

# Conceptual Design and Performance Analysis Tool for Rotorcraft Systems

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## Abstract

The availability of a free open source tool for the design and performance evaluation of rotary wing aircrafts is scarce, this work intends to provide a solution for engineering students, aircraft engineers or anyone interested in aircraft design with such a tool. The tool applies the Momentum theory and the Blade Element theory to execute the analysis of the rotor performance and presents the results in a clear and simple way (power against airspeed plots, rotor disk distribution plots of the calculated variables, as well as numerical results explicitly identified). A small data base with airfoils (and their aerodynamic performances) is included in the tool. The tool's results validation is done and presented to assure the user of the reliability of the analyses done.

**Keywords:** Momentum Theory, Blade Element Theory, aircraft performance, preliminary design, design space

## 1. Introduction

### 1.1. Motivation

A helicopter is an aircraft that uses rotary wing systems (rotors) to produce lift, thrust and control forces. This generation of force does not require forward velocity, so the aircraft can lift vertically and hover. Motion can be induced by tilting the rotor, depending on the direction of this action the helicopter can move forwards or backwards, left or right. The wide range of flight movements even at low speeds give the helicopter characteristics like no other aircraft making it suitable for all kinds of operation.

Versatility doesn't come without a cost, for this kind of aircraft can have very different flight conditions, each having specific requirements, making it delicate (or near impossible) to have an optimal performance for all the different design points (hovering, cruise flight, maximum speed flight, climb). Rotorcraft design is a multidisciplinary problem that includes fields as aerodynamics, aeroelasticity, thermodynamics, structures and materials, flight dynamics and controls.

Many software tools for conceptual and preliminary rotorcraft design have been developed throughout the last 40 to 50 years, but a free and open source tool is still not easy to come across. The work developed here aims to provide such a tool for anyone interested in rotary wing aircrafts' design. The simple and direct nature of the Mat-

Lab tool will allow the user to easily make design modifications and calculate their impact on the aircraft performance as to verify theoretical concepts, empirical models or any other performance related aspect of the rotorcraft.

### 1.2. State of the art

A lot of rotorcraft design tools have been developed throughout the years to evaluate flight performance (considering rotor and fuselage aerodynamics, structural and weight analysis, and control and stability), environmental impact (fuel consumption, noise, and exhaust gas emissions) and aircraft manufacturing, maintenance and operational costs.

Previous to this work several tools were already developed in Instituto Superior Técnico. Roman Vasyliovych Rutsky [1], developed a tool for the preliminary design of a conventional helicopter, Anatol Conjocari [2], expanded the tool to other helicopters configurations (co-axial and tandem), and Miguel Ponte [3], further developed the complexity of the tool introducing the possibility of a more detailed rotor blade design (airfoil change along the blade, as well as chord and twist distributions).

Developed by the Israel Institute of Technology RAPID/RaTE is a software package for rotorcraft preliminary design analysis, it models general configurations (conventional helicopters and

tilt-rotors) based on existing aircrafts by extracting common features, or "design trends", which are then used in the first sizing stage of the helicopter design, later it performs trim response, mission, vibration, and stability analysis as well as flight mechanics and aeroelastic simulations, see [4].

## 2. Theoretical Background

Two main theories have been developed to evaluate the helicopter rotor's performance, the Momentum Theory (MT), and the Blade Element Theory (BET). Each of these theories will be described in this section following the content presented in [5].

### 2.1. Momentum Theory

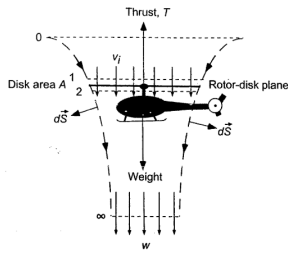


Figure 1: Flow model for momentum theory in hovering flight ([5], pg. 61)

The generic approach of this problem assumes that the flow is one-dimensional, quasi-steady, incompressible and inviscid, and although it does not take into account the complex vortical wake structure associated with the rotor aerodynamics or the actual details of the flow environment (local flow around the rotor blade), it allows for a first-order prediction of the thrust generated and power required for a given flight condition. Applying basic conservation laws of fluid mechanics to the control volume of Figure 1 (such as conservation of mass, equation 1, momentum, equation 2 and energy, equation 3) to the rotor flow, as a whole, estimations of the performance can be made, this simple approach became known as the Rankine-Froude momentum theory.

$$\int_S \rho \vec{V} \cdot d\vec{S} = 0 \quad (1)$$

Where  $\rho$  is the specific mass,  $\vec{V}$  the flow velocity, and  $d\vec{S}$  the unit vector normal to the control volume surface  $S$ .

$$\int_S p d\vec{S} + \int_S (\rho \vec{V} \cdot d\vec{S}) \vec{V} = \vec{F} \quad (2)$$

Where  $p$  is the pressure and  $\vec{F}$  the resultant force.

$$\int_S \frac{1}{2} (\rho \vec{V} \cdot d\vec{S}) |\vec{V}|^2 = E \quad (3)$$

Where  $E$  is the work done on the fluid by the rotor.

