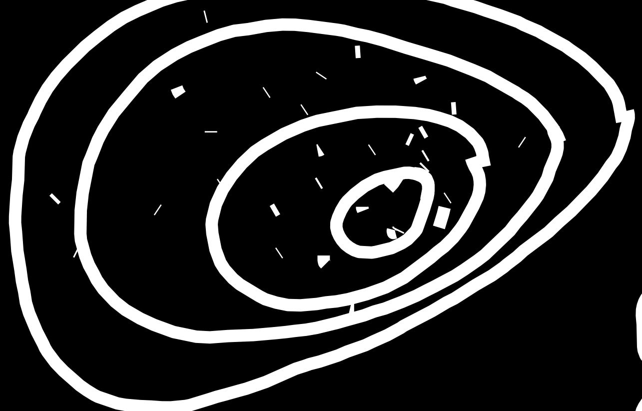

Markov Chain

stochastic

model for a sequence of possible events where future events depend only on the current but not on previous events

$$\mathbb{P}(X_{n+1} \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \mathbb{P}(X_{n+1} \mid X_n = x_n)$$

if $\mathbb{P}(X_1 = x_1, \ldots, X_n = x_n) > 0$, $X_1, \ldots, X_{n+1}$ RVs

Time homogeneous Markov chain

$$\mathbb{P}(X_{n+1} = x \mid X_n = y) = \mathbb{P}(X_n = x \mid X_{n-1} = y)$$

π stationary distribution

$$\Leftrightarrow (X_1 \sim \pi \Rightarrow X_2 | X_1 \sim \pi)$$

$$\Leftrightarrow (X_1 \sim \pi \Rightarrow X_n | X_1 \sim \pi)$$

for all n

irreducible Markov chain

nonzero probability of transitioning (possibly in more than one step) from any state to any other

irreducible Markov chain
$\Rightarrow$ stationary distribution is unique

stationary distribution is usually the limiting distribution of Markov chain start with $\theta \sim \pi$, assume $\theta \neq \theta^\circ$

$$\pi(\theta) \underbrace{q(\theta,\theta^\circ)\ \alpha(\theta,\theta^\circ)}_{\text{transition probability from } \theta \text{ to } \theta^\circ}$$

$$= \pi(\theta)\, q(\theta,\theta^\circ) \min\left(1, \frac{\pi(\theta^\circ)\, q(\theta^\circ,\theta)}{\pi(\theta)\, q(\theta,\theta^\circ)}\right)$$

$$= \min\left(\pi(\theta)\, q(\theta,\theta^\circ),\ \pi(\theta^\circ)\, q(\theta^\circ,\theta)\right)$$

$$= \pi(\theta^\circ)\, q(\theta^\circ,\theta)\ \alpha(\theta^\circ,\theta)$$

$$\sum_{\theta} \pi(\theta') \bar{q}(\theta',\theta) = \pi(\theta') \underbrace{\sum_{\theta} \bar{q}(\theta',\theta)}_{=1} = \pi(\theta')$$

$$= \sum_{\theta} \pi(\theta)\, q(\theta,\theta')$$

π is invariant distribution

(stationary $\hat{=}$ invariant)

Assignment 1, Exercise 4

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} m \\ Hm+u \end{pmatrix}, \begin{pmatrix} P & PH' \\ HP & HPH'+R \end{pmatrix} \right)$$

$$\Rightarrow \quad Y \sim N(Hm+u,\ HPH'+R)$$

match to equation (2)

$$\pi(\theta) = \frac{p(x|\theta)\, p(\theta)}{\int p(x|\theta)\, p(\theta)\, d\theta} = c\, p(x|\theta)\, p(\theta)$$

$$\frac{\pi(\theta^0)}{\pi(\theta)} = \frac{\cancel{c}\, p(x|\theta^0)\, p(\theta^0)}{\cancel{c}\, p(x|\theta)\, p(\theta)}$$

$$\nabla \log \pi(\theta) = \nabla \log \left(c\, p(x|\theta)\, p(\theta)\right)$$

$$= \nabla \log(c) + \nabla \log\left(p(x|\theta)\, p(\theta)\right)$$

$$= \nabla \log\left(p(x|\theta)\, p(\theta)\right)$$