# Selected solutions Module 2

**Exercise 2.3.**

Introduce decision variables $x_{ij}$ where $x_{ij} = 1$ indicates that read $i$ get assigned to location $j$ and $x_{ij} = 0$ otherwise.

Introduce also decision variables $y_j$ with $y_j = 1$ if at least one read is assigned to location $j$ and $y_j = 0$ otherwise.

Objective function:

$$\max \sum_{i=1}^{n} \sum_{j=1}^{m} s_{ij} x_{ij}$$

Now the constraints. First, each read should be assigned to exactly one location:

$$\sum_{j=1}^{m} x_{ij} = 1 \quad \forall i = 1, \ldots, n$$

(You may also use $\leq$.)

Second, two overlapping locations may not both be assigned reads:

$$y_j + y_k \leq 1 \quad \forall j, k = 1, \ldots, m \text{ met } o_{jk} = 1 \text{ en } j \neq k$$

this constraint can alternatively we replaced by the following:

$$y_j + y_k \leq 2 - o_{jk} \quad \forall j, k = 1, \ldots, m \text{ met } j \neq k$$

In addition, we need to make sure that $y_j = 1$ whenever at least one read is assigned to location $j$:

$$x_{ij} \leq y_j \quad \forall i = 1, \ldots, n \text{ en } j = 1, \ldots, m$$

Finally, all variables are binary:

$$x_{ij}, y_j \in \{0,1\} \quad \forall i = 1, \dots, n \text{ en } j = 1, \dots, m$$

**Exercise 2.4.**

*Decision variables:*

Let $x_{ij} = 1$ if student $i$ is assigned to project $j$ and $x_{ij} = 0$ otherwise.
Let $y_{jk} = 1$ if project $j$ is assigned to room $k$ and $y_{jk} = 0$ otherwise.
Let $d$ be the lowest number assigned by a student to a project that he/she needs to do.

A possible ILP formulation is then as follows:

$$\begin{array}{lllll}
\max & d & & & \\
\text{s.t.} & d + Mx_{ij} & \leq c_{ij} + M & & \text{for } i = 1, \dots, n \text{ and } j = 1, \dots, p \\
& \sum_{i=1}^{n} x_{ij} + My_{jk} & \leq m_k + M & & \text{for } j = 1, \dots, p \text{ and } k = 1, \dots, p \\
& \sum_{j=1}^{p} x_{ij} & = 3 & & \text{for } i = 1, \dots, n \\
& \sum_{k=1}^{p} y_{jk} & = 1 & & \text{for } j = 1, \dots, p \\
& \sum_{j=1}^{p} y_{jk} & = 1 & & \text{for } k = 1, \dots, p \\
& x_{ij}, y_{jk} \in \{0,1\} & & & \text{for } j = 1, \dots, p \\
& & & & \text{and } k = 1, \dots, p \text{ and } i = 1, \dots n
\end{array}$$

with $M$ a large number.

*Explanation:* The first group of constraints makes sure that $d \leq c_{ij}$ when $x_{ij} = 1$.
The second group of constraints makes sure that at most $m_k$ students are assigned to project $j$ when $y_{jk} = 1$.
The third group of constraints makes sure that each student is assigned to exactly three projects.
The fourth group of constraints makes sure that each project is assigned to exactly one room.
The fifth group of constraints makes sure that each room is assigned exactly one project.

The second group of constraints may also be replaced by

$$\sum_{i=1}^{n} x_{ij} \leq \sum_{k=1}^{p} m_k y_{jk} \quad \text{for } j = 1, \dots, p$$