Applying these three equations for the different flight conditions stages (typically hover, axial climb and level flight) a series of relations can be derived for the analysis and comprehension of the helicopter characteristics.

### 2.2. Blade Element Theory

Rotor aerodynamics analysis has its foundation on Blade Element Theory as it allows to calculate radial and azimuthal distributions of the aerodynamic loads over the rotor disk. The base assumption of this theory is that each blade section acts as a 2D airfoil to produce forces. The rotor performance is calculated integrating the airloads of each section along the blade and averaging the results over a complete revolution. This theory, unlike the momentum theory, can be used as a basis to design the rotor blade in terms of twist and chord distribution, as well as in terms of the airfoil, or airfoils, to be used.

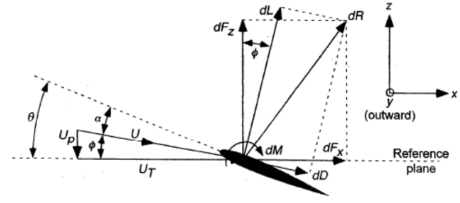


Figure 2: Blade element - Force diagram

The force analysis gives us:

$$dL = \frac{1}{2} \rho U^2 c C_l dy, \quad \text{and} \quad dD = \frac{1}{2} \rho U^2 c C_d dy \quad (4)$$

Where  $L$  is the aerodynamic lift,  $U$  the inflow velocity,  $c$  the blade chord,  $C_l$  the airfoil lift coefficient,  $D$  the aerodynamic drag and  $C_d$  the airfoil drag coefficient.

The forces aligned with the perpendicular ( $z$ ) and parallel ( $x$ ) directions (in relation with the rotor plane) can be expressed:

$$\begin{aligned} dF_z &= dL \cos \phi - dD \sin \phi, \quad \text{and} \\ dF_x &= dL \sin \phi + dD \cos \phi \end{aligned} \quad (5)$$

With  $\phi$  being the inflow angle, see Figure 2.

And consequently the thrust, torque and power contributions are:

$$\begin{aligned} dT &= N_b dF_z, \quad dQ = N_b dF_x r \quad \text{and} \\ dP &= N_b dF_x \Omega r \end{aligned} \quad (6)$$

With  $N_b$  being the rotor number of blades.

### 2.3. Linear inflow models

When considering a forward flight condition the assumption of axisymmetric flow is not valid, nonetheless simple models can be used to estimate the basic effects of the inflow. The simplicity of these models has made them widely used in helicopter rotor aerodynamics problems.

The general form of these models is:

$$\lambda_i = \lambda_0(1 + k_x r \cos \Psi + k_y r \sin \Psi) \quad (7)$$

With  $\lambda_i$  being the inflow ratio at a given position,  $k_x$  and  $k_y$  are, respectively, the longitudinal and lateral inflow slopes,  $r$  is the nondimensional radial position and  $\Psi$  is the azimuthal coordinate.  $\lambda_0$  is the inflow ratio at the center of the rotor.

The different models considered in this work are as follows:

- Coleman et al. (1945), [6]
- Drees (1949), [7]
- Payne (1959), [8]
- White & Blake (1979), [9]
- Pitt & Peters (1981), [10]
- Howlett (1981), [11]

### 2.4. Flapping blade

The motion resulting from the coupling of aerodynamic forces acting on the rotor is paramount to the understanding of the blade motion as to allow for the pilot to successfully control the helicopter. Rotors commonly have articulations near the blade root in the form of flapping and lead-lag hinges.

The equilibrium position of a flapping blade will depend on the relation between aerodynamic and centrifugal forces.

