Exercises
Lecture 4
4.4, 4.6, 4.7

4.2 $\min z = 3x_1 - 6x_2$

s.t. $5x_1 + 7x_2 + s_1 = 35$

$-x_1 + 2x_2 + s_2 = 2$

basis	$\bar{b}$	x_1	x_2	s_1	s_2	
s_1	35	5	7	1		
← s_2	2	-1	2		1	
$-z$	0	3	-6 ↑			
← s_1	28	$17/2$		1	$-7/2$	$r_1 - 7/2\, r_2$
x_2	1	$-1/2$	1		$1/2$	$r_2/2$
$-z$	6	↑			3	$r_0 + 3r_2$
x_1	$56/7$	1		$2/17$	$-14/34$	$r_1 \cdot 2/17$
x_2	$45/17$		1	$1/17$	$-10/34$	$r_2 + 1/17\, r_1$
$-z$	6				3	r_0

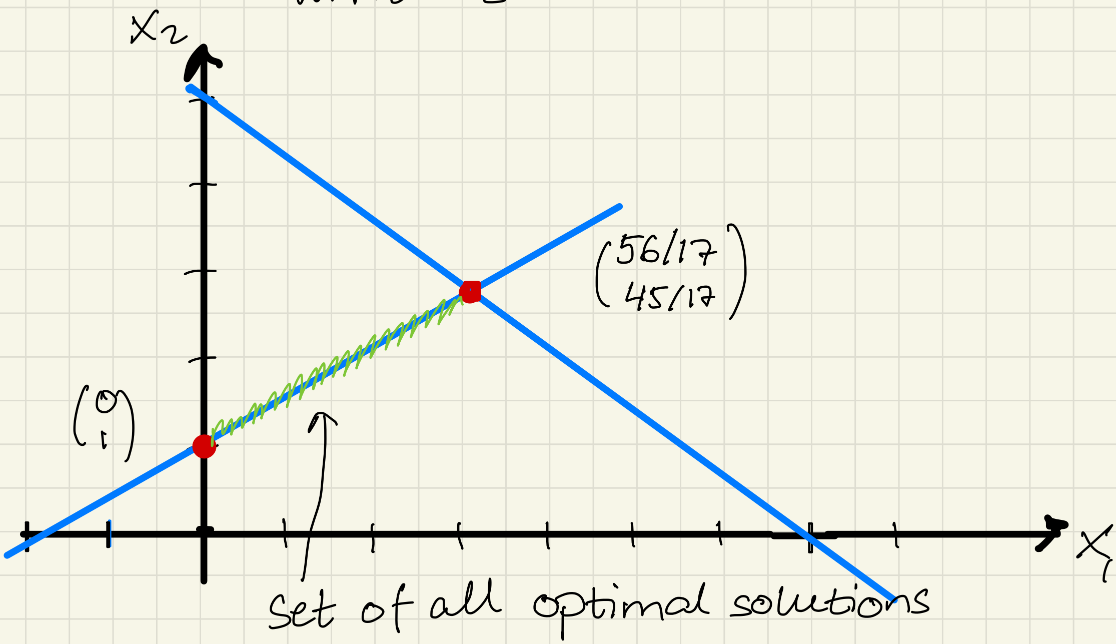
The solution in the second tableau is optimal, since $\bar{c}_j \geq 0$ for all j, but since

$\bar{c}_1 = 0$ and x_1 is non-basic, we can bring x_1 into the basis to get another solution with the same objective value

The two extreme points are

$$\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} = \begin{pmatrix} 56/17 \\ 45/17 \end{pmatrix}$$

with $z^* = -6$



The set of optimal solutions is the line segment between the two extreme points, including the points, i.e.,

$$x^* = \lambda_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 56/17 \\ 45/17 \end{pmatrix}, \quad \lambda_1, \lambda_2 \geq 0, \lambda_1 + \lambda_2 = 1$$

with $z^* = -6$

4.6

$$\min\ z = 2x_1 + x_2$$

s.t.

$$x_1 + x_2 - s_1 + x_1^a = 1$$

$$3x_1 + 2x_2 + s_2 = 6$$

$$x_1, x_2, s_1, s_2, x_1^a \geq 0$$

Phase 1 $\quad \min\ w = x_1^a$

basis	$\bar{b}$	x_1	x_2	s_1	s_2	x_1^a
x_1^a	1	1	1	-1	0	1
s_2	6	3	2	0	1	0
$-w$	0					1

Express the objective row in non-basic variables

basis	$\bar{b}$	x_1	x_2	s_1	s_2	x_1^a
$\leftarrow$ x_1^a	1	1	1	-1	0	1
s_2	6	3	2	0	1	0
$-w$	-1	-1	-1	1		
		$\uparrow$				

$r_0 - r_1$

basis	$\bar{b}$	x_1	x_2	s_1	s_2	x_1^a	
x_1	1	1	1	-1			r_1
s_2	3		-1	3	1	-3	$r_2 - 3r_1$
$-w$	0					1	$r_0 + r_1$

Optimal solution phase 1, since $\bar{c}_j \geq 0$ for all j. $x_1^a = 0$ and non-basic, and $w = 0$.

