Selected solutions Module 1

Exercise 1.2.

Define first the decision variables: $x_{ij} = 1$ if surgery i is executed in OR j and $x_{ij} = 0$ otherwise. The ILP becomes:

$$
\begin{array}{lccccccccccl}
\max & 5x_{11} & +5x_{12} & +4x_{21} & +4x_{22} & +3.5x_{31} & +3.5x_{32} & +1.5x_{41} & +1.5x_{42} & +3x_{51} & +3x_{52} & \\
s.t. & 5x_{11} & & +4x_{21} & & +3.5x_{31} & & +1.5x_{41} & & +3x_{51} & & \leq 8 \\
& & 5x_{12} & & +4x_{22} & & +3.5x_{32} & & +1.5x_{42} & & +3x_{52} & \leq 8 \\
& x_{11} & +x_{12} & & & & & & & & & \leq 1 \\
& & & x_{21} & +x_{22} & & & & & & & \leq 1 \\
& & & & & x_{31} & +x_{32} & & & & & \leq 1 \\
& & & & & & & x_{41} & +x_{42} & & & \leq 1 \\
& & & & & & & & & x_{51} & +x_{52} & \leq 1 \\
& \multicolumn{11}{l}{x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52} \in \{0, 1\}}
\end{array}
$$

Exercise 1.3.

We can describe the input as follows. Define $b_{ij} = 1$ if the i-th male student and the j-th female student are on each others preference list, and $b_{ij} = 0$ otherwise.

Decision variables: $x_{ij} = 1$ if the i-th male student will collaborate with the j-th female student and $x_{ij} = 0$ otherwise.

The ILP formulation is as follows:

$$
\begin{array}{llll}
\max & \displaystyle\sum_{i=1}^{62}\sum_{j=1}^{34} x_{ij} & & \\
s.t. & \displaystyle\sum_{i=1}^{62} x_{ij} & \leq 1 & \text{voor } j = 1,\dots,34 \\
 & \displaystyle\sum_{j=1}^{34} x_{ij} & \leq 1 & \text{voor } i = 1,\dots,62 \\
 & x_{ij} & \leq b_{ij} & \text{voor } i = 1,\dots,62 \text{ en } j = 1,\dots,34 \\
 & x_{ij} & \geq 0 & \text{voor } i = 1,\dots,62 \text{ en } j = 1,\dots,34 \\
 & x_{ij} & \text{integer} & \text{voor } i = 1,\dots,62 \text{ en } j = 1,\dots,34
\end{array}
$$

(It can be shown that the integrality constraints can be omitted, obtaining an LP formulation with integral extreme points.)

Exercise 1.4.

First define the sets $B_j := \{i \in \{1,\dots,n\} \mid t_{ij} \leq 10\}$, i.e. the locations from which house j can be reached within 10 minutes.

Step 1: what are the decision variables?

$x_i = 1$ if at location i an ambulance will be located and $x_i = 0$ otherwise (note that in this problem it does not make sense to place more than one ambulance at the same location).

$y_j = 1$ if house j is covered and $y_j = 0$ otherwise.

Step 2: formulate the objective function.

We want to cover as many houses as possible, so:

$$\max \sum_{j=1}^{m} y_j$$

Step 3: formulate the constraints.

House j is only covered if an ambulace is located at at least one location from B_j:

$$y_j \leq \sum_{i \in B_j} x_i \quad \text{for all } j \in \{1, \ldots, m\}$$

There are only p ambulances available:

$$\sum_{i=1}^{n} x_i \leq p$$

Do not forget that the decision variables may only take the values 0 and 1. We have already mentioned that when introducing the variables, but we also need to put it in the ILP formulation.

$$x_i \in \{0, 1\} \quad \text{for all } i \in \{1, \ldots, n\}$$
$$y_j \in \{0, 1\} \quad \text{for all } i \in \{j, \ldots, m\}$$