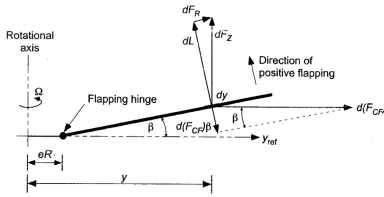


Figure 3: Blade forces equilibrium about the flapping hinge, small angle approximation ([5], pg. 175)

The equation of motion for a flapping blade results in:

$$\beta^{**} + \nu_\beta^2 \beta = \frac{1}{I_b \Omega^2} M_\beta \quad (8)$$

With  $\beta$  being the flapping angle,  $\beta^{**}$  its second derivative in respect to the azimuth angle,  $\nu_\beta$  the nondimensional flapping frequency,  $I_b$  the mass moment of inertia,  $\Omega$  the blade rotational velocity and  $M_\beta$  the aerodynamic moment.

## 3. Software Implementation

The computational tool is intended to allow the user to make the conceptual and preliminary design of a rotary wing aircraft (conventional, co-axial or tandem helicopters) at two different levels of complexity, these will be henceforth called the *basic design* and the *detailed design*. Additionally another tool was developed to the design of small dimensions unmanned rotorcrafts, *drone tool*. One peripheral tool was programmed so that the user could compare airfoil performances when designing the rotor blades *airfoil comparison tool*.

The main idea that dictates the behaviour of the tools is that the user defines the flight conditions (and top level aircraft requirements, in the *detailed tool*) in which the aircraft is to operate and calculations are made in order to assess the feasibility of the design. The user will have the freedom to conduct alterations (namely to the rotor geometry) and compare the performances of the different designs.

### 3.1. Basic tool

The information flow and user experience will be described in this chapter. First the user will have to choose between three different helicopter configurations, conventional, co-axial or tandem. After this decision the *basic tool* operation will have minimal differences for each configuration in terms of the user experience, only the results will be significantly different.

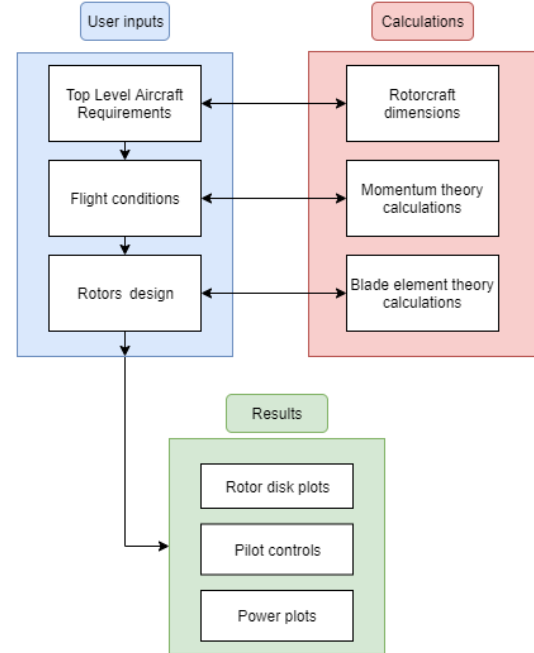


Figure 4: Basic tool structure

The user will be asked one single input to start

the design of the chosen aircraft this input will be one of the following three (which one to choose is up to the user):

- Aircraft total mass -  $m$  in  $[kg]$
- Number of passengers -  $N_{pass}$
- Main rotor radius -  $R$  in  $[m]$

With the one input the other two will be calculated. Based on the rotor radius the helicopter dimensions (both for the fuselage as for the rotors) are estimated using the relations from [4].

Following the dimensions estimations the user will define three flight conditions for the helicopter. The first calculations will follow the momentum theory, and the user will have the opportunity to generate a plot where the power variation with the forward velocity is shown.

After this, the rotors geometries might be set so that the blade element theory can be used and calculate a more accurate performance analysis of the helicopter. The variables that the user will be able to control in terms of the rotor design are:

- Number of blades -  $N_b$
- Rotor radius -  $R$
- Rotor blade chord -  $c$
- Rotor blade airfoil
- Rotor rotational velocity -  $\Omega$
- Rotor root-cut-out -  $r_0$

The blade element theory calculation are used to trim the helicopter (in the conventional case taking into account the tail rotor contribution, and on the co-axial and tandem case taking into account the rotors' interaction).

The user has control over some of the calculations parameters:

- Number of radial segments (radial discretization)
- Number of angular positions (azimuthal discretization)
- Convergence criteria
- Maximum number of iterations
- Linear inflow model

The results are demonstrated as rotor disk plots of the variable that the user wishes to see, and as power plots (showing the power variation with the helicopter flight velocity).

