

1. You are given three sequences of n positive integers: a_1, \dots, a_n , b_1, \dots, b_n , and c_1, \dots, c_n , together with an integer D . The question is to determine whether there exist three permutations π, ψ, ϕ (i.e., bijections that map an index $\{1, \dots, n\}$ to another index $\{1, \dots, n\}$) such that for all $i = 1, \dots, n$ it holds that $a_{\pi(i)} + b_{\psi(i)} + c_{\phi(i)} = D$.

For example, you are given $a = (20, 23, 49)$, $b = (22, 25, 27)$ and $c = (19, 40, 45)$ and $D = 90$, then the answer is yes, because with $\pi(i) = i$ (identity), $\psi(1) = 2$, $\psi(2) = 3$, $\psi(3) = 1$, and $\phi(1) = 3$, $\phi(2) = 2$, and $\phi(3) = 1$, it holds that $a_1 + b_2 + c_3 = 20 + 25 + 45 = 90$, $a_2 + b_3 + c_2 = 23 + 27 + 40 = 90$ and $a_3 + b_1 + c_1 = 49 + 22 + 19 = 90$.

- (a) (3 points) Use the technique of preprocessing the data to get an exact algorithm with time complexity $O^*(n!)$ instead of $O^*(n!^3)$.
- (b) (1 point) If by preprocessing the runtime for solving a problem is reduced from $O^*(n!^3)$ to $O^*(n!)$, can we say that this problem is *kernelizable*? Explain why or why not.