1. You are given three sequences of n positive integers: $a_1, \ldots, a_n$, $b_1, \ldots, b_n$, and $c_1, \ldots, c_n$, together with an integer D. The question is to determine whether there exist three permutations π,ψ,ϕ (i.e., bijections that map an index $\{1, \ldots, n\}$ to another index $\{1, \ldots, n\}$) such that for all $i = 1, \ldots, n$ it holds that $a_{\pi(i)} + b_{\psi(i)} + c_{\phi(i)} = D$.

   For example, you are given $a = (20, 23, 49)$, $b = (22, 25, 27)$ and $c = (19, 40, 45)$ and $D = 90$, then the answer is yes, because with $\pi(i) = i$ (identity), $\psi(1) = 2$, $\psi(2) = 3$, $\psi(3) = 1$, and $\phi(1) = 3$, $\phi(2) = 2$, and $\phi(3) = 1$, it holds that $a_1+b_2+c_3 = 20+25+45 = 90$, $a_2+b_3+c_2 = 23+27+40 = 90$ and $a_3 + b_1 + c_1 = 49 + 22 + 19 = 90$.

   (a) (3 points) Use the technique of preprocessing the data to get an exact algorithm with time complexity $O^*(n!)$ instead of $O^*(n!^3)$.

   Solution:
   First observe that if such permutations exist, there also exist two permutations π and ψ when we just take $\phi(i) = i$.
   Next, sort the list $b_1, \ldots, b_n$. Then, for each permutation π, for each $i = 1, \ldots, n$ search for $D - c_i - a_{\pi(i)}$ in this list b. If such an element exists, assign the original index to $\psi(i)$ and delete this element from the list b. Otherwise, reset ψ and b and try the next permutation π. Return success when the list b is empty.

   - 1 point for sorting one of the lists and searching in it with $D-$ two other elements
   - 1 point for observing that one permutation is completely obsolete

   (b) (1 point) If by preprocessing the runtime for solving a problem is reduced from $O^*(n!^3)$ to $O^*(n!)$, can we say that this problem is *kernelizable*? Explain why or why not.

   Solution: No, the reduced runtime is not because of a problem instance being reduced, and certainly not to a size less than $g(k)$.
   BTW: A decision problem with input (I, k) is *kernelizable* if every such input can be reduced to an instance (I', k') s.t.:

   1. $k' \leq k$
   2. $|I'|$ is smaller than $g(k)$ for some function g only depending on k
   3. (I', k') has a solution if and only if (I, k) has one, and
   4. the reduction from (I, k) to (I', k') must be computable in polynomial time.

End of assignment