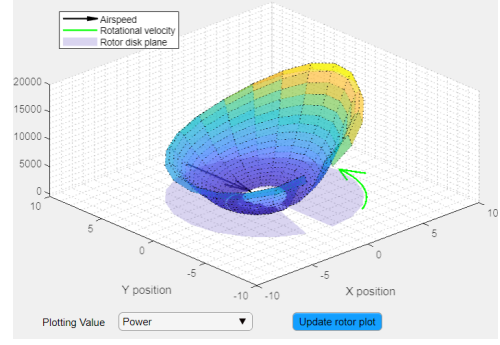


Figure 5: Basic tool - Main rotor disk plot

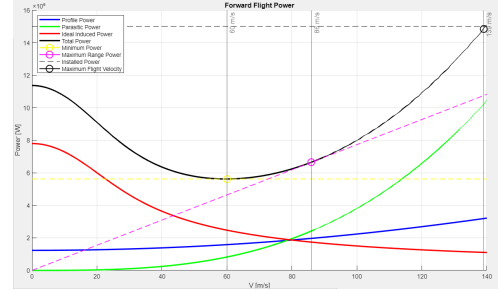


Figure 6: Basic tool - Power plot

### 3.2. Detailed tool

This tool offers a much deeper level of involvement in the helicopter design, more specific top level requirements can be defined, rotor blade geometry is now editable in terms of chord, twist and airfoil distribution along the blade, and a more complete rotor trim is introduced as the blade motion now also considers flapping (the pilot's cyclic controls will be calculated).

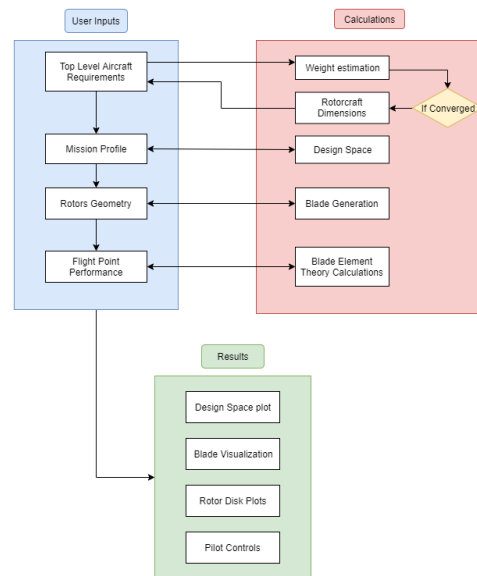


Figure 7: Detailed tool structure

The inputs required in regards to the top level aircraft requirements for this tool are:

- Aircraft payload  $[kg]$ :  $m_{payload}$
- Crew and passengers:  $N_{pass}$
- Cruise minimum range  $[km]$ :  $R_{min}$
- Cruise endurance  $[minutes]$ :  $E$
- Cruise speed  $[m/s]$ :  $V_{cruise}$
- Maximum gross weight  $[kg]$ :  $m_{gross}^{max}$

After having set all these values a series of calculations follow to assess the feasibility of the design.

The user will again be able to set some flight conditions and the result will be the generation of a design space where the disk loading ( $DL$ ) *versus* the power loading ( $PL$ ) will be plotted:

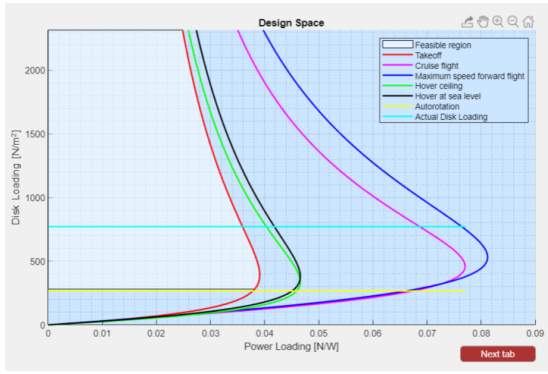


Figure 8: Design space

Following this the rotors geometry definition, the user might change the variables shown below:

- Number of blades:  $N_b$
- Rotor radius:  $R$  in  $[m]$
- Rotor root cut-out:  $r_0$  [% of the rotor radius]
- Flapping hinge position  $eR$  [% of the rotor radius]
- Blade mass distribution:  $[kg/m^2]$
- Rotor rotational velocity:  $\Omega$  in  $[rpm]$
- Chord distribution:  $c(y)$  in  $[m]$
- Twist distribution:  $\theta_{tw}(y)$  in degrees
- Airfoil distribution

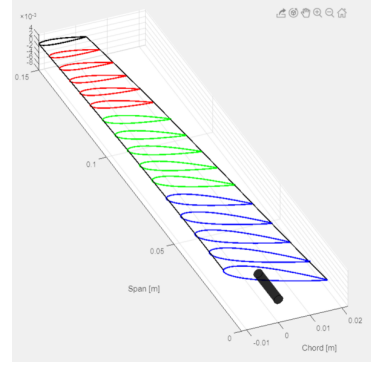


Figure 9: Rotor blade geometry example

The calculations parameters and the results representation in this tool are identical to what happens in the *basic tool*.

