CS4070: ASSIGNMENT 3: GAUSSIAN PROCESS CLASSIFICATION

Hand in before 12 January, 23.59.

(1) (a) Write a script that samples from a Gaussian process $f\colon \mathbb{R} \to \mathbb{R}$, for given mean functions μ and kernel K.

(b) Consider the following 3 kernel functions: squared exponential kernel, polynomial kernel of degree 2, neuronal kernel.
For each of these kernel functions, draw 5 realisations from the Gaussian process and overlay these in one plot. Consider $x \in [-5, 5]$. Play around a bit with the hyperparameters in these kernels so that "typical" paths are displayed. Include the plots in your report. Note that for numerical stability you may wish to add a small multiple of the identity matrix to the Gram-matrix (which is the matrix with elements $K_{ij} = K(x_i, x_j)$).

(2) Assume data $\{(x_i, y_i)\}$, where $x_1 = -1$, $x_2 = -2$, $x_3 = 1$, $x_4 = 2$, $y_1 = 1$, $y_2 = 4$, $y_3 = 1$, $y_4 = 4$. Assume the relationship $y_i = f(x_i) + \varepsilon_i$ (default non-parametric regression model). Apply Gaussian process regression to predict the value at zero. Use the squared exponential kernel, where hyperparameters are estimated using empirical Bayes (type II maximum likelihood). You may either implement this yourself, or use a package in R, Matlab, Python, Julia... (choose the language that is convenient for you) for this. Include a plot of the predictive density for $x \in [-3, 3]$ (note that the data were generated using $f(x) = x^2$ with no additive noise superimposed).

Date: December 21, 2023.