are for autoregressive lag length $k = 4$.

Looking at the results for price-to-fundamental ratios, we observe a similar pattern to real house prices. Perhaps not surprisingly, however, the periods of explosive dynamics in the price-to-fundamental ratios are somewhat shorter. For the U.S., in particular, we observe that the house-price-to-income ratio did not become explosive until the early 2000s. The fact that U.S. real house prices were explosive in the late 1990s but the house-price-to-income ratio was not implies that explosiveness may have been driven in part by economic fundamentals. This conclusion is in line with

Real House Price Index



Price-to-Income Ratio



NOTE: Shaded areas indicate periods of exuberance determined by the GSADF test.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

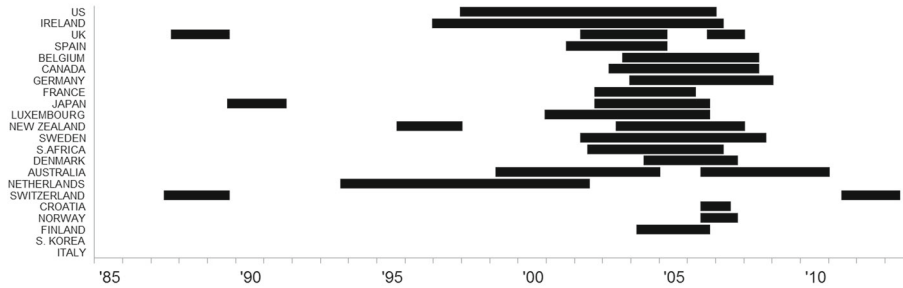
Fig. 4 Date-stamping with real Hhouse prices and the price-to-income ratios

the strong growth (partly due to the new information technologies) in U.S. income in the 1990s.

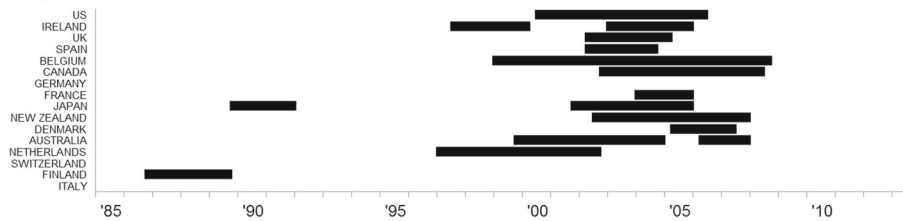
There is an unusual synchronization in the episodes of explosive behavior across most of the countries in the sample since the early-to-mid-2000s. This period of near-simultaneous exuberance was pervasive across very different housing markets whose fundamentals were not necessarily similar. This episode is also unprecedented given the sample period and country coverage in our dataset. An overall picture of the behavior of international housing markets is given by the results for the panel *GSADF* test. The panel *GSADF* statistics are statistically significant for both real house prices and price-to-fundamental ratios, which provides strong evidence in favor of global exuberance in our sample.

What is more interesting is the time evolution of the three panel *BSADF* statistics displayed in Fig. 6. Irrespective of the variable under examination (real house prices, the price-to-income ratio and the price-to-rent ratio), the results demonstrate in a clear manner the three phases of the housing market: the *BSADF* statistics start below their critical values at the beginning of the sample, they increase rapidly after the mid-1990s and they eventually exceed their critical values during the second phase starting in the early 2000s, with the period of global exuberance in housing markets continuing until 2006–07 at which point it quickly collapsed in all three statistics (near-)simultaneously.

Real House Price Index



Price-to-Rent Ratio



NOTE: Shaded areas indicate periods of exuberance determined by the GSADF test.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; OECD; authors' calculations.

Fig. 5 Date-stamping with real house prices and the price-to-rent ratios

The Cases of the U.S., the U.K. and Spain. Having established the emergence of global exuberance in housing markets, we now return to the experiences of individual countries and, specifically, the U.S., the U.K. and Spain. This choice is based on the economic size and significance of these countries, and because they exemplify the distinct patterns observed during the three main phases of the timeline of events that we describe in our paper. As can be seen from Figs. 7, 8 and 9, the real house price appreciation has been very significant for these three countries since the mid-1990s, with the run-up in the U.K. and Spain being larger over time than that in the U.S. This sets Spain and the U.K. apart, but as our evidence shows, it does not mean that explosive behavior is somehow a weaker phenomenon for the U.S.

