CS4400
DEEP REINFORCEMENT LEARNING

Lecture 10: Deep Multi-agent RL

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Content of this lecture

10.1

Deep Multi-agent RL

Multi-agent games

10.1 Single- vs. Multi-agent environments

- Single agents assume *stationary* environment
  - all other objects are *passive*
  - always react the same to player actions
- Real world has many different actors
  - actors have *intentions* (reward functions)
  - actors can change behavior (in response)
  - environment no longer stationary
- Multi-agent RL formulates environment as a *game*

10.1 Terminology of games

- single/two/multi-player game
  - Solitaire/Chess/Settlers-of-Catan

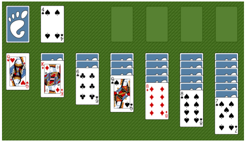

Image sources: wikimedia.org

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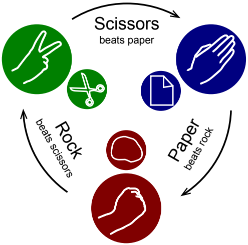

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- discrete/continual state/actions/time
  - turn-based/real-time strategy

Image sources: wikimedia.org, Samvelyan et al. (2019)

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- discrete/continual state/actions/time
  - turn-based/real-time strategy
- zero-sum/general-sum/cooperative game
  - Chess/Settlers-of-Catan/Hanabi

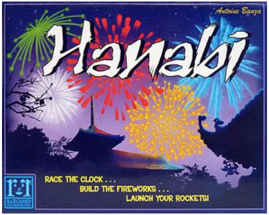

Image sources: wikimedia.org

10.1 Partially observable stochastic games

- POSG $\langle \mathcal{S}, \{\mathcal{A}^i\}_{i=1}^N, \{\mathcal{O}^i\}_{i=1}^N, \rho, P, \{R^i\}_{i=1}^N, \{O^i\}_{i=1}^N \rangle$
  - state space $s \in \mathcal{S}$ of the game
  - action space $a^i \in \mathcal{A}^i$ for each agent $1 \leq i \leq N$
    - joint actions denoted as $\boldsymbol{a} \in \mathcal{A} := \mathcal{A}^1 \times \ldots \times \mathcal{A}^N$
  - observation space $o^i \in \mathcal{O}^i$ for each agent $1 \leq i \leq N$
  - initial state distribution $s_0 \sim \rho(\cdot)$
  - transition probability $s_{t+1} \sim P(\cdot|s_t, \boldsymbol{a}_t)$
  - reward probability $r_t^i \sim R^i(\cdot|s_t, \boldsymbol{a}_t, s_{t+1})$ for each agent $1 \leq i \leq N$
  - observation function $o_t^i \sim O^i(\cdot|s_t)$ for each agent $1 \leq i \leq N$
    - action-observation histories $\tau_t^i := [o_0^i, a_0^i, \ldots, o_t^i] \in (\mathcal{O}^i \times \mathcal{A}^i)^t \times \mathcal{O}^i$
    - joint action-observation history $\boldsymbol{\tau}_t := \{\tau_t^i\}_{i=1}^N$
    - joint history is sufficient belief: $r(\boldsymbol{\tau}_t, \boldsymbol{a})$ and $P(\boldsymbol{\tau}_{t+1}|\boldsymbol{\tau}_t, \boldsymbol{a}_t)$ exist
- Decentralized policy $\boldsymbol{\pi}(\boldsymbol{a}|\boldsymbol{\tau}_t) = \prod_{i=1}^N \pi^i(a^i|\tau_t^i)$

see Oliehoek and Amato (2016) for a sound introduction

core concept: Independent Q-learning (IQL)

- Simply ignore non-stationarity induced by other agents
  - estimate decentralized Q-values $\{q^i_{\theta_i}(\tau^i_t, a^i)\}_{i=1}^N$

$$\mathcal{L}^{\text{IQL}} := \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=0}^{n-1} \Big(r^i_t + \gamma \max_{a^i} q^i_{\theta'_i}(\tau^i_{t+1}, a^i) - q^i_{\theta_i}(\tau^i_t, a^i_t)\Big)^2 \Big| \langle \tau^i_t, a^i_t, r^i_t, \tau^i_{t+1}\rangle \in \mathcal{D}^i\Big]$$

