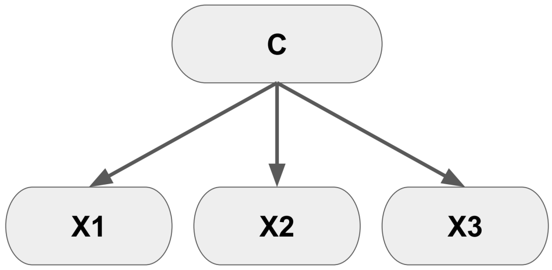

Figure 1: Bayesian network for exercise 4

Exercise Sheet - Reasoning under Uncertainty

Exercise 1. Show from the first principles that $P(a|b \wedge a) = 1$.

Exercise 2. For each of the following statements, either prove that it is true or give a counterexample.

a) If $P(a|b, c) = P(b|a, c)$, then $P(a|c) = P(b|c)$.

b) If $P(a|b, c) = P(a)$, then $P(b|c) = P(b)$.

c) If $P(a|b) = P(a|b, c)$, then $P(a|b, c) = P(a|c)$.

Exercise 3. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

Exercise 4. We have a bag of three biased coins a, b. and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1 , X_2, and X_3.

a) Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

b) Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

Exercise 5. From Russell & Norvig, ex. 14.15. This question is about section 14.4
Consider the variable elimination algorithm above.

(a) Perform variable elimination to the query:
$P(Burglary|JohnCalls = true, MaryCalls = true)$

(b) Count the number of arithmetic operations performed, and compare it with the number performed by the enumeration algorithm.

(c) Suppose a network has the form of a chain: a sequence of Boolean variables $X_1, \ldots, X_n$, where $Parents(X_i) = \{X_{i-1}\}$ for $i = 2, \ldots, n$. What is the complexity of computing $P(X_1|X_{n=true})$ using enumeration? Using variable elimination?