Figures 7 to 9 show the estimated *BSADF* statistics for the three countries together with 95 % critical values. Focusing on real house prices, we observe that during the period of global exuberance leading to the 2008–09 global recession, the U.S. played the leading role with the U.K. and Spain following.

In spite of the differences between the housing markets of Spain and the U.K., our findings show that both countries went through a near-simultaneous period of exuberance during the second phase of the timeline—although the U.K. also experienced an echo before the final collapse ahead of the 2008–09 recession. This pattern of strong synchronization in housing exuberance across countries is in line with the view that exuberance in real house prices in the U.S. migrated and amplified the effect of domestic factors in these two (otherwise very different and distant) housing markets.

Panel BSADF Indicators

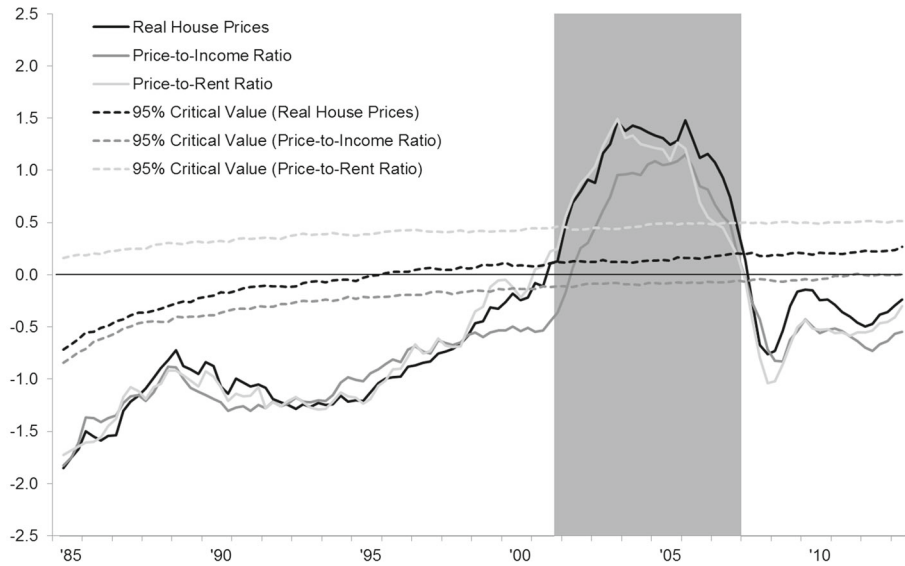


Fig. 6 Date-stamping global periods of exuberance

The Predictive Ability of Macro and Financial Variables

Having established a timeline of exuberance in international housing markets, we employ a pooled probit model to assess the in-sample predictive ability of macroeconomic and financial variables. The probit model is described by

$$P(EXU_{i,t} = 1) = \Phi(x_{i,t}\beta), \quad (36)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function, $x_{i,t}$ is a vector of predictors for all $i = 1, \dots, N$, and $EXU_{i,t}$ is the dependent binary variable for all $i = 1, \dots, N$ that takes the value of 1 if there is evidence of exuberance in country i and 0 otherwise, i.e.,²⁴

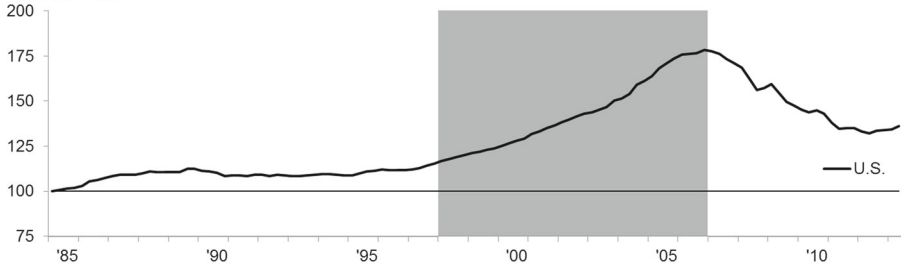
$$EXU_{i,t} = \begin{cases} 0, & \text{if } BSADF_{i,t}(r_0) > scu_t^\alpha, \\ 1, & \text{if } BSADF_{i,t}(r_0) < scu_t^\alpha. \end{cases} \quad (37)$$