- Convergence for *joint training* with small learning rates
  - other agents' policies almost stationary

- Similar for all flavors of independent policy-gradient methods

IQL first by Tan (1993); see Schröder de Witt et al. (2020) for an example of independent PPO

10.1 Summary

- Multiple non-stationary agents are formalized by games
- POSG formalize discrete time games:
  - multi-agent
  - simultaneous-move
  - stochastic
  - partial-information
- IQL learns POSG by assuming stationary players

Learning Objectives

LO10.1: Classify a game in the given terminology
LO10.2: Explain and implement IQL

10.2

Deep Multi-agent RL
Game theory

core concept: Nash equilibria

- How *should* rational agents/players behave?
- **Rational**: players **maximize** *only* their *own outcome*
  - choosing (silent, silent) appears optimal

Prisoner's dilemma

A \ B	confess	silent
confess	A: -5, B: -5	A: 0, B: -10
silent	A: -10, B: 0	A: -1, B: -1

core concept: Nash equilibria

- How *should* rational agents/players behave?
- **Rational**: players **maximize** *only* their *own outcome*
  - choosing (silent, silent) appears optimal
  - assume A remains silent and B remains silent
  - if A remains silent, B should confess

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  - assume A remains silent and B remains silent
  - if A remains silent, B should confess
  - if B remains confess, A should confess
  - if A confess and B confess,
    neither should change

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en.wikipedia.org/wiki/Nash_equilibrium

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- **Rational**: players **maximize** *only* their *own outcome*
  - choosing (silent, silent) appears optimal
  - assume A remains silent and B remains silent
  - if A remains silent, B should confess
  - if B remains confess, A should confess
  - if A confess and B confess,
    neither should change
- Definition **Nash equilibrium** (a^*, b^*):
  - $r^A(a^*, b^*) \geq r^A(a, b^*), \quad \forall a$
  - $r^B(a^*, b^*) \geq r^B(a^*, b), \quad \forall b$
- (silent, silent) is not a Nash equilibrium!

Prisoner's dilemma

A \ B	confess	silent
confess	-5 / -5	0 / -10
silent	-10 / 0	-1 / -1

en.wikipedia.org/wiki/Nash_equilibrium

10.2 General-sum multi-player games

- Every agent has its own centralized value function $Q_i^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a})$
  - depends on other agents' actions $\boldsymbol{a}^{-i} \in \mathcal{A}^{-i}, \boldsymbol{a} = \{a^i\} \cup \boldsymbol{a}^{-i}$
- Differently valued *Nash equilibria* $\boldsymbol{a}_* \in \mathcal{A}$ (or none) can exist
  - no other action a^i of agent i is better if all other actions $\boldsymbol{a}_*^{-i}$ remain
  - $Q_i^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}_*) \geq Q_i^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \{a^i\} \cup \boldsymbol{a}_*^{-i}), \forall a^i \in \mathcal{A}^i \setminus \{a_*^i\}, \forall i \in \{1, \dots, N\}$

A \ B	stag	hare
stag	2 / 2	1 / 0
hare	0 / 1	1 / 1

the stag-hunt game

A \ B	head	tail
head	0 / 1	1 / 0
tail	1 / 0	0 / 1

matching pennies

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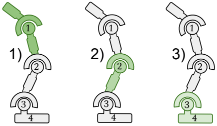
- Requires extensive search for all Nash equilibria (NE)
  - there might not be a NE
  - not all NE are equally good for everyone
  - not all information is available to decentralized agents
  - agents might play different NE (bad for everyone)

