Exercises, lecture 2, 2.6

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a) We have n jobs
m machines
p_{ij} = processing time of job i on machine j

Variables:

$$x_{ij} = \begin{cases} 1 & \text{if job } i \text{ is assigned to machine } j \\ 0 & \text{otherwise} \end{cases}$$

C_j = makespan on machine j

$C = \max_j C_j$

Problem formulation:

$$\min \quad C$$

$$\text{s.t} \quad C \geq \sum_{i=1}^{n} p_{ij} x_{ij} \qquad j = 1, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, \ldots, n$$

$$x_{ij} \in \{0,1\} \qquad i = 1, \ldots, n, \ j = 1, \ldots, m$$

b) Here, we need to make sure that all jobs using the same resource are assigned to the same machine
The goal is to minimize the total processing time

This is a valid formulation:

$$\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} p_{ij} x_{ij}$$

$$\text{s.t} \quad \sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, \dots, n$$

$$x_{i_1 j} - x_{i_2 j} = 0 \qquad j = 1, \dots, m \text{ and all } i_1, i_2 \in N_r \text{ for all } r$$

$$x_{ij} \in \{0,1\} \qquad i = 1, \dots, n, \; j = 1, \dots, m$$

One can for instance also introduce a new variable

$$y_{jr} = \begin{cases} 1 & \text{if machine } j \text{ has resource } r \\ 0 & \text{otherwise} \end{cases}$$

In that case we can substitute the second constraint above by

$$x_{ij} - y_{jr} \leq 0 \qquad j = 1, \dots, m \quad i \in N_r$$

and add constraints

$$\sum_{j=1}^{m} y_{jr} \le 1 \quad \text{for all } r$$

$$y_{jr} \in \{0,1\} \quad j=1,\dots,m, \text{ all } r$$

(c)

Define

$$x_{ijt} = \begin{cases} 1 & \text{if job } i \text{ is starting on machine } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

$i=1,\dots,n, \quad j=1,\dots,m, \quad t=0,\dots,T$

$$y_{jrt} = \begin{cases} 1 & \text{if resource } r \text{ is used on machine } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$\min \quad z$$

$$z \geqslant (t+1)\cdot x_{ijt} \qquad t=0,\dots,T, \ i=1,\dots,n, \ j=1,\dots,m \qquad (1)$$

$$\sum_{j=1}^{m}\sum_{t=0}^{T} x_{ijt} = 1 \qquad i=1,\dots,m \qquad (2)$$

$$\sum_{i=1}^{n} x_{ijt} \le 1 \qquad j=1,\dots,m, \ t=0,\dots,T \qquad (3)$$

$$x_{ijt} - y_{jrt} \le 0 \qquad \forall\, t, j, \ i \in N_r \ \forall\, r \qquad (4)$$

$$\sum_{j=1}^{m} y_{jrt} \leq 1 \qquad r=1,\dots,R, \quad t=0,\dots,T \qquad (5)$$

$$x_{ijt} \in \{0,1\} \qquad i=1,\dots,n, \; j=1,\dots,m, \; t=0,\dots,T$$

$$y_{jrt} \in \{0,1\} \qquad j=1,\dots,m, \; r=1,\dots,R, \; t=0,\dots,T$$

Constraint (1): We need to know when the last job has finished.

(2): Each job should be done at one point in time on one machine

(3): max 1 job at the same time on a machine

(4): If a job needs a resource on a machine it should be available

(5): Each resource is available on one machine at any point in time