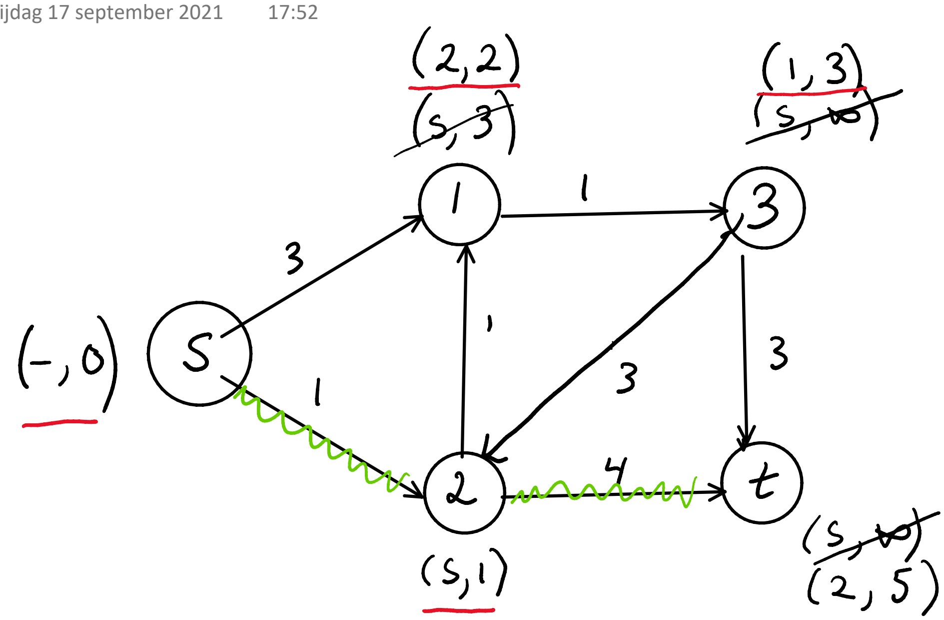
Exercise 10.1

vrijdag 17 september 2021 17:52



Iteration 1:

$W = \{s, 2\}$ $\quad \rho(1) = \min\{3, 1+1\} = 2$

$\rho(t) = \min\{\infty, 1+4\} = 5$

Iteration 2:

$W = \{s, 2, 1\}$ $\quad \rho(3) = \min\{\infty, 2+1\} = 3$

Iteration 3:

$W = \{s, 2, 1, 3\}$ No updates

Iteration 4:

$W = V$, stop

The shortest path is $s \to 2 \to t$, with length 5

Next, verify that $\pi = -p$ is a feasible solution to the dual.

The dual problem:

$$
\begin{array}{llr}
\max & \pi_s - \pi_t & \\
\text{st} & \pi_s - \pi_1 \le 3 & (1) \\
& \pi_s - \pi_2 \le 1 & (2) \\
& \pi_1 - \pi_3 \le 1 & (3) \\
& -\pi_1 + \pi_2 \le 1 & (4) \\
& -\pi_2 + \pi_3 \le 3 & (5) \\
& \pi_2 - \pi_t \le 4 & (6) \\
& \pi_3 - \pi_t \le 3 & (7) \\
& \pi_s, \pi_1, \pi_2, \pi_3, \pi_t \in \mathbb{R} &
\end{array}
$$

At the termination of the algorithm we have $p(s) = 0$, $p(1) = 2$, $p(2) = 1$, $p(3) = 3$, $p(t) = 5$

Set $\pi_v = -p(v)$, and check all constraints:

(1): $0-(-2) = 2 < 3$
(2): $0-(-1) = 1 \leq 1$
(3): $-2-(-3) = 1 \leq 1$
(4): $2 - 1 = 1 \leq 1$
(5): $1 - 3 = -2 < 3$
(6): $-1-(-5) = 4 \leq 4$
(7): $-3-(-5) = 2 < 3$

All constraints are satisfied, so the dual solution $\pi_v = -p(v)$ is feasible.

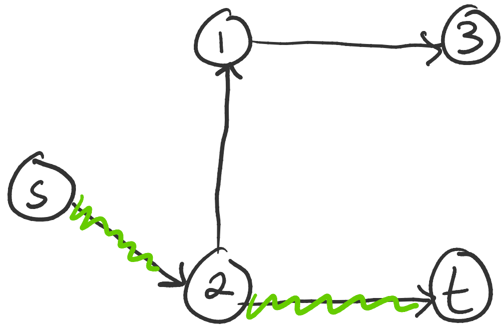
Verify that $\pi_s - \pi_t$ is equal to the length of the shortest path: $\pi_s - \pi_t = 0 - \pi_t = 0 - (-p(t)) = p(t) = 5$, which is precisely the length of the shortest path from s to t.

Finally, determine a shortest path using complementary slackness.

We may set $f_{uv} > 0$ if $\pi_u - \pi_v = l_{uv}$

Which dual constraints are satisfied with equality? (2), (3), (4), (6)

So, f_{s2}, f_{13}, f_{21}, and f_{2t} may take values > 0



Only the arcs $(s,2)$ and $(2,t)$ form a path from s to t, so they form a shortest path.