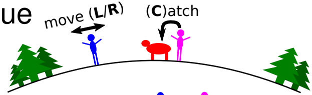
Cooperative multi-player games



- Very common engineering setup
  - independent components need to work together
- Reward functions r^i of all agents are identical

$$r^i(\boldsymbol{\tau}_t, \boldsymbol{a}) \; := \; r^j(\boldsymbol{\tau}_t, \boldsymbol{a}), \quad \forall i, j, \boldsymbol{\tau}_t, \boldsymbol{a}$$

- No conflict of interest: unique centralized value



$$Q_i^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}) \; = \; Q_j^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}), \quad \forall i, j, \boldsymbol{\tau}_t, \boldsymbol{a}$$

- Independent value is decentralized expectation
  - $q^i(\tau_t^i, a^i; \boldsymbol{\pi}) \; := \; \mathbb{E}_{\boldsymbol{\pi}}\Big[Q_i^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}) \,\Big|\, \tau_t^i, a^i \Big], \quad \forall i, \tau_t^i, a^i$
  - $\pi^i(a^i | \tau_t^i) := 1$ iff $a^i = \arg\max_{a^i \in \mathcal{A}^i} q^i(\tau_t^i, a^i; \boldsymbol{\pi})$

	C	L	R
C	+r	-p	-p
L	-p	0	0
R	-p	0	0

- Optimal centralized $\neq$ optimal decentralized policy
  - decentralization may require more *information gathering actions*

assignment sheet 4

called Decentralized POMDP, see Oliehoek and Amato (2016) for a sound derivation

10.2 Zero-sum two-player games

- Very common board game setup
- Reward functions always add to zero (or a constant)

$$r^1(\boldsymbol{\tau}_t, \boldsymbol{a}) \; := \; -r^2(\boldsymbol{\tau}_t, \boldsymbol{a}), \quad \forall \boldsymbol{\tau}_t, \boldsymbol{a}$$

- Which previously discussed games are zero-sum?

assignment sheet 4

for a more formal definition see e.g. Raghavan (1994)

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- Which previously discussed games are zero-sum?

A \ B	head	tail
head	1 / 0	0 / 1
tail	0 / 1	1 / 0

matching pennies

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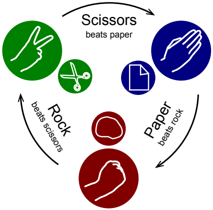
- Very common board game setup
- Reward functions always add to zero (or a constant)

$$r^1(\boldsymbol{\tau}_t, \boldsymbol{a}) \;:=\; -r^2(\boldsymbol{\tau}_t, \boldsymbol{a}), \quad \forall \boldsymbol{\tau}_t, \boldsymbol{a}$$

- Unique value for *equal information* games
  - mirrored value function: $Q_1^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}) = -Q_2^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}),\ \forall \boldsymbol{\tau}_t, \boldsymbol{a}$
  - all Nash equilibria have the same value

$$V(\boldsymbol{\tau}_t) \;=\; \max_{a^1 \in \mathcal{A}^1} \min_{a^2 \in \mathcal{A}^2} Q_1^{\boldsymbol{\pi}}(\boldsymbol{\tau}_t, \boldsymbol{a}), \quad \forall \boldsymbol{\tau}_t$$

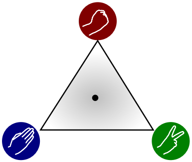


- Can be learned by *self-play*
  - agent plays against mirrored self ($\max \to \min$)
  - unequal information $\tau_t^1 \neq \tau_t^2$ can lead to cycles

A4.2 self-play became popular with AlphaGo (Silver et al., 2016, 2017, 2018); see e.g. Vinyals et al. (2019) for cyclic games

10.2 Cyclic games

- Optimal move can depend on other player's *policy/strategy*
  - sometimes no Nash equilibrium exists
  - simultaneous moves or unequal information