### 3.3. Drone tool

Through this tool the user will be able to design a multirotor unmanned aircraft of small dimensions.

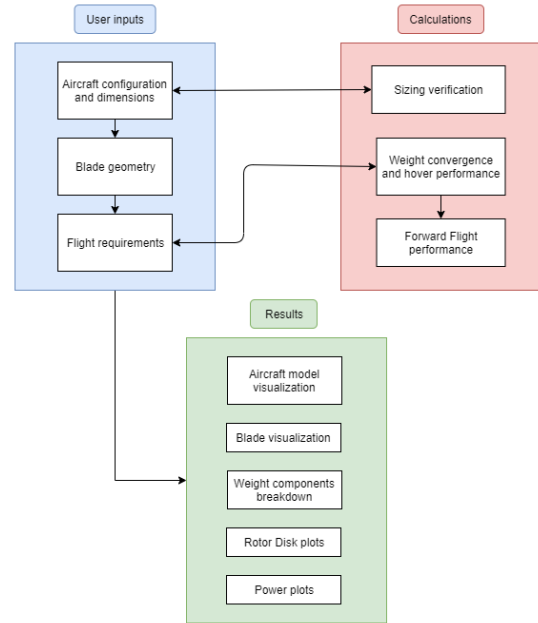


Figure 10: Drone tool structure

The user inputs are:

- Fuselage body shape
- Number of rotors
- Coaxial or "single" rotor configuration
- Vertical spacing between coaxial rotors
- Rotor arm
- Rotor radius
- Characteristic dimensions 1 and 2
- Airframe material

The result of the user design is shown as a preliminary sketch:

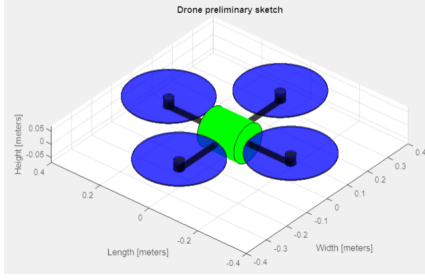


Figure 11: Drone preliminary sketch

The rotor design is identical to what has been described previously in section 3.2.

After the geometry has been set the flight performance requirements are needed:

- Payload and fuselage:  $m_{payload}$
- Hover Endurance:  $End_h$  in minutes
- Range
- Maximum velocity
- Flight ceiling

An assessment for the total mass of the aircraft based on the inputs is done and after having defined all geometric and weight characteristics of the drone the performance analysis might begin. The results will be shown as rotor disk plots and power plots (as is done in the *basic tool* and in the *detailed tool*).

### 3.4. Airfoil Comparison tool

This is an extremely simple tool where the user can compare the 2-D aerodynamic performance of the available airfoils in the software database. Two different airfoils can be plotted as well as their  $C_l$  and  $C_d$  curves when varying the angle of attack.

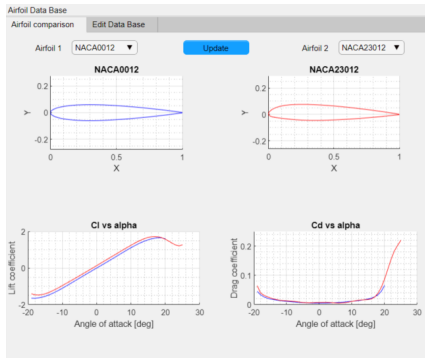


Figure 12: Airfoil Comparison Tool

## 4. Validation

### 4.1. Sizing Validation

The helicopter chosen for the validation process is the Sikorsky UH-60 Black Hawk. The geometric characteristics of the main and tail rotor used were obtained in [12].

To compare the results of the software for the helicopter dimensions the maximum take off weight (MTOW) in kilograms was used as an input and the calculations were made from that starting point.