Our choice of predictors is based partly on the theoretical model of house price determination discussed in Section “[House Prices, Fundamentals and Rational Bubbles](#)”—particularly in regards to the role of time variation in interest rates and related

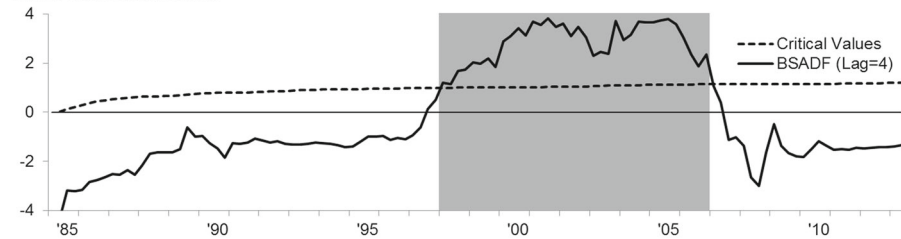
²⁴The variability of EXU_i within a country is limited because the *GSADF* methodology does not detect many episodes of exuberance. An advantage of the pooled probit model is that, by incorporating the full variability across countries, it increases the number of episodes leading to more tightly identified results.

Exuberance in Real House Prices

Real House Price Index
1985 Q1 = 100



BSADF and Critical Values



NOTE: Shaded area indicates period when real house prices surpass critical value.

SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Fig. 7 Date-stamping with U.S. real house prices

financial variables. We also include a number of macro factors which the existing literature has found to be relevant for predicting booms and busts (or turning points of the housing cycle). A detailed description of the dataset is provided in Grossman et al. (2014). This dataset is complemented with OECD and BIS data, also with national and other sources, whenever necessary.

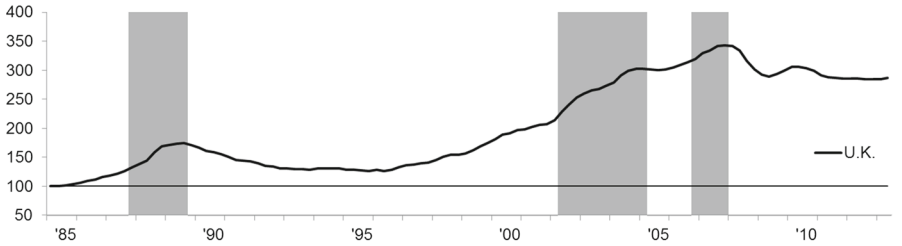
Among the financial predictors, we consider the spread between the long- and short-term interest rates. The spread proxies the slope of the yield curve and indicates market expectations of future policy rates. We also include long-term interest rates to proxy for mortgage rates and the effect they may have on the likelihood of a period of exuberance. Long-term rates also provide a financial measure of the opportunity costs of investing in housing.

In addition, using OECD data, we consider quarter-over-quarter changes in stock markets in part because we expect stock prices to be forward-looking and, moreover, to reflect the profitability of alternative asset classes. Furthermore, stock market appreciation is related to changes in households' financial wealth.

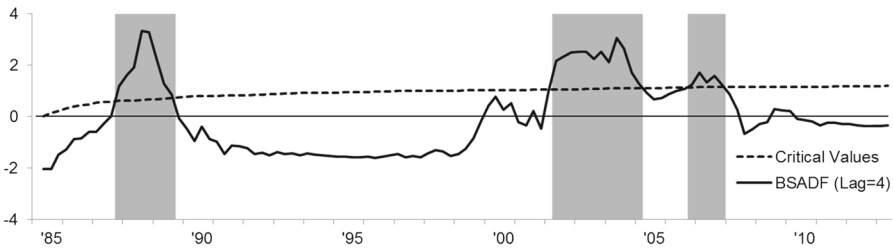
We incorporate measures of domestic nominal credit growth to the private sector from the BIS in quarter-over-quarter growth rates as well as the current-account-to-GDP ratio and the quarter-over-quarter changes in the current account. An expansion of private credit can lead to asset price and housing booms, and capital inflows from abroad can fuel a rapid expansion of domestic credit. Hence, we aim to determine whether periods of exuberance can be predicted not just from the extent of the preceding credit expansion, but also from whether the funding arises externally.