A \ B	rock	paper	scissors
rock	0 / 0	+1 / -1	-1 / +1
paper	-1 / +1	0 / 0	+1 / -1
scissors	+1 / -1	-1 / +1	0 / 0

images modified from wikimedia.org

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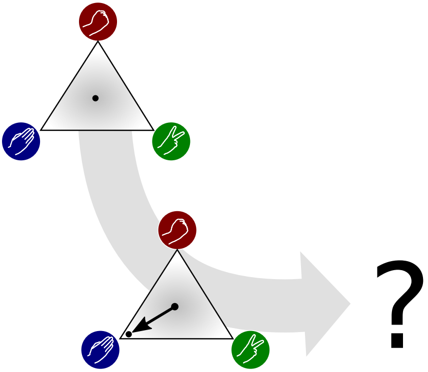

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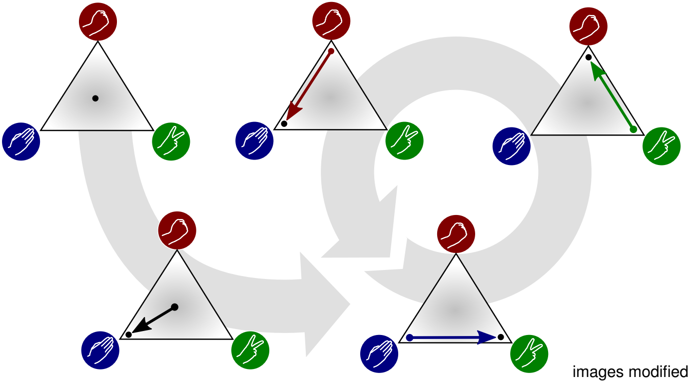
10.2 Cyclic games

- Optimal move can depend on other player's *policy/strategy*
  - sometimes no Nash equilibrium exists
  - simultaneous moves or unequal information
- Self-play assumes opponent uses the same policy!



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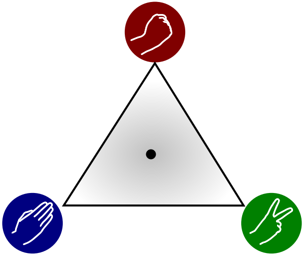
10.2 Leagues

- What is the optimal response in a zero sum cyclic game?
  - mixed Nash equilibrium $\max_{\pi^1} \min_{\pi^2} \mathbb{E}\Big[Q(\boldsymbol{\tau}_t, \boldsymbol{a}) \,\Big|\, \substack{a^1 \sim \pi^1(\cdot|\tau_t^1) \\ a^2 \sim \pi^2(\cdot|\tau_t^2)} \Big]$
  - average case response $\max_{\pi^1} \mathbb{E}\Big[Q(\boldsymbol{\tau}_t, \boldsymbol{a}) \,\Big|\, \substack{a^1 \sim \pi^1(\cdot|\tau_t^1) \\ a^2 \sim \pi'(\cdot|\tau_t^2)}, \pi' \sim \Pi \Big]$



A4.3

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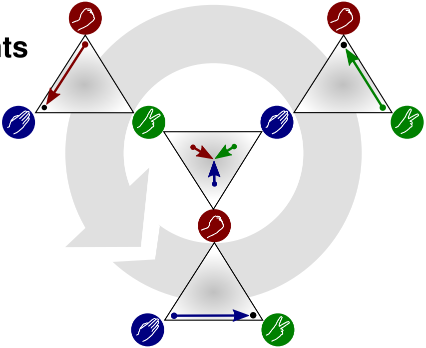
- Can be trained in a **league of agents**
  - keep old policies around
  - play against all of them
  - use either worst or average loss



A4.3 this idea has been recently popularized by AlphaStar (Vinyals et al., 2019); images modified from `wikimedia.org`

10.2 Leagues

- What is the optimal response in a zero sum cyclic game?
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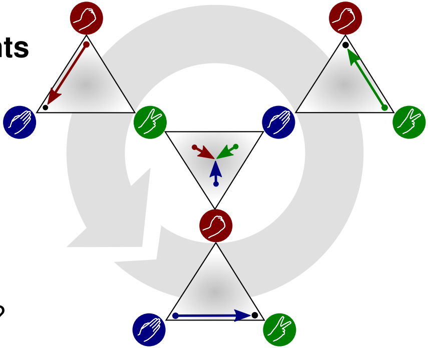
  - average case response $\max_{\pi^1} \mathbb{E}\Big[Q(\boldsymbol{\tau}_t, \boldsymbol{a}) \,\Big|\, \substack{a^1 \sim \pi^1(\cdot|\tau_t^1) \\ a^2 \sim \pi'(\cdot|\tau_t^2)}, \pi' \sim \Pi\Big]$
- Can be trained in a **league of agents**
  - keep old policies around
  - play against all of them
  - use either worst or average loss
- Open research questions:
  - *which* policies should be kept?
  - let *adversary* choose who to play?



