

1. (4 points) Consider the following problem. We have a set of items  $A$  (called the universe), and we are given  $m$  subsets  $B_1, \dots, B_m \subseteq A$ . The problem we study here is whether there exists a subset of items  $H \subseteq A$  such that 1) each of the sets  $B_i$  has at least one element in common with this  $H$  (also called a *hit*) and 2) the size of  $H$  is at most  $k$ .

Give *two* rules to reduce an instance of this problem *that are as general as you can think of* (without loss of optimality), and for each of these explain briefly why it is correct.