

Exercise Sheet - Reasoning over Time

Exercises are obtained from Russell & Norvig.

Exercise 1. A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.

Exercise 2. For the DBN specified in Exercise 1 and for the evidence values:

- e_1 = not red eyes, not sleeping in class
- e_2 = red eyes, not sleeping in class
- e_3 = red eyes, sleeping in class

perform the following computations:

- a) State estimation: Compute $P(\text{EnoughSleep}_t | e_{1:t})$ for each of $t = 1, 2, 3$.
- b) Smoothing: Compute $P(\text{EnoughSleep}_t | e_{1:3})$ for each of $t = 1, 2, 3$.
- c) Compare the filtered and smoothed probabilities for $t = 1$ and $t = 2$.