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10.2 Summary

- Nash equilibria (NE) are stable for rational agents
- General-sum games can have many or no NE
- Cooperative and zero-sum games have unique centralized values
- Without NE, cyclic games must be solved by leagues

Learning Objectives

LO10.3: Define, explain and find Nash equilibria
LO10.4: Explain general-sum, zero-sum and collaborative games

10.3

Deep Multi-agent RL
Centralized training

10.3 Centralized training and decentralized execution

- More information available during *centralized training*
  - other agents' actions $\boldsymbol{a}_t^{-i}$ and histories $\boldsymbol{\tau}_t^{-i}$
  - sometimes true state s_t, e.g. for value functions $V^\pi(s_t, \boldsymbol{\tau}_t)$
- Centralized training allows *parameter sharing*
  - all agents have the same architecture and parameters θ
  - extending input with class/role/id differentiates agents
- Effectively reuses training data of all agents
  - enforces *permutation invariance* between agents
  - can dramatically improve sample efficiency
- Example: IQL with centralized training and parameter sharing
  - DRQN implementation where `dim=-2` stacks agents

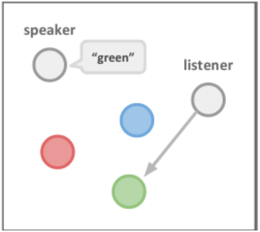
see e.g. `pymarl` for a collaborative IQL implementation (Samvelyan et al., 2019)

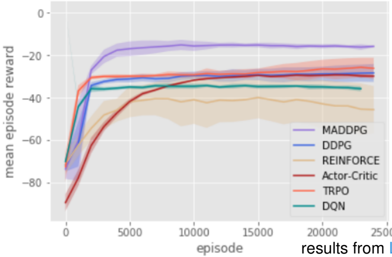
Multi-agent DDPG (MADDPG)

- Independent DDPG with centralized Q-value functions $Q^{\boldsymbol{\pi}}_{\phi_i}$
  - parameter sharing only for collaborative games
  - fixes all other agents' behavior to $\boldsymbol{a}_t^{-i}$ from replay buffer

$$\mathcal{L}^{\text{MADDPG}}_{\mu[\boldsymbol{\theta}]} := -\sum_{i=1}^{N} \mathbb{E}_\mu\Big[\sum_{t=0}^{n-1} Q^{\boldsymbol{\pi}}_{\phi_i}\big(s_t, \{\pi^i_{\theta_i}(\tau^i_t)\} \cup \boldsymbol{a}_t^{-i}\big)\Big]$$

$$\mathcal{L}^{\text{MADDPG}}_{Q[\boldsymbol{\phi}]} := \sum_{i=1}^{N} \mathbb{E}_\mu\Big[\sum_{t=0}^{n-1}\Big(r^i_t + \gamma Q^{\boldsymbol{\pi}}_{\phi'_i}\big(s_{t+1}, \{\pi^j_{\theta'_j}(\tau^j_{t+1})\}_{j=1}^N\big) - Q^{\boldsymbol{\pi}}_{\phi_i}(s_t, \boldsymbol{a}_t)\Big)^2\Big]$$





results from Lowe et al. (2017)

Counterfactual multi-agent learning (COMA)

