Exercise Sheet - Rational Decisions

Exercise 1. Suppose you abandon AI and become a farmer. For that, you must buy a tractor. On the local marketplace there are only two tractors: a new tractor (€20 000) and a used tractor (€15 000). The engine of the old tractor may be defective: you Google the fact that 15% of the used tractors have a defective engine. If the engine is defective, he will have to buy the new tractor, and get only €2000 back for selling again the used one. However, before buying, you have the option to take the old tractor to a garage for an expert evaluation. The expert evaluation will cost you 500€. If the engine is defective, there is a 22% chance that the garage does not notice it (22% false positives, but no false negatives)[1].

HINTS: For your own use: start by drawing the decision network of the problem, and then compute the following values:

$P(ok) =$	$P(pass \mid ok) =$	$P(pass \mid \neg ok) =$
$P(\neg ok) =$	$P(\neg pass \mid ok) =$	$P(\neg pass \mid \neg ok) =$

where 'ok' means that the engine of the used tractor is not defective, and 'pass' means that the used tractor passes the evaluation.

Also, you will need the following formula for answering some of the questions:

Bayes' Theorem: $$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Solution: See Figure 1 for an example decision network. Note that another way to do it is to create 2 separate decision diagrams, one for the case you decide to do the evaluation and the other for the case where you do no evaluation.

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1. The probability that the used tractor will pass its evaluation, $P(pass)$, is:

a) 0,5	b) 0,157	c) 0,883	d) 0,981	e) 0,777

Solution: $p(pass|ok) = 1$

$p(\neg pass|ok) = 0$
$p(pass|\neg ok) = .22$
$p(\neg pass|\neg ok) = .78$

$p(pass) = p(pass|ok) * p(ok) + p(pass|\neg ok) * p(\neg ok)$
$1 * 0.85 + 0.22 * 0.15 =$
$0.85 + 0.033 = 0.883$:

c)

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[1] In other words, if the used tractor is broken, there is a 22% chance that the expert evaluation will tell you it's ok.

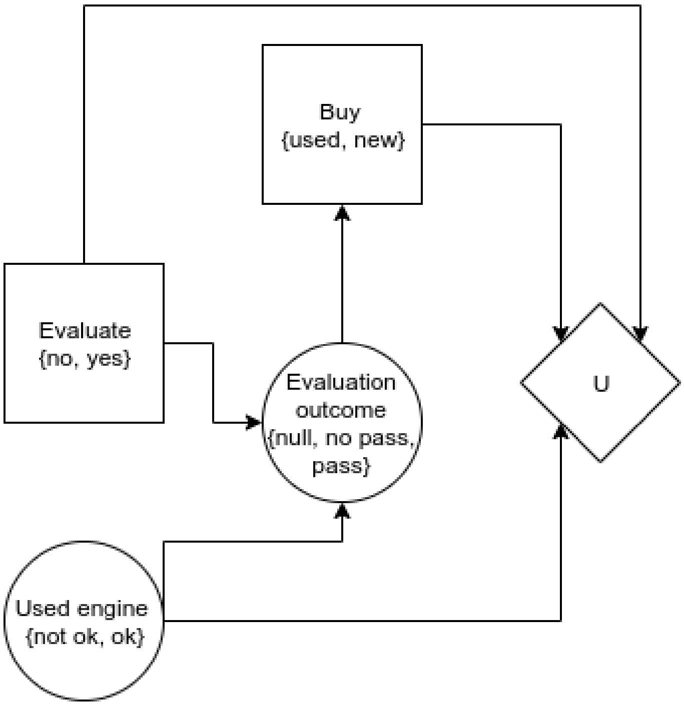

Figure 1: Example decision network.

2. Calculate the expected cost of buying the used tractor given that you choose not to have any expert evaluation: $EC(buyused)$. Given you choose not to have any evaluation, which of the following is the best option?

a) buy the new tractor	b) buy the used tractor	c) buy both

Solution:

Payoff if buy a broken used one (15000) and sell it (-2000) and buy a new one (20000) is 33000.

$EC(buyused) = p(ok) * 15.000 + p(\neg ok) * 33.000$
$EC(buyused) = 0.85 * 15.000 + 0.15 * 33.000 = 17.700$

which is less than a new one. So used is better

b)

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3. Given that the used tractor failed the evaluation, what is the expected cost of buying the used tractor anyway? (Hint: what is $P(ok|\neg pass)$?)

a) 500	b) 2500	c) 15500	d) 20500	e) 33500

Solution: Given that you got a not pass from the evaluation, to calculate the expected cost of buying the used tractor, we must calculate the probability that it is ok given that we got a not pass.

Simply, one can reason that: true positives are 100%. If ok, then pass. Contraposing, if not pass, then not ok.