	Actual	Software
MTOW [kg]	8329.0	8329.0
Radius [m]	8.1778	7.878
Height [m]	3.7592	4.151
Length [m]	15.4305	15.150
Tip to tip length [m]	19.7612	18.655
Width [m]	2.9464	2.979
Tail Rotor arm [m]	9.8908	9.520

Table 1: Conventional Helicopter dimensions validation

	Actual	Software
Radius [m]	8.1778	7.878
Blade chord [m]	0.5273	0.5209
Angular velocity [rad/s]	27.00	28.47

Table 2: Conventional Helicopter Main Rotor dimensions validation

	Actual	Software
Radius [m]	1.6764	1.539
Blade chord [m]	0.2469	0.206
Angular velocity [rad/s]	124.62	127.339

Table 3: Conventional Helicopter Tail Rotor dimensions validation

The dimensions calculated using the software show a good agreement with the actual helicopter values.

### 4.2. Basic tool - performance

The rotor performance will be validated comparing the software results with the flight test data from [13].

The test flight were done considering an weight coefficient  $C_W = 0.0065$  which is equivalent to a MTOW of 8329 kilograms at sea level.

$$C_W = \frac{W}{\rho A V_{tip}^2} \quad (9)$$

With  $W$  being the aircraft weight,  $A$  the rotor disk area, and  $V_{tip}$  the blade tip velocity.

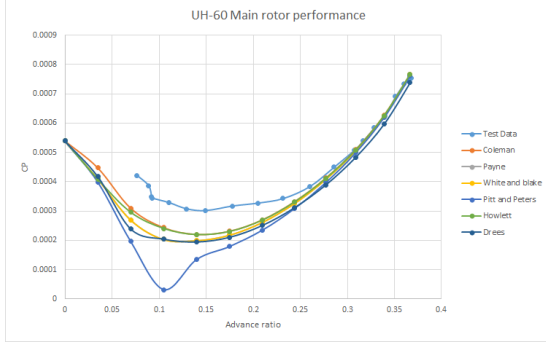


Figure 13: Main rotor power coefficient

In Figure 13 it is seen that all the inflow models present very similar results with the exception of the Pitt and Peters model for the range  $0.07 \leq \mu \leq 0.17$  where the power coefficient presents very low values. This is a result of an error occurring in the solving of the inflow equations, the inflow reaches very negative values (flow going upwards through the rotor) in a large region of the rotor disk resulting in a negative power contribution much greater than what would be expected thus reducing the total power of the rotor.

Remembering that the inflow models are valid for  $\mu > 0.15$  it is seen that the results of the software are fairly acceptable given the low level of detail of the *basic tool*.

#### 4.3. Detailed tool - Performance

The same flight test data used above will be applied here to validate the values obtained using the *detailed tool*.

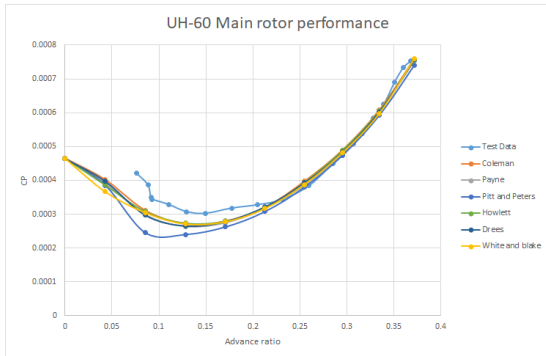


Figure 14: Main rotor power coefficient

It can be seen that the results when considering blade flapping (displayed in Figure 14) are in better concordance with the flight data when compared to the results in Figure 13, this was expected as was stated before. The greatest improvements occur for low advance ratios making the results very close to the actual flight test values throughout the whole velocity range considered.

#### 4.4. Co-axial configuration validation

To validate the analysis of a co-axial system a similar approach will be taken. Values from wind tunnel testing, see [14], will be compared to the software results.

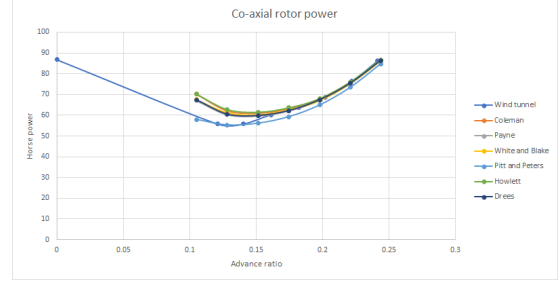


Figure 15: Co-axial rotor power

The results for the coaxial configuration (geometry described in [14]) show a good agreement between the software values and the wind tunnel test values.

#### 4.5. Tandem configuration validation

Now performing a same type of analysis to a tandem rotor configuration and using wind tunnel test values from [14] the validation will be done.

