List of exercises from 'Probability and Stochastic Processes', Yates and Goodman

2020-2021

September 17, 2021

1 Week 1: refresher random variables

	2nd edition	3rd edition
Event space	1.4.1, 1.5.1	1.5.1, 1.4.1
Conditional probability	1.5.5	1.4.7
Bayes; rule	1.7.6	2.1.6
CDF	2.4.3	3.4.3
Expected value	2.5.5	3.5.8
Variance	2.8.4	3.8.4
Conditional PMF	2.9.3	In Problem 3.4.3, find $P_{X\|B}(x)$, where the condition $B = \{X > 0\}$. What are $E[X\|B]$ and $Var[X\|B]$?
Uniform distribution	3.4.5	4.5.10

2 Week 2: multivariate distributions

	2nd edition	3rd edition
CDF	4.1.1	5.1.1
Events	4.2.1	5.2.1
Marginal pdf	4.2.2, 4.3.2	5.2.2, 5.3.2
Joint pdf	4.4.1, 4.5.2, 4.5.6	5.4.1, 5.5.3, 5.5.9
Correlation/covariance	4.7.9	5.7.12

3 Week 3: random processes

	2nd edition	3rd edition
Random processes	10.2.3, 10.10.3 NOTE: in some pdf-versions of the 2nd edition there is a typo. The process is defined as: $W(t) = X\cos(2\pi f_0 t) + Y\sin(2\pi f_0 t)$.	13.1.4, 13.9.8
IID processes	10.4.1, 10.4.2	13.3.1, 13.3.2
Gaussian processes	4.11.1, 4.11.4	5.9.1, 5.9.6
Poisson processes	10.5.1, 10.5.6	13.4.1, 13.4.8

4 Week 4: autocorrelation function

	2nd edition	3rd edition
Autocorrelation function	10.8.1, 10.8.3	13.7.1, 13.7.3
Stationary	10.9.1, 10.10.4	13.8.1, 13.9.9
Valid autocorr.	10.10.1	13.9.1
linear filtering	11.1.1, 11.1.3	1.1, 1.3
extra	E1 (see below)	

5 Week 5: random signal processing

	2nd edition	3rd edition
Random sequences	11.2.1, 11.2.3(a)-(c)	2.1, 2.3(a)-(c)
NOTE, in exercise 11.2.3/2.3 there is an error. $R_Y[n]$ should be $C_Y[n]$.		
	11.2.8	2.8
extra	E2,E3 (see below)	E2,E3 (see below)
Power spectral density	11.5.1	5.1
Frequency domain	11.8.2, 11.8.5	8.2, 8.5

6 Week 6: statistical estimation

	2nd edition	3rd edition
Expected value of sums	6.1.5	see below
2D random variables	6.2.1	see below
Central limit theorem	6.6.1	9.4.1
Sample mean	7.1.2	10.1.2
Markov and Chebychev ineq.	7.2.4 (a)-(b)	10.2.5 (a)-(b)
Optimal estimation	9.1.2	12.1.2

7 Week 7: Markov chains

	2nd edition	3rd edition
Discrete-Time Markov Chain	12.1.1, 12.2.2	1.3, 2.2
Limiting State Probabilities	12.5.2	5.2
Discrete-Time Markov Chain	12.1.2, 12.2.1	1.4, 2.1
	12.1.5	1.5
Limiting State Probabilities	12.5.1	5.1
	12.5.6	5.12

8 Extra Exercises

E1 The input to a digital filter is a sequence of random variables $..., X_{-1}, X_0, X_1, ...$ The output is also a sequence of random variables $..., W_{-1}, W_0, W_1, ...$ The relationship between input and output is

$$W_n = \frac{1}{2}(X_n + X_{n+1}) \tag{1}$$

Let the input be a sequence of iid random variables with $E[X_n] = 0$ and $Var[X_n] = 1$. Find the following properties of the output sequence: $E[W_n]$, $Var[W_n]$, $Cov[W_{n+1}, W_n]$ and ρ_{W_{i+1},W_i}.

E2 The input $..., X_{-1}, X_0, X_1, ...$ denote an iid random sequence with $E[X_n] = 0$ and $Var[X_n] = 1$. The output of a digital filter is $..., Y_{-1}, Y_0, Y_1, ...$ defined by

$$Y_n = X_{n+1} + X_n + X_{n-1} \tag{2}$$

Find the autocovariance function $C_Y[m,k]$.

E3 Let $...,X_{-1},X_0,X_1,...$ and the output $...,Y_{-1},Y_0,Y_1,...$ of a digital filter obey

$$Y_n = \frac{1}{2}\left(X_n + Y_{n-1}\right) \tag{3}$$

(note the Y_{n-1} on the right side of the equation!).

Let the input be a sequence of iid random variables with $E[X_n] = \mu_X = 0$ and $Var[X_n] = \sigma^2$. Find the following properties of the output sequence: $E[Y_n]$, $Var[Y_n]$, $Cov[Y_{n+1}, Y_n]$ and ρ_{Y_{i+1},Y_i}.

9 Exercises from the old book

6.1.5 The input to a digital filter is a random sequence $...,X_{-1},X_0,X_1,...$ with $E[X_i] = 0$ and autocovariance function

$$C_X[m,k] = \begin{cases} 1 & k = 0 \\ 1/4 & |k| = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

A smooting filter produces the output sequence

$$Y_n = (X_n + X_{n-1} + X_{n-2})/3 \tag{5}$$

Find the following properties of the output sequence: $E[Y_n]$, $Var[Y_n]$.

6.2.1 Find the PDF of $W = X + Y$ when X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 \le x \le y \le 1 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$