Otherwise we apply bayes' rule,

$p(ok|\neg pass) = \frac{p(\neg pass|ok) * p(ok)}{p(\neg pass)}$

but $p(\neg pass|ok) = 0$

so *if you had a not pass result, you know for sure it is broken.*

So the expected cost is 33.500.

e)

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4. Calculate $EC(buyused|pass)$, that is, the expected cost of buying the used tractor given that you have received a 'pass' from the expert evaluation.

a) 19460	b) 15676	c) 22533,78	d) 16169,85	e) 33500

Solution: $EC(buyused|pass) = p(ok|pass) * U(buyused, eval, ok) + p(\neg ok|pass) * U(buyused, eval, \neg ok)$

Where $p(ok|pass) = \frac{p(pass|ok)*p(ok)}{p(pass)} = \frac{1*0.85}{.883} = 0.9626$
$p(\neg ok|pass) = 1 - p(ok|pass) = 0.0373$

So

$EC(buyused|pass) = 0.9626 * 15500 + 0.0373 * 33500 = 16169,85$

d), a) if the student thought that $p(\neg ok|pass) = .22$

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5. What is the expected cost of first evaluating the used tractor and then performing the best action $(EC(eval, bestaction))$?

(If you're unsure of your previous calculation of $EC(buyused|pass)$,
you can use $EC(buyused|pass) = 17000,55$)

a) 18931,478	b) 18817,978	c) 19664,98	d) 16676.48	e) 17409,99

Solution:

$EC(buynew|\neg pass) = p(ok|\neg pass) * U(new) + p(\neg ok|\neg pass) * U(new)$

But $p(\neg ok|\neg pass) = 1$ and $p(ok|\neg pass) = 0$, so

$EC(buynew|\neg pass) = 0 * U(buynew, eval) + 1 * U(buynew, eval)$

$EC(buynew|\neg pass) = 20500$ (if you forget to add the 500, you get solution **b)**)

Now we can compute:

$EC(eval, best) = P(pass) \cdot EC(buyused|pass) + P(\neg pass) \cdot EC(buynew|\neg pass)$
$= 0.883 * 16169.85 + 0.117 * 20500$

$EC(eval, best) = 16676.48 ==$ **d)**

If unsure of $EC(buyused|pass)$, and uses 17000,55: (still have to compute $EC(buynew|\neg pass)$ from scratch...)

$EC(eval, best) = P(pass) \cdot EC(buyused|pass) + P(\neg pass) \cdot EC(buynew|\neg pass)$

$= 0.883 * 17000, 55 + 0.117 * 20500 = 17409.9$ **e)**

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6. In point 2 you calculated $EC(buyused)$. In point 6 you calculated $EC(eval, bestaction)$. Use these values to determine what is the *value of information* v of the evaluation of the used tractor.

(If you're unsure of your previous calculation of $EC(eval, bestaction)$ you can
assume $EC(eval, bestaction) = 18777, 77$)

a) 0<v<1118	b) v<0	c) 1700<v<2000	d) 1000<v<1100

Solution:

The value of the information is $EC(eval, best) - EC(buyused)$ since buyused is the current(=previous to evaluation) best action.

$EC(buyused) = 17.700$

$EC(eval, bestaction) = 16676.48$

The value of the information is $16676.48 - 17700 = -1023.52$ –> **(b)**

Solution for $EC(eval, bestaction) = 18777,77$: 1077,77 –> **(d)**

Solution for $EC(eval, bestaction) = 19664,98$: 1964,98 –> **(c)**

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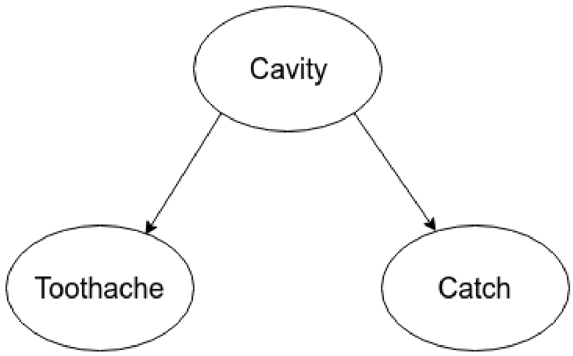

Figure 2: Bayesian Network for exercise 2.

Exercise 2. Suppose I'm going to the dentist. The Bayesian Network is shown in Figure 2. I know that the probability that I have a cavity is $P(cavity) = 0.2$, the probability that I have a toothache is $P(toothache) = 0.2$. Furthermore I know that if I have a cavity, the probability that the probe from the dentist catches behind this tooth is $P(catch|cavity) = 0.9$, the probability that I have a cavity given that I have a toothache is $P(cavity|toothache) = 0.6$. I also know $P(catch, toothache) = 0.124$.

(a) Give the probability of $P(catch|cavity, toothache)$.

Solution: Since catch is conditionally independent of toothache given cavity, we have that $P(catch|cavity, toothache) = P(catch|cavity) = 0.9$

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(b) What is the probability of having a cavity, given that you have a toothache and caught, $P(cavity|catch, toothache)$?

Solution: We need to find

$$P(cavity|catch, toothache = \frac{P(catch|cavity, toothache)P(cavity|toothache)}{P(catch|toothache)}.$$

From question a we have that $P(catch|cavity, toothache) = P(catch|cavity) = 0.9$. We can calculate: $P(catch|toothache) = \frac{P(catch,toothache)}{P(toothache)} = \frac{0.124}{0.2} = 0.62$. By combining all our information we get:

$$P(cavity|catch, toothache) = \frac{0.9 * 0.6}{0.62} = 0.871.$$

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