

$$1_{[0,1]}(\theta) = \begin{cases} 1 & 0 \leq \theta \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P(\theta | y) \propto P(y, \theta) \propto \theta^s (1-\theta)^{n-s} \cdot \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1} 1_{[0,1]}(\theta)$$

$$\propto \frac{1}{B(\alpha, \beta)} \theta^{(s+\alpha)-1} (1-\theta)^{(n-s+\beta)-1} 1_{[0,1]}(\theta)$$

$$\propto \frac{1}{B(s+\alpha, n-s+\beta)} \theta^{(s+\alpha)-1} (1-\theta)^{(n-s+\beta)-1} 1_{[0,1]}(\theta)$$

Conjugate prior

$$\log\left(\frac{\theta}{1-\theta}\right) = z \quad \text{natural logarithm}$$

$$\frac{\theta}{1-\theta} = \exp(z)$$

$$\theta = \exp(z) - \exp(z) \theta$$

$$\theta = \frac{e^z}{1+e^z} \quad f: z \mapsto \frac{e^z}{1+e^z} = \frac{1}{1+e^{-z}}$$

$$f: \mathbb{R} \rightarrow (0, 1)$$

$$\theta_{ij} = f(\alpha_i + \beta_j x_{ij})$$

$$f_x(x, \gamma) = \int f(x|\theta) p(\theta, \gamma) d\theta$$

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmax}} f_x(x, \gamma)$$

$$y_{\text{new}} = x_{\text{new}}^T \theta + \epsilon_{\text{new}}$$