Solutions to exercises from Lecture 3

Exercise 4.1 a(i)

$$\begin{array}{lcrcrcrcrcrcr}
\min z & = & -3x_1 & - & 4x_2^+ & + & 4x_2^- & & & & & & \\
\text{s.t.} & & 2x_1 & + & 3x_2^+ & - & 3x_2^- & + & s_1 & & & = & 8 \\
 & & & - & x_2^+ & + & x_2^- & & & + & s_2 & = & 2 \\
\end{array}$$
$$x_1, x_2^+, x_2^-, s_1, s_2 \geq 0$$

Exercise 4.1 a(ii)

$$\begin{array}{lcrcrcrcr}
\max z & = & 5x_1' & + & 7x_2 & & & & \\
\text{s.t.} & & -2x_1' & + & 9x_2 & & & = & 13 \\
 & & 5x_1' & + & 3x_2 & + & s_2 & = & 20 \\
\end{array}$$
$$x_1', x_2, s_2 \geq 0$$

Exercise 4.1 b) Basic solutions:

1. Non-basic variable: $x_1' = 0$

$$\begin{pmatrix} x_1' \\ x_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 13/9 \\ 47/3 \end{pmatrix} \text{ feasible}$$

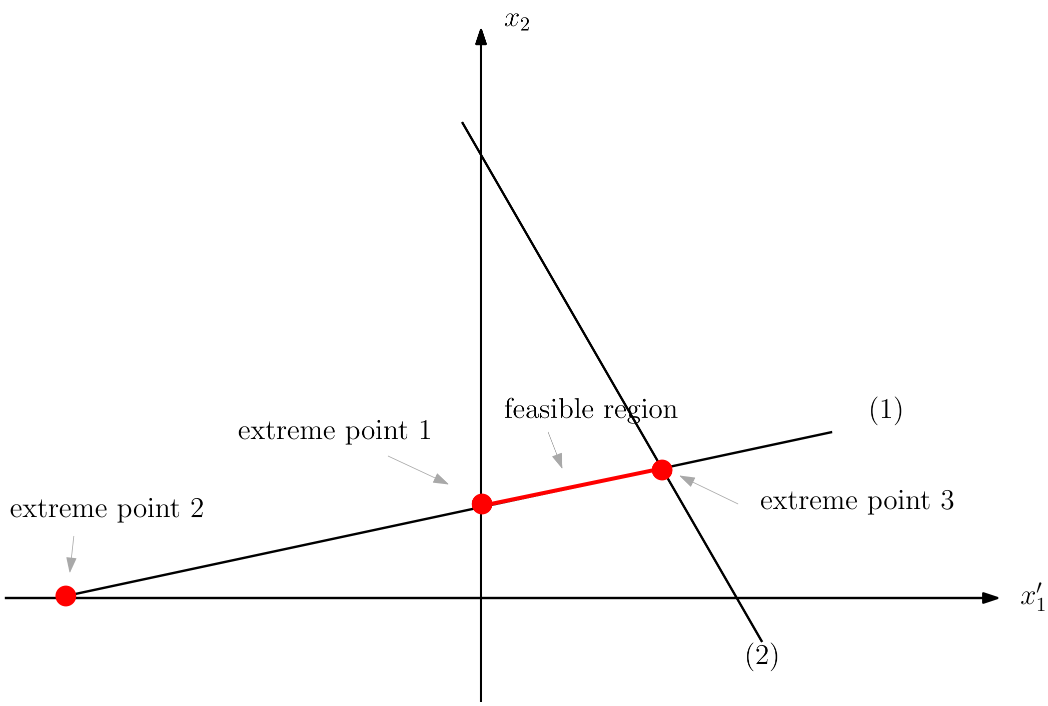
2. Non-basic variable: $x_2 = 0$

$$\begin{pmatrix} x_1' \\ x_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} -13/2 \\ 0 \\ 105/2 \end{pmatrix} \text{ infeasible}$$

3. Non-basic variable: $s_2 = 0$

$$\begin{pmatrix} x_1' \\ x_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} 47/17 \\ 105/51 \\ 0 \end{pmatrix} \text{ feasible}$$

Exercise 4.1 c) This is a graphical illustration of the problem instance:



Exercise 4.2 Take for instance the problem instance

$$
\begin{array}{rrrrl}
\max z = & & & = & 0 \\
\text{s.t.} & x_1 & & = & 0 \\
& & x_2 & = & 1 \\
& & x_1,\ x_2 & \geq & 0
\end{array}
$$

The basic feasible solution

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

This bfs is degenerate, but has only one corresponding basis.