- Stochastic policy-gradients in cooperative games
  - centralized training with parameter sharing
  - centralized value $Q^{\boldsymbol{\pi}}_{\phi}(s, \boldsymbol{a})$, decentr. policy $\boldsymbol{\pi}_\theta(\boldsymbol{a}|\boldsymbol{\tau}_t) = \prod_{i=1}^{N} \pi^i_\theta(a^i|\tau^i_t)$

$$\mathcal{L}^{\text{C-QV}}_{\pi[\theta]} := -\sum_{i=1}^{N} \mathbb{E}_{\pi_\theta}\Big[A_t \ln \pi^i_\theta(a^i_t|\tau^i_t)\Big], \quad A_t := Q^{\boldsymbol{\pi}}_{\phi}\big(s_t, \boldsymbol{a}_t\big) - V^{\boldsymbol{\pi}}(s_t)$$

$$= -\mathbb{E}_{\pi_\theta}\Big[A_t \ln \boldsymbol{\pi}_\theta(\boldsymbol{a}_t|\boldsymbol{\tau}_t)\Big],$$ centralized = sum of independent

StarCraft 2 results of COMA from Förster et al. (2018)

Counterfactual multi-agent learning (COMA)

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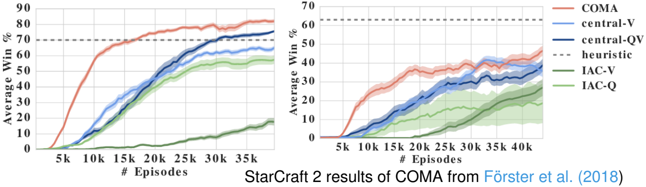
$$\mathcal{L}^{\text{COMA}}_{\pi[\theta]} \;:=\; -\sum_{i=1}^{N} \mathbb{E}_{\pi_\theta}\Big[A^i_t \ln \pi^i_\theta(a^i_t|\tau^i_t)\Big], \qquad A_t \;:=\; Q^{\boldsymbol{\pi}}_{\phi}\big(s_t, \boldsymbol{a}_t\big) - V^{\boldsymbol{\pi}}(s_t)$$

- Same baseline $V^{\boldsymbol{\pi}}(s_t)$ for all joint actions $\boldsymbol{a}_t$ has high variance
  - different counterfactual baseline in advantage A^i_t for each $\boldsymbol{a}^{-i}_t$

$$A^i_t \;:=\; Q^{\boldsymbol{\pi}}_{\phi}(s_t, \boldsymbol{a}_t) - \sum_{a'^i \in \mathcal{A}^i} \pi^i_\theta(a'^i|\tau^i_t)\, Q^{\boldsymbol{\pi}}_{\phi}\big(s_t, \{a'^i\} \cup \boldsymbol{a}^{-i}_t\big)$$



StarCraft 2 results of COMA from Förster et al. (2018)

10.3 Summary

- During training we might have centralized information
- MADDPG extends DDPG with centralized Q-values
- COMA extends AC additionally with low-variance bias

Learning Objectives

LO10.5: Explain centralized training and decentralized execution
LO10.6: Explain how MADDPG and COMA exploit centralized training

10.3 Next lecture

- Next lecture: **advanced MARL**!
- Remember assignment sheet 4 (and exercise sheet 4)!
- Questions? Ask them here: answers.ewi.tudelft.nl

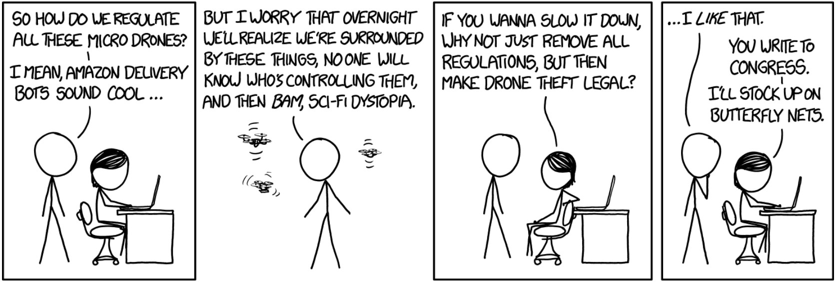


image source: xkcd.com

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