Exercise Sheet - Game Theory

Exercise 1 (MiniMax Algorithm)**.** The Minimax algorithm assumes that the opponent plays optimally. Suppose the opponent *does not play optimally*. Show the following:

1. For non-optimally playing opponents the Minimax value is at least as good as the computed Minimax value if played against an optimal opponent.
2. For non-optimally playing opponents, the Minimax value may not be optimal.

Exercise 2 (Heuristic Minimax)**.** This exercise is about the board game Tic-Tac-Toe, played on a 3x3 grid.

(a) Given a game tree, we also refer to a branch in the game tree as a *game* (thus, a game captures a possible play of Tic-Tac-Toe). Give an upper bound on how many possible games of Tic-Tac-Toe there are?

(b) Draw the game tree starting from an empty board down to depth 2 (i.e., 2-plies). Take symmetry into account; that is, do not draw any branches in the game that are symmetric.

(c) Define an appropriate *evaluation function* for Tic-Tac-Toe. Give arguments why it is a good one and compute the utility of the states at depth 2 from part (b).

(d) Based on the utility of the evaluation function of (c) use the Heuristic Minimax algorithm to compute the Heuristic Minimax value of each state of the game tree. Also, give the corresponding optimal actions.

Exercise 3 (Zero sum game)**.** Consider the following game (Bombers and Fighters):

		Bomber Crew	
		Look Up	Look Down
Fighter Pilots	Hun-in-the-Sun	*0.95,0.05*	*1,0*
	Ezak-Imak	*1,0*	*0,1*

1. Compute the maximin strategies of both players using Von Neumann's Maximin technique.
2. Compute all (mixed) Nash equilibria deriving the answer from your solution to 1.
3. Compute the maximin value of the Fighter Pilots directly, using the alternative methods shown on slide 25 of the lecture.
4. Show that the value of the "Fighters and Bombers" game (i.e. the maximin value of the Fighter Pilots) is *strictly greater than* 0.95 (and thus that there is a mixed strategy that is better than the pure Hun-in-the-sun strategy).

Exercise 4 (Pirates and Gold)**.** We are given the following puzzle. Five rational *pirates* a, b, c, d, e negotiate about how to share 100 *gold coins*. The pirates are ranked from a *(highest)* to e *(lowest)* The task is to distribute the coins such that:

1. the highest ranked pirate proposes a distribution.
2. each pirate can *accept* or *reject* the proposal, *majority* decides (highest ranked pirate *breaks ties*, otherwise the pirate takes part in the negotiation as any other pirate).
3. if the proposal is rejected the proposing pirate is *killed* and the next highest ranked pirate makes a proposal, and so forth.

The *preferences* of the pirates (in this order and additive) are as follows: (i) stay alive, (ii) maximize the number of gold coins the pirate gets, and (iii) kill other pirates.

1. Model this as an extensive form game (informally) and explain how strategies are defined.
2. Compute the unique subgame perfect Nash equilibrium.