Exuberance in Real House Prices

Real House Price Index
1985 Q1=100



BSADF and Critical Values



NOTE: Shaded areas indicate period when real house prices surpass critical value.

SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Fig. 8 Date-stamping with U.K. real house prices

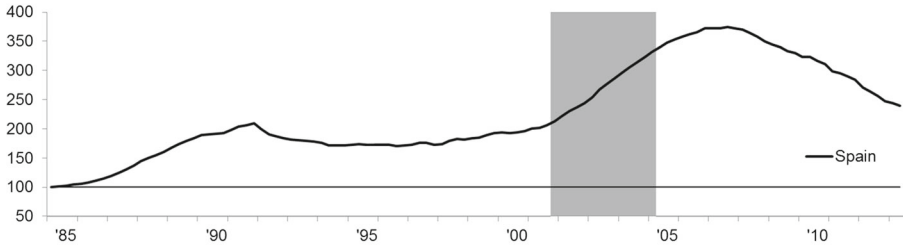
We also examine quarter-over-quarter changes in real personal disposable income per capita and, based on OECD data, of real housing rents too. Housing rents proxy for time variation in the expected returns on housing—the relationship between house prices and rents is at the core of our asset-pricing model of housing determination. Regarding real personal disposable income, this variable is meant to capture key demand-side fundamentals that are conventionally viewed as anchoring the housing market over the long run.

As indicators of the state of the business cycle, we use the unemployment rate from national sources, real GDP growth (quarter-over-quarter) and CPI inflation (quarter-over-quarter). These variables are not generally viewed as leading indicators, but they can have predictive power on future consumption and investment on housing. Moreover, we use the updated quarterly average of the global indicator of real economic activity proposed by Kilian (2009) and oil prices on the West Texas Intermediate from the Wall Street Journal to account for global economic developments.

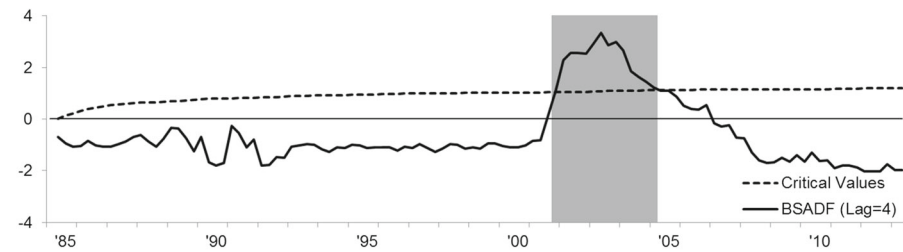
Due to the fact that we do not have data on rent and private credit growth for all countries, our final dataset consists of a subset of 14 countries that have experienced at least one period of exuberance in-sample and for which we have all available data: Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the U.K., Ireland, Japan, Netherlands and the U.S. To specify the pooled probit model, we start with a general specification including all our explanatory variables. This model is sequentially reduced by deleting the insignificant variable

Exuberance in Real House Prices

Real House Price Index
1985 Q1=100



BSADF and Critical Values



NOTE: Shaded area indicates period when real house prices surpass critical value.

SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Fig. 9 Date-stamping with Spain real house prices

with the highest p-value after each iteration until all remaining variables appear statistically significant.

The estimation results for the final specification of the pooled probit model with and without random effects can be found in Table 2. Overall, there is evidence that financial and macro variables, at least in-sample, have predictive power for exuberance. Our findings suggest that the factors that matter for predicting an episode of exuberance are: First, mortgage costs and access to mortgage credit, which are proxied by long-term rates and credit growth to the private sector. Foreign savings/dissavings and the current account also play a role. Second, demand-side factors of the housing market and, in particular, per capita real disposable income growth and the unemployment rate. Third, domestic and global macroeconomic conditions, where domestic conditions are proxied by domestic real GDP growth, while global economic conditions are proxied with the index of global economic activity of Kilian (2009).