Phase 2, re-introdue original objective function:

$$z = 2x_1 + x_2$$

Exress z in non-basic variables
From the tableau we see that

$$x_1 + x_2 - s_1 = 1, \text{ i.e } \quad x_1 = 1 - x_2 + s_1$$

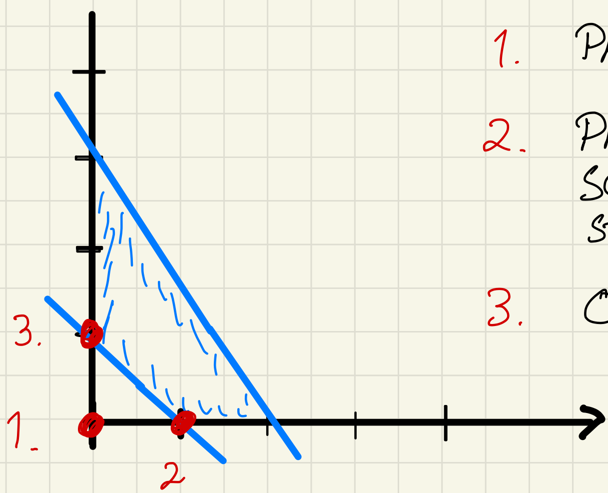
We obtain: $$\begin{aligned} z &= 2x_1 + x_2 \\ &= 2(1 - x_2 + s_1) + x_2 \\ &= 2 - x_2 + 2s_1 \end{aligned}$$

basis	$\bar{b}$	x_1	x_2	s_1	s_2
x_1	1	1	1	-1	
s_2	3		-1	3	1
$-z$	-2		-1	2	

x_2	1	1	1	-1		r_1
s_2	4	1		2	1	r_2+r_1
$-z$	-1	1		1		r_0+r_1

Optimal solution since $\bar{c}_j \geq 0$ for all j

$$x^* = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad z^* = 1$$



1. Phase 1 starting bfs
2. Phase 1 optimal solution = Phase 2 starting solution
3. Optimal bfs

4.7

$$\min \quad z = -3x_1 + x_2 - 20$$

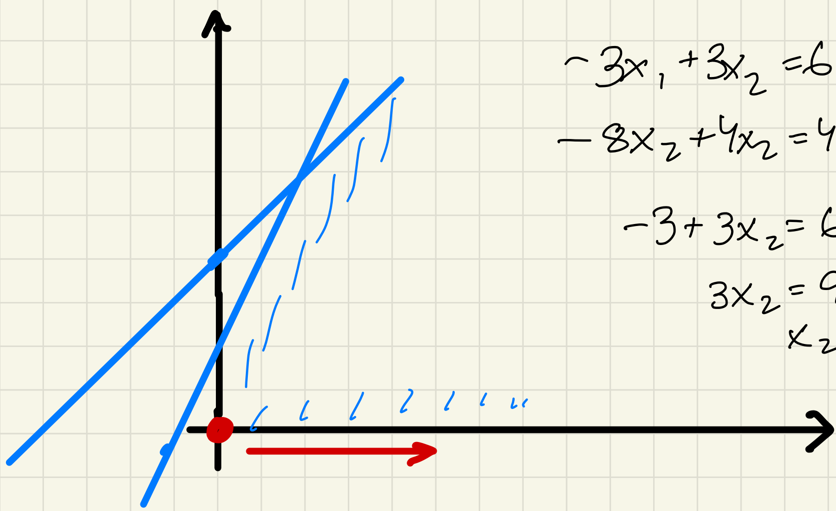
$$\text{s.t} \quad -3x_1 + 3x_2 + s_1 = 6$$

$$-8x_1 + 4x_2 + s_2 = 4$$

$$x_1, x_2, s_1, s_2 \geq 0$$

basis	$\bar{b}$	x_1	x_2	s_1	s_2
s_1	6	-3	3	1	0
s_2	4	-8	4	0	1
$-z$	20	-3	1		

Since $\bar{a}_{i1} < 0$ for $i = 1, 2$, the problem is unbounded, i.e., $z^* = -\infty$



$-3x_1 + 3x_2 = 6$

$-8x_2 + 4x_2 = 4$

$-3 + 3x_2 = 6$

$3x_2 = 9$

$x_2 = 3$