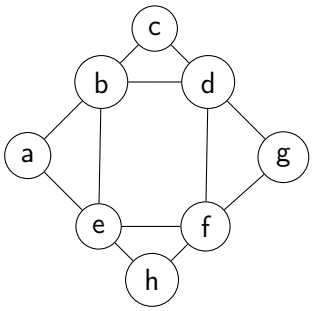
1. (4 points) Give a tree decomposition of the following graph that has the lowest width you can find, *and* explain why this is a correct tree decomposition (hint: you don't need to give the definition itself, but you may use it for your explanation).



Solution: The answer maybe given by a tree like displayed below, or given in the set notation where both the tree structure and the bags need to be defined: $T = (V, E)$ with $V = \{1, 2, 3, 4\}$, $E = \{(1, 2), (2, 3), (1, 4)\}$ and the bags $\{cbdf\}, \{abef\}, \{efh\}, \{dfg\}$.

or $V = \{1, 2, 3, 4, 5, 6\}$, $E = \{(1, 2), (1, 3), (1, 4), (2, 5), (2, 6)\}$ and the bags $\{def\}, \{bde\}, \{dfg\}, \{efh\}, \{bcd\}, \{abe\}$.

Explanation: this tree meets the three other properties of a tree decomposition:

- every vertex is represented in one of the bags of the tree nodes
- for every edge, there is a tree node with both end points of that edge
- for every vertex, the bags of tree nodes containing this vertex is a connected subtree

End of assignment