These results support the important role of declining long-term interest rates in the synchronization of domestic housing markets, in line with the predictions of our stylized asset-pricing model. Current account changes also matter, lending some empirical support to the view that international capital flows may contribute to the propagation of exuberance across countries. Moreover, our evidence highlights that global economic conditions contribute positively to the likelihood of exuberance. This finding seems to conform with the anecdotal evidence that during the boom years prior to the 2008–09 global recession, international investors—reaping the

Table 2 Estimation results for the pooled probit model

Long-term interest rates	-0.166***	-0.221***
Private credit growth	0.222***	0.207***
Current account growth	-0.004*	-0.005**
Per capita personal disposable income growth	0.209***	0.223***
Unemployment rate	-0.073**	-0.130**
Real GDP growth	0.169***	0.176***
Global economic conditions	0.014***	0.014**
Random-effects	No	Yes
McFadden's R^2	0.230	0.252

Note: *, ** and *** denote statistical significance at the 10, 5 and 1 percent significance levels, respectively. We fit a probit model with constant term (first column) and a random-effects probit model (second column). The balanced panel includes 14 countries with at least one period of exuberance in-sample and complete data from the first quarter of 1975 to the second quarter of 2013 on interest rate spreads, long-term rates, changes in stock market valuations, private credit growth, the current-account-to-GDP ratio, growth in the current account, growth in real per capita disposable income, growth in real rents, the unemployment rate, real GDP growth, headline CPI inflation, changes in the West Texas Intermediate index of oil prices, and the updated measure of global economic conditions of Kilian (2009). We only report the results of a parsimonious specification of each model after recursively eliminating the non-statistically significant variables with the highest p-values. Hence, the specification reported in the table only includes long-term rates, private credit growth, current account growth, real per capita personal disposable income growth, the unemployment rate, real GDP growth and Kilian (2009)'s global conditions measure. Standard errors are adjusted for country-clusters.

benefits of strong global growth—turned their attention increasingly towards housing markets for investment, fueling housing price run-ups across many different countries.

The last row of Table 2 also reports McFadden's pseudo R^2 for the probit model with and without random effects. The estimates for the pseudo R^2 are 0.230 and 0.252, respectively, which suggests that although the macro and financial variables examined have strong predictive content, there is ample room for other explanatory factors. According to the theoretical analysis of section “[House Prices, Fundamentals and Rational Bubbles](#)”, unobserved fundamentals and rational housing bubbles may well be such factors.

Concluding Remarks

In this paper, we employed two econometric procedures, namely the *SADF* and *GSADF*, developed recently by Phillips and Yu (2011, 2015) in order to examine whether house prices and house-price-to-fundamental ratios exhibited explosive behavior during the last four decades and, if so, to identify the exact periods of exuberance. An appealing feature of our study is the use of data from the International House Price Database of the Federal Reserve Bank of Dallas, which has a large cross-sectional dimension and, therefore, allows a unique, international look at the behavior of housing markets.

A consistent timeline of events emerges from our empirical evidence, suggesting that the latest boom-bust cycle in international housing markets was unusually widespread and evolved in three phases: One of origination that can be related to the U.S. experience primarily during the second half of the 1990s; a second phase of propagation that is characterized by widespread and synchronized episodes of exuberance across very different housing markets during the first half of the 2000s; and a final phase where this episode of global exuberance burst within a short period of time right before the severe contraction in economic activity of the 2008–09 global recession.

In order to exploit the cross-sectional dimension of our dataset, we also propose an extension of the *GSADF* test to a panel setting. This extension is appealing because it allows us to draw general conclusions about international housing markets. Moreover, it is straightforward to implement and, like the *SADF* and *GSADF*, allows dating periods of exuberance. The results of the panel *GSADF* test provide further support to the view that exuberance in housing markets prior to the 2008–09 global recession was an international phenomenon.

