1. (4 points) Consider the following problem. We have a set of items A (called the universe), and we are given m subsets $B_1, \ldots, B_m \subseteq A$. The problem we study here is whether there exists a subset of items $H \subseteq A$ such that 1) each of the sets B_i has at least one element in common with this H (also called a *hit*) and 2) the size of H is at most k.

   Give *two* rules to reduce an instance of this problem *that are as general as you can think of* (without loss of optimality), and for each of these explain briefly why it is correct.

Solution:

1. If there is an item $x \in A$ which occurs in more than k subsets B_j whose pairwise intersection is $\{x\}$, add x to the hitting set H, remove all those subsets from the problem, and reduce k by 1. Under this condition on the pairwise intersection being exactly x, there are no other elements shared between these subsets, so if x is not included in the hitting set, we would need one separate element for each of these –more than k– subsets B_j, which is not possible given the limit k.
2. For any pair $i \neq j$, if $B_i \subseteq B_j$ remove B_j, because hitting i implies hitting j.
3. For any subset B_i of size 1, include the item in B_i in H, because to hit i there is no other option.
4. Identify a crown structure: an (independent) subset $I \subseteq A$ and a (neighbor) subset $N \subseteq \{B_1, \ldots, B_m\}$ where each set B_i has only items from I, with $|I| \leq |N|$. Then the items from I included in the maximal matching of (I, N) should be in H, because some subset of I is needed...

- Assign (up to) 2 points for each correct non-trivial rule including explanation.
- Naive rules such as removing duplicate sets give 1 point.