

Exercise 8.1



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$$8.1 \quad \min z = 3x_1 - 6x_2$$

$$\text{s.t.} \quad 5x_1 + 7x_2 \leq 35$$

$$-x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

basis	\bar{b}	x_1	x_2	s_1	s_2
s_1	35	5	7	1	
$\leftarrow s_2$	2	-1	2		1
$-z$	0	3	-6		

$$7 - \frac{7}{2} \cdot 2$$

basis	\bar{b}	x_1	x_2	s_1	s_2
s_1	28	$17/2$		1	$-7/2$
x_2	1	$-1/2$	1		$1/2$
$-z$	6				3

$$r_1 - \frac{7}{2}r_2$$

$$r_2 / 2$$

$$r_0 + 3r_2$$

Optimal solution since $\bar{c}_j \geq 0$ for all j

$$x^* = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad z^* = -6$$

basis	\bar{b}	x_1	x_2	s_1	s_2
s_1	28	$17/2$		1	$-7/2$
x_2	1	$-1/2$	1		$1/2$
$-z$	6				3

Add constraint $2x_2 \leq 1$

$$2x_2 + s_3 = 1 \quad (*)$$

Express x_2 in non-basic variables:

Using this row, we see that

$$x_2 - 1/2 x_1 + 1/2 s_2 = 1 \Rightarrow x_2 = 1 + 1/2 x_1 - 1/2 s_2$$

$$\text{Insert in } (*) \Rightarrow 2(1 + 1/2 x_1 - 1/2 s_2) + s_3 = 1$$

$$x_1 - s_2 + s_3 = -1$$

Add $x_1 - s_2 + s_3 = -1$ to the current Simplex tableau:

basis	\bar{b}	x_1	x_2	s_1	s_2	s_3
s_1	28	$17/2$		1	$-7/2$	
x_2	1	$-1/2$	1		$1/2$	
s_3	-1	1			-1	1
$-z$	6				3	

← (green arrow pointing to s_3 row)
↑ (green arrow pointing to 3 in $-z$ row)

basis	\bar{b}	x_1	x_2	s_1	s_2	s_3	
s_1	$63/2$	5		1		$-7/2$	$r_1 + \frac{7}{2}r_3$
x_2	$1/2$	0	1			$1/2$	$r_2 + 1/2 r_3$
s_2	1	-1			1	-1	$r_3 \cdot (-1)$
$-z$	3	3				3	$r_0 + 3r_3$

Optimal solution. All \bar{c}_j are still nonnegative, and now all $\bar{b}_i \geq 0$ as well

Optimal solution:

$$\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} \quad z^* = -3$$