Selected solutions Module 4

Exercise 4.9.

Starting tableau first phase: $\min w = x_2^a$.

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3	x_2^a
s_1	5	1	1	1	1	0	0	0
x_2^a	2	0	1	1	0	-1	0	1
s_3	1	2	0	1	0	0	1	0
$-w$	0	0	0	0	0	0	0	1

Subtract the second row from the objective function row to create a 0 in the column of basic variable x_2^a.

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3	x_2^a
s_1	5	1	1	1	1	0	0	0
x_2^a	2	0	1	1	0	-1	0	1
s_3	1	2	0	1	0	0	1	0
$-w$	-2	0	-1	-1	0	1	0	0

Bring x_2 into the basis. $\min\{5/1, 2/1\} = 2/1$ so x_2^a leaves the basis (min ratio test). Subtract row 2 from row 1 and add row 2 to the objective function row.

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3	x_2^a
s_1	3	1	0	0	1	1	0	-1
x_2	2	0	1	1	0	-1	0	1
s_3	1	2	0	1	0	0	1	0
$-w$	0	0	0	0	0	0	0	1

Reduced costs are all nonnegative, so we have found an optimal solution to the first phase.

x_2^a is not in the basis, so we can remove the column of x_2^a and put the original objective function back.

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3
s_1	3	1	0	0	1	1	0
x_2	2	0	1	1	0	-1	0
s_3	1	2	0	1	0	0	1
$-z$	0	-4	1	-1	0	0	0

Subtract row 2 from the objective function row to create a zero in the column of basic variable x_2.

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3
s_1	3	1	0	0	1	1	0
x_2	2	0	1	1	0	-1	0
s_3	1	2	0	1	0	0	1
$-z$	-2	-4	0	-2	0	1	0

Bring x_1 into the basis. s_3 leaves the basis (min ratio test).

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3
s_1	$2\frac{1}{2}$	0	0	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$
x_2	2	0	1	1	0	-1	0
x_1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$-z$	0	0	0	0	0	1	2

Optimal solution found (reduced costs nonnegative) $x_1 = \frac{1}{2}$, $x_2 = 2$, $x_3 = 0$.

We can find more optimal solutions by bringing x_3 into the basis (reduced costs are zero). x_1 leaves the basis.

basis	$\bar{b}$	x_1	x_2	x_3	s_1	s_2	s_3
s_1	3	1	0	0	1	1	0
x_2	1	-2	1	0	0	-1	-1
x_3	1	2	0	1	0	0	1
$-z$	0	0	0	0	0	1	2

Other optimal solution found: $x_1 = 0$, $x_2 = 1$, $x_3 = 1$.

These are the only optimal basic solutions.

All optimal solutions can now be found by taking all convex combinations of the two optimal basic solutions:

All optimal solutions: $\lambda \begin{bmatrix} \frac{1}{2} \\ 2 \\ 0 \end{bmatrix} + (1 - \lambda) \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ with $0 \leq \lambda \leq 1$.