1. Given a graph $G = (V, E)$ with vertex weights $w_v > 0$, we consider the problem of finding an independent set $S \subseteq V$ with the maximum total weight, i.e., $w(S) := \sum_{v \in S} w_v$ is maximized.

   Let also a *nice* tree decomposition $(Tr = (T, F), \{V_t : t \in T\})$ of G be given.

   (a) (1 point) Define for a leaf node t of Tr the maximum total weight $OPT_t(U)$ for each possible subset U of the bag V_t.

   (b) (1 point) Define for a forget node t of Tr the maximum total weight $OPT_t(U)$ for each possible independent subset U of the bag V_t.

   (c) (2 points) Define for an introduce node t of Tr the maximum total weight $OPT_t(U)$ for each possible independent subset U of the bag V_t.

   (d) (1 point) Define for a join node t of Tr the maximum total weight $OPT_t(U)$ for each possible independent subset U of the bag V_t.

   (e) (1 point) Suppose that for all tree nodes t and for all arguments U the values defined in the above recursive function have been stored in a table $M_t[U]$. Explain how to determine the maximum total weight of an independent set of G from this table.

End of assignment