$$p = \frac{1}{1+e^{-\theta^T x}}$$

$$\Leftrightarrow \log \frac{p}{1-p} = \theta^T x$$

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$\psi(z) = \frac{1}{1+e^{-z}}$$

$$\psi'(z) = -\frac{1}{(1+e^{-z})^2}\left(-e^{-z}\right) = \frac{e^{-z}}{(1+e^{-z})^2} = e^{-z}\,\psi(z)^2$$

$$= \psi(z)\left(1-\psi(z)\right)$$

$$p_i = \psi(\theta^T x_i)$$

$$\frac{\partial p_i}{\partial \theta_j} = \psi'(\theta^T x_i)\,(x_i)_j = \psi(\theta^T x_i)\left(1-\psi(\theta^T x_i)\right)(x_i)_j$$

$$= p_i(1-p_i)(x_i)_j$$

$$\psi'(\theta^T x_i) = \underbrace{\psi(\theta^T x_i)}_{p_i} \underbrace{(1 - \psi(\theta^T x_i))}_{1 - p_i} = p_i (1 - p_i)$$

p_i depends on θ

$$\log L(\theta) = \log \prod_{i=1}^{n} p_i^{y_i} (1 - p_i)^{1 - y_i}$$

$$= \sum_{i=1}^{n} \log p_i^{y_i} (1 - p_i)^{1 - y_i} = \sum y_i \log p_i + \sum (1 - y_i) \log (1 - p_i)$$

$$\frac{\partial \log L(\theta)}{\partial \theta_j} = \sum \frac{y_i}{p_i} \frac{\partial p_i}{\partial \theta_j} + \sum \frac{1 - y_i}{1 - p_i} \frac{\partial (1 - p_i)}{\partial \theta_j}$$

$$= \sum_i \left(\frac{y_i}{p_i} - \frac{1-y_i}{1-p_i}\right)\frac{\partial p_i}{\partial \theta_j}$$

$$= \sum_i \left(\frac{y_i}{p_i} - \frac{1-y_i}{1-p_i}\right) p_i(1-p_i)(x_{ij})$$

$$= \sum_i \left(y_i(1-p_i) - (1-y_i)p_i\right)(x_{ij})$$

$$= \sum_i (y_i - p_i)(x_{ij}) = \sum (y_i - p_i)\, x_{ij}$$

$$= X^T(y-p)$$

$$\frac{\partial}{\partial \theta_k} \frac{\partial \log L(\theta)}{\partial \theta_j} = \frac{\partial}{\partial \theta_k} \sum_i (y_i - p_i) x_{ij}$$

$$= - \sum_i \frac{\partial p_i}{\partial \theta_k} x_{ij} = - \sum_i p_i(1-p_i) x_{ik} x_{ij}$$

$$= - X^T \Lambda X$$

$$\Lambda = \begin{pmatrix} p_1(1-p_1) & & \\ & \ddots & \\ & & p_n(1-p_n) \end{pmatrix}$$

Trick –
- Calculate posterior density without constants
- Recognize certain distribution
- Fill in constants from known distribution