Finally, we examined whether macroeconomic and financial variables impact the likelihood of exuberance in housing markets. In particular, we employed a panel probit model with a large number of predictors that have been suggested in the literature to be important determinants of housing cycles. Our results suggest that long-run interest rates, credit growth, demand-side factors (such as per capita real disposable income growth) and domestic and global macroeconomic conditions impact the likelihood of a housing market being in a state of exuberance. These findings provide (at least partially) an explanation for the widespread exuberance detected in the early- and mid-2000s.

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Appendix A: Demand Equation for Rental Housing

Consider the maximization of the Stone-Geary utility function with housing units rented, H_t , and consumption of other goods, C_t , i.e.,²⁵

$$U(H_t, C_t) = (H_t - \theta_H)^\alpha (C_t - \theta_C)^{1-\alpha}, \quad 0 < \alpha < 1,$$

subject to the intratemporal budget constraint,

$$C_t + x_t H_t = Y_t,$$

²⁵ While the Stone-Geary reduces to the Cobb-Douglas utility function whenever the parameters θ_H and θ_C are both set equal to zero, the specification permits both the rental rate elasticity and the income elasticity to vary with both rental rates and income—unlike the Cobb-Douglas where both elasticities are constant or the constant elasticity of substitution utility function for which the income elasticity is constant.

where the price of the consumption good is normalized to 1. $X_t \equiv x_t H_t$ is the housing rents—rental expenditures—paid and x_t the rental rate per unit rented, Y_t refers to disposable income, and α , θ_H and θ_C are preference parameters.

From first-order conditions, the Stone-Geary utility function subject to the standard intratemporal budget constraint gives a linear expenditure system where the demand for rental housing takes the following form:

$$H_t = \theta_H + \frac{\alpha}{x_t} (Y_t - x_t \theta_H - \theta_C), \quad (38)$$

or in expenditure terms,

$$X_t \equiv x_t H_t = \alpha Y_t + (1 - \alpha) \theta_H x_t - \alpha \theta_C. \quad (39)$$

Under the assumption that in equilibrium the units rented are constant (i.e., $H_t = H$) and normalized to 1, the demand equation that determines housing rents in Eq. 39 reduces to an affine transformation of disposable income (Y_t), i.e.,

$$X_t = x_t = \theta_F + \delta Y_{t+1}, \quad (40)$$

where $\delta \equiv \frac{\alpha}{1 - (1 - \alpha)\theta_H}$ and $\theta_F \equiv -\frac{\alpha}{1 - (1 - \alpha)\theta_H}\theta_C$.

B The Panel GSADF Test

The bootstrap procedure consists of the following steps:

1. For each country, impose the null hypothesis of a unit root and fit the restricted *ADF* regression equation,

$$\Delta y_{i,t} = a_{i,r_1,r_2} + \sum_{j=1}^k \psi_{i,r_1,r_2}^j \Delta y_{i,t-j} + \epsilon_{i,t},$$

to obtain coefficient estimates (\hat{a}_{i,r_1,r_2} , and $\hat{\psi}_{i,r_1,r_2}^j$ for $j = 1, \dots, k$) and residuals ($\hat{\epsilon}_i$).

2. Create a residual matrix with typical element $\hat{\epsilon}_{i,i}$.
3. In order to preserve the covariance structure of the error term, generate bootstrap residuals, $\epsilon_{i,t}^b$, by sampling with replacement draws from the residual matrix.
4. Use the bootstrap residuals and the estimated coefficients to recursively generate bootstrap samples for first differences,

$$\Delta y_{i,t}^b = \hat{a}_{i,r_1,r_2} + \sum_{j=1}^k \hat{\psi}_{i,r_1,r_2}^j \Delta y_{i,t-j}^b + \epsilon_{i,t}^b,$$

and for levels,

$$y_{i,t}^b = \sum_{p=1}^t \Delta y_{i,p}^b.$$

5. Compute the sequence of panel *BSADF* statistics and the panel *GSADF* statistic for $y_{i,t}^b$.
6. Repeat steps (3) to (5) a large number of times to obtain the empirical distribution of the test statistics under the null of a unit root.

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