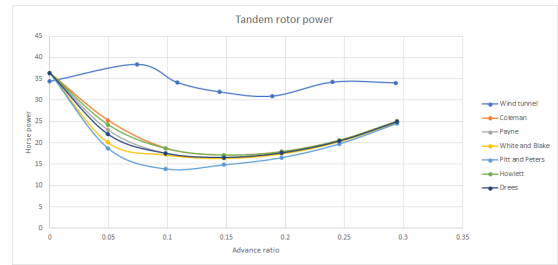


Figure 16: Tandem rotor power

The tandem configuration wind tunnel test results are somewhat scattered as is stated in [14], and the software calculated results are not close to the wind tunnel test ones, they follow a similar behaviour as do the results for a single or coaxial rotor.

This difference in the results might due to a low capability of analysing the interaction between the rotors, the presented configuration has no overlap and the two rotors are on the same plane, this means that in the software calculations no rotor wake will be shedded into the other rotor thus rendering them non interfering with which other.

## 5. Results

In this section a direct software application will be demonstrated. Two different aircrafts (one conventional helicopter and one co-axial) with the same

top level requirements will be designed and the performances will be compared in order to choose the more viable solution.

Setting the aircraft requirements as:

Top level aircraft requirements	
Payload	10000 [kg]
Crew and passengers	20
Range	600 [km]
Endurance	300 [minutes]

Table 4: Top level aircraft requirements

For both configuration the initial weight estimation calculations are equal and the results for the total aircraft weight and fuel weight are the following:

Fuel weight	5469 [kg]
Total aircraft weight	41500 [kg]

Table 5: Weight estimation results

The general aircraft dimensions calculated for both configurations are:

	Conventional	Co-axial
Main rotor radius [m]	12.96	10.90
Height [m]	5.82	6.04
Length [m]	25.63	18.52
Tip to tip length [m]	31.15	24.50
Width [m]	4.22	4.76
Tail rotor arm [m]	16.14	N/A
Rotor vertical spacing [m]	N/A	1.50

Table 6: Dimensions results for the aircraft (with N/A meaning not applicable)

The main rotor dimensions are:

	Conventional	Co-axial
Main rotor radius [m]	12.956	10.90
Blade chord [m]	1.24	1.24
Angular Velocity [rad/s]	18.85	21.76
Airfoil	NACA4412	NACA4412
Number of blades	4	4

Table 7: Dimensions results for the main rotor

The blades were designed as rectangular, untwisted and with the same airfoil throughout the span.

The tail rotor dimensions are:

	Conventional
Tail rotor radius [m]	2.89
Blade chord [m]	0.77
Angular velocity [rad/s]	75.53
Airfoil	NACA4412
Number of blades	2

Table 8: Dimensions results for the tail rotor

Both aircrafts will be required to flight under the same set of conditions, which are:

	Velocity [m/s]	Altitude [m]
Sea level hover	0	0
Maximum altitude hover	0	1250
Vertical climb	8	20
Cruise flight	35	500
Maximum velocity flight	45	50
Autorotational flight	10 (forward)	20

Table 9: Flight conditions

Now both configurations will be analysed, using BET and considering the flapping motion, for each of the flight conditions presented and their performances will be compared.

	Conventional power [kW]	Co-axial power [kW]
Sea level hover	9652.7	10756.5
Maximum altitude hover	10062.4	11087.6
Vertical climb	14623.0	14749.1
Cruise flight	5098.8	4767.2
Maximum velocity flight	4616.1	4237.3

Table 10: Flight performance comparison

For the autorotational flight the power is null (by definition of the flight condition) so the result to be looked at is the descent velocity that allows for the helicopter to operate in autorotation. For the conventional case this result was a descent velocity of 19.24 m/s and for the co-axial case is was of 25.32 m/s, which represents a difference of 31.6 %.

It is noted hat the co-axial configuration is less efficient when looking at axial flight conditions,



but for these specific cases (geometry, dimensions, and flight characteristics) the co-axial helicopter requires less power for the level flight operations (cruise and maximum velocity).

It can also be seen that the power required for the maximum velocity flight is lower than the power for the cruise flight (for both configurations) which means that that velocity value is on the descendent part of the power curve and the minimum power flight conditions is yet to be found.

Now comparing the power requirements variation with the forward flight velocity at an altitude of 500 meters and an installed power value of 15000 kW for both helicopters, the calculations will use MT.

