Exercise Sheet - Rational Decisions

Exercise 1. Suppose you abandon AI and become a farmer. For that, you must buy a tractor. On the local marketplace there are only two tractors: a new tractor (€20 000) and a used tractor (€15 000). The engine of the old tractor may be defective: you Google the fact that 15% of the used tractors have a defective engine. If the engine is defective, he will have to buy the new tractor, and get only €2000 back for selling again the used one. However, before buying, you have the option to take the old tractor to a garage for an expert evaluation. The expert evaluation will cost you 500€. If the engine is defective, there is a 22% chance that the garage does not notice it (22% false positives, but no false negatives)[1].

HINTS: For your own use: start by drawing the decision network of the problem, and then compute the following values:

$P(ok) =$ $P(pass \mid ok) =$ $P(pass \mid \neg ok) =$

$P(\neg ok) =$ $P(\neg pass \mid ok) =$ $P(\neg pass \mid \neg ok) =$

where 'ok' means that the engine of the used tractor is not defective, and 'pass' means that the used tractor passes the evaluation.

Also, you will need the following formula for answering some of the questions:

$$\text{Bayes' Theorem:} \qquad P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

1. The probability that the used tractor will pass its evaluation, $P(pass)$, is:

a) 0,5	b) 0,157	c) 0,883	d) 0,981	e) 0,777

2. Calculate the expected cost of buying the used tractor given that you choose not to have any expert evaluation: $EC(buyused)$. Given you choose not to have any evaluation, which of the following is the best option?

a) buy the new tractor	b) buy the used tractor	c) buy both

3. Given that the used tractor failed the evaluation, what is the expected cost of buying the used tractor anyway? (Hint: what is $P(ok|\neg pass)$?)

a) 500	b) 2500	c) 15500	d) 20500	e) 33500

4. Calculate $EC(buyused|pass)$, that is, the expected cost of buying the used tractor given that you have received a 'pass' from the expert evaluation.

a) 19460	b) 15676	c) 22533,78	d) 16169,85	e) 33500

5. What is the expected cost of first evaluating the used tractor and then performing the best action ($EC(eval, bestaction)$)?

   (If you're unsure of your previous calculation of $EC(buyused|pass)$, you can use $EC(buyused|pass) = 17000, 55$)

a) 18931,478	b) 18817,978	c) 19664,98	d) 16676.48	e) 17409,99

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[1] In other words, if the used tractor is broken, there is a 22% chance that the expert evaluation will tell you it's ok.

6. In point 2 you calculated $EC(buyused)$. In point 6 you calculated $EC(eval, bestaction)$. Use these values to determine what is the *value of information* v of the evaluation of the used tractor.

(If you're unsure of your previous calculation of $EC(eval, bestaction)$ you can assume $EC(eval, bestaction) = 18777, 77$)

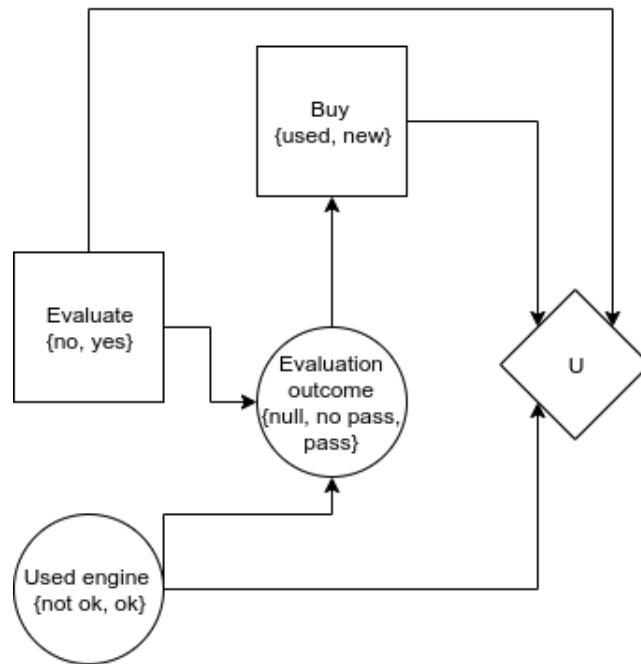| a) 0<v<1118 | b) v<0 | c) 1700<v<2000 | d) 1000<v<1100 |

Figure 1: Example decision network.

Exercise 2. Suppose I'm going to the dentist. The Bayesian Network is shown in Figure **??**. I know that the probability that I have a cavity is $P(cavity) = 0.2$, the probability that I have a toothache is $P(toothache) = 0.2$. Furthermore I know that if I have a cavity, the probability that the probe from the dentist catches behind this tooth is $P(catch|cavity) = 0.9$, the probability that I have a cavity given that I have a toothache is $P(cavity|toothache) = 0.6$. I also know $P(catch, toothache) = 0.124$.

(a) Give the probability of $P(catch|cavity, toothache)$.

(b) What is the probability of having a cavity, given that you have a toothache and caught, $P(cavity|catch, toothache)$?