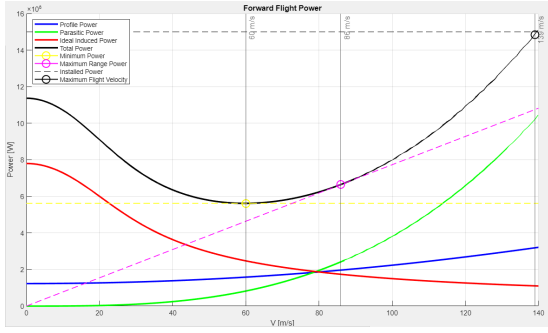


Figure 17: Conventional helicopter configuration - Power curve

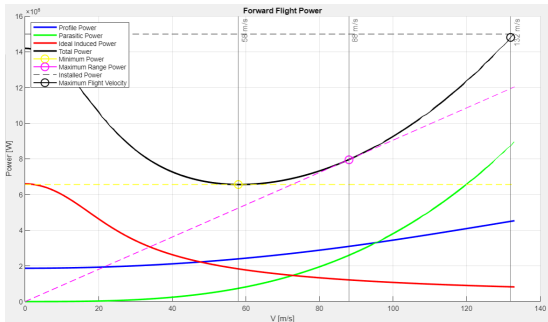


Figure 18: Co-axial helicopter configuration - Power curve

	Conventional	Co-axial
Minimum power velocity [m/s]	60	58
Minimum power for level flight [kW]	5619.4	6560.7
Maximum range velocity [m/s]	86	88
Maximum forward velocity [m/s]	139	132

Table 11: Performance results using MT for both configurations

The two configurations show similar results for the minimum power flight velocity, maximum range velocity and maximum forward velocity. The major difference is in terms of the power for maximum range where a difference of almost 17 % with the co-axial configuration consuming more power.

The velocity values calculated for the minimum power flight are both higher than the velocity for the maximum velocity flight conditions specified in the table 10.

## 6. Conclusions

### 6.1. Achievements

This work set out to provide a free, open source, user friendly computational tool for the design of rotary wing aircrafts of different configurations.

The results presented in section 4 are the proof that the software is capable of providing quality results very quickly and of presenting them in a clear manner so that the user can evaluate the design impacts of the aircraft performance.

The results obtained for the UH-60 were extremely good, as is shown in Figure 14, for all inflow models available in the tool, with the Pitt and Peters [10] model having some minor issues for a small velocities range, and there was a significant improvement when comparing with the results where no flapping was considered, see Figure 13.

The work developed here is shown to be a very viable and useful tool and its open source nature makes it all the more attractive for the people interested.

### 6.2. Further work

The modular nature of the tool makes it possible for other analysis to be added to the software without compromising the work already developed and increasing the depth of the aircraft design. Features like blade structural and aeroelastic analysis can be introduced based on the force and moments distributions already calculated through the tool. A gas turbine module can be used to assess for the power plant efficiency and performance, as well as as for emissions calculations. Noise and vibrations analysis is another component that was not

addressed in this work and can be useful given the regulation limitations for certain aircrafts. Fuselage parameterization in terms of passenger distribution and general outer shape might be of some interest. The generation of a flight envelope (*V-n diagram* plotting the Load factor against the airspeed) is another feature that could be studied.

In terms of the blade motion and aerodynamic performance developments can easily be introduced. More complex blade motions like lead-lag and feathering can be applied. Wake swirl considerations for a more comprehensive rotor interaction. Dynamic stall which can have major impacts on helicopter blades' performance can be introduced (this would required a more complete aerodynamic performance analysis of the airfoils used). A trim function taking into account the aerodynamic forces and moments of the fuselage (the aircraft attitude can then be calculated), this would allow for more complex maneuvers to be examined (turn maneuvers).

When considering multi rotor aircrafts (helicopters or drones) the rotor geometry is assumed to be identical throughout this work, the possibility for different rotor geometries in a single aircraft might be of some used, for example in a co-axial configuration, given the different airflow conditions of the top and bottom rotor, it might be more efficient to have different designs for the different rotors.

Compound aircrafts have been developed to expand the range of operations and might be added to the tool in further versions. Also rotor performance optimization is something to be considered, as a helicopter has very different flight conditions and cannot be optimally design for all with a single design it could be interesting to do some work in this regard (knowing all the mission profile and the flight conditions trying to optimize the rotor for fewest fuel consumption or emission, or for least amount of time to perform the mission). Even blade morphing could be introduced to adapt the rotor geometry